A NEW AND

COMPLETE SYSTEM

OF

ARITHMETIC.

COMPOSED FOR THE

USE OF THE CITIZENS OF THE UNITED STATES.

BY NICOLAS PIKE, A. M.

QUID MUNUS REIPUBLICÆ MAJUS MELIUSVE AFFERRE POSSUMUS, QUAM SI JUVENTUTEM DOCEMUS, ET BENE ERUDIMUS.

——E VARIIS SUMENDUM EST OPTIMUM.——*CICERO.*

SECOND EDITION, ENLARGED.

REVISED AND CORRECTED,
BY EBENEZER ADAMS, A. M.
PRECEPTOR OF LEICESTER ACADEMY.

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1797.

R E C O M M E N D A T I O N S.

Dartmouth Univerſity, A. D. 1786.

AT the requeſt of Nicolas Pike, Eſq. we have inſpected his Syſtem of Arithmetic, which we cheerfully recommend to the public, as eaſy, accurate and complete. And we apprehend there is no treatiſe of the kind extant, from which ſo great utility may ariſe to Schools.

B. WOODWARD, Math. and Phil. Prof.

JOHN SMITH, Prof. of the Learned Languages.

I do moſt ſincerely concur in the preceding recommendation.

J. WHEELOCK, Preſident of the Univerſity.

Providence, State of Rhodeiſland, 1785.

WHOEVER may have the peruſal of this treatiſe on Arithmetic may naturally conclude I might have ſpared myſelf the trouble of giving it this recommendation, as the work will ſpeak more for itſelf than the moſt elaborate recommendation from my pen can ſpeak for it : But as I have always been much delighted with the contemplation of mathematical ſubjects, and at the ſame time fully ſenſible of the utility of a work of this nature, was willing to render every aſſiſtance in my power to bring it to the public view : And ſhould the ſtudent read it with the ſame pleaſure with which I peruſed the ſheets before they went to the Preſs, am perſuaded he will not fail of reaping that benefit from it which he may expect, or wiſh for, to ſatisfy his curioſity in a ſubject of this nature. The Author, in treating on numbers, has done it with ſo much perſpicuity and ſingular addreſs, that I am convinced the ſtudy thereof will become more a pleaſure than a taſk.

The arrangement of the work, and the method by which he leads the *tyro* into the firſt principles of numbers, are novelties I have not met with in any book I have ſeen. Wingate, Hatton, Ward, Hill, and many other Authors, whoſe names might be adduced, if neceſſary, have claimed a conſiderable ſhare of merit ; but when brought into a comparative point of veiw with this treatiſe, they are inadequate and defective. This volume contains, beſides what is uſeful and neceſſary in the common affairs of life, a great fund for amuſement and entertainment. The Mechanic will find in it much more than he may have occaſion for ; the Lawyer, Merchant and Mathematician will find an ample field for the exerciſe of their genius ; and I am well aſſured it may be read to great advantage by ſtudents of every claſs, from the loweſt School to the Univerſity. More than this need not be ſaid by me, and to have ſaid leſs, would be keeping back a tribute juſtly due to the merit of this Work.

BENJAMIN WEST.

University in Cambridge, A. D. 1786.

HAVING, by the desire of Nicolas Pike, Esq. inspected the following volume in manuscript, we beg leave to acquaint the public, that in our opinion it is a work well executed, and contains a complete system of Arithmetic. The rules are plain, and the demonstrations perspicuous and satisfactory; and we esteem it the best calculated, of any single piece we have met with, to lead youth, by natural and easy gradations, into a methodical and thorough acquaintance with the science of figures. Persons of all descriptions may find in it every thing, respecting numbers, necessary to their business; and not only so, but if they have a speculative turn and mathematical taste, may meet with much for their entertainment at a leisure hour.

We are happy to see so useful an American production, which, if it should meet with the encouragement it deserves, among the inhabitants of the United States, will save much money in the country, which would otherwise be sent to Europe, for publications of this kind.

We heartily recommend it to schools, and to the community at large, and wish that the industry and skill of the Author may be rewarded, for so beneficial a work, by meeting with the general approbation and encouragement of the public.

JOSEPH WILLARD, D. D. President of the University.
E. WIGGLESWORTH, S. T. P. Hollis.
S. WILLIAMS, L. L. D. Math. et Phil. Nat. Prof. Hollis.

Yale College, 1786.

UPON examining Mr. Pike's System of Arithmetic and Geometry in manuscript, I find it to be a work of such mathematical ingenuity, that I esteem myself honored in joining with the Rev. President Willard, and other learned gentlemen, in recommending it to the public as a production of genius, interspersed with originality in this part of learning, and as a book suitable to be taught in schools—of utility to the merchant, and well adapted even for the university instruction. I consider it of such merit, as that it will probably gain a very general reception and use throughout the republic of letters.

EZRA STILES, President.

Boston, 1785.

FROM the known character of the Gentlemen who have recommended Mr. Pike's System of Arithmetic, there can be no room to doubt, that it is a valuable performance; and will be, if published, a very useful one. I therefore wish him success in its publication.

JAMES BOWDOIN.

P R E F A C E

To the FIRST EDITION.

IT may, perhaps, by ſome, be thought needleſs, when Authors are ſo multiplied, to attempt publiſhing any thing further on Arithmetic, as it may be imagined there can be nothing more than the repetition of a ſubject already exhauſted. It is however the opinion of not a few, who are conſpicuous for their knowledge, in the Mathematics, that the books, now in uſe among us, are generally deficient in the illuſtration and application of the rules; of the truth of which, the general complaint among Schoolmaſters is a ſtrong confirmation. And not only ſo, but as the United States are now an independent nation, it was judged that a Syſtem might be calculated more ſuitable to our meridian, than thoſe heretofore publiſhed.

Although I had ſufficient reaſon to diſtruſt my abilities for ſo arduous a taſk, yet not knowing any one who would take upon himſelf the trouble, and apprehending I could not render the public more eſſential ſervice, than by an attempt to remove the difficulties complained of, with diffidence I devoted myſelf to the work.

I have availed myſelf of the beſt Authors which could be obtained, but have followed none particularly, except Bonnycaſtle's Method of Demonſtration.

Although I have arranged the Work in ſuch order as appeared to me the moſt regular and natural, the ſtudent is not obliged to pay a ſtrict adherence to it; but

may

may paſs from one Rule to another, as his inclination, or opportunity for ſtudy, may require.

The Federal Coin, being purely decimal, moſt naturally falls in after Decimal Fractions.

I have given ſeveral methods of extracting the Cube Root, and am indebted to a learned friend, who declines having his name made public, for the inveſtigation of two very conciſe Algebraic Theorems for the extraction of all Roots, and of a particular Theorem for the Surſolid.

Among the Miſcellaneous Queſtions, I have given ſome of a philoſophical nature, as well with a view to inſpire the pupil with a reliſh for philoſophical ſtudies, as to the uſefulneſs of them in the common buſineſſes of life.

The ſhort introduction to Algebra, which is ſubjoined, was abſtracted principally from Bonnycaſtle, and that of Conic Sections, from Emerſon's Works.

Being ſenſible the following Treatiſe will ſtand or fall, according to its real merit or demerit, I ſubmit it to the judgment of the *candid*.

With pleaſure I embrace this opportunity, to expreſs my gratitude to thoſe learned Gentlemen, who have honored this Treatiſe with their approbation, as well as to ſuch Gentlemen as have encouraged it by their ſubſcriptions; and to requeſt the reader to excuſe any errors he may meet with; for although great pains have been taken in correcting, yet it is difficult to prevent errors from creeping into the preſs, and ſome may have eſcaped my own obſervation; in either caſe, a hint from the candid will much oblige their

Moſt obedient,

And humble Servant,

THE AUTHOR.

P R E F A C E

To THIS EDITION.

THE demand for the former Edition of Mr. Pike's New and Complete Syſtem of Arithmetic, *has induced the Proprietor of that work to republiſh it.*

The numerous Recommendations, by characters celebrated for their knowledge of the Mathematics, prefixed to this and the former Edition, and its introduction as a Claſſic into ſeveral of our Univerſities, ſpeak its eulogy; and the eſtimation in which it is now held is ſuperior to any publication of the kind extant.

Explanatory of Decimal Calculation in the Federal Currency, the Rules which are adduced under this head, ſtamp an additional value on this Edition. Dollars and Cents are coming gradually into uſe; but pounds, ſhillings and pence will continue to be in practice, and be the baſis of many Arithmetical Queſtions, both in the United States and elſewhere.

Much attention has been given to the Reviſion and Correction of this valuable work, at the requeſt of the Proprietor, and recommendation of the Author, by the ingenious Mr. Ebenezer Adams, *Preceptor of Leiceſter Academy; and, as far as his other engage-*

ments

ments would permit, the Author himſelf has aided in making the work as perfect as poſſible, and in the detection of many errors which eſcaped notice in the former, and which are corrected in this, edition, now preſented to the public.

With reſpect, the Public's moſt obedient and humble ſervant,

ISAIAH THOMAS.

Worceſter, February, 1797.

CONTENTS.

Prob.

Art.

EXPLANATION *of the* CHARACTERS *made Use of in this* TREATISE.

$=$	THE ſign of equality: As, 12 pence $=$ 1 ſhilling, ſignifies that 12 pence are equal to 1 ſhilling; and, in general, that whatever precedes it is equal to what follows.
$+$	The ſign of addition: As $5+5=10$, that is, 5 added to 5 is equal to 10. Read 5 plus 5, or 5 more 5 equal to 10.
$-$	The ſign of ſubtraction: As, $12-4=8$, that is, 12 leſſened by 4 is equal to 8, or 4 from 12 and 8 remains. Read 12 minus 4, or 12 leſs 4 equal to 8.
$\times$	The ſign of multiplication: As $6\times5=30$, that is, 6 multiplied by 5 is equal to 30. Read 6 into 5 equal to 30.
$\div$ *or* 6)30(	The ſign of diviſion: As, $30\div5=6$, that is, 30 divided by 5 is equal to 6. Read 30 by 5 equal to 6.
$\frac{875}{25}$	Numbers placed fractionwiſe, do likewiſe denote diviſion, the numerator or upper number being the dividend, and the denominator or lower number, the diviſor, thus, $\frac{875}{25}$ is the ſame as $875\div25=35$.
$: \; :: \; :$	The ſign of proportion, thus, $2:4::8:16$, that is, As 2 is to 4 ſo is 8 to 16.
$\overline{9-2}+6=13$	Shews that the difference between 2 and 9 added to 6 is equal to 13. Read 9 minus 2 plus 6 equal to 13. And that the line atop (called a *Vinculum*) connects all the numbers over which it is drawn.
$12-\overline{3+5}=4$	Signifies that the ſum of 3 and 5 taken from 12 leaves or is equal to 4.
$\overline{\quad}\vert^{2}$	Signifies the ſecond power, or ſquare.
$\overline{\quad}\vert^{3}$	Signifies the third power, or cube.
$\overline{\quad}\vert^{m}$	Signifies any power in general, as $\overline{6}\vert^{2}=$ ſquare of 6; and $\overline{50}\vert^{3}=$ cube of 50, &c. thus m ſignifies either the ſquare or cube, or any other power.

√,

$\surd$, or $\overline{\quad}|^{\frac{1}{2}}$ — Prefixed to any number or quantity, ſignifies that the ſquare root of that number is required. It likewiſe (as alſo the character for any other root) ſtands for the expreſſion of the root of that number or quantity to which it is prefixed. As $\surd 36 = 6$, and $\sqrt{108+36} = 12$, *or* $\overline{36}|^{\frac{1}{2}} = 6$, &c.

$\sqrt[3]{\ }$, or $\overline{\quad}|^{\frac{1}{3}}$ — Prefixed to any number, ſignifies that the cube root of that number is required, or expreſſed. As $\sqrt[3]{216} = 6$, and $\sqrt[3]{513+216} = 9$, &c.—or $\overline{216}|^{\frac{1}{3}} = 6$, &c.

$\sqrt[n]{\ }$, or $\overline{\quad}|^{\frac{n}{m}}$ — Signifies any root in general. As $\overline{36}|^{\frac{1}{2}} =$ ſquare root, $\overline{216}|^{\frac{1}{3}} =$ cube root, &c. Thus, $\frac{n}{m}$ ſignifies either the ſquare root, cube root, or any other root whatever,

a b c d — When ſeveral letters are ſet together, they are ſuppoſed to be multiplied into each other; as thoſe in the margin are the ſame as $a \times b \times c \times d$, and repreſent the continual product of quantities or numbers.

$\frac{1}{a}$ — Is the reciprocal of a, and $\frac{a}{b}$ is the reciprocal of $\frac{b}{a}$.

If a be the root, then $a \times a = aa$ or a^2 is the ſquare of a, and $a \times a \times a = aaa$ or a^3 is the cube of a, &c.

Note, The figure atop is called the index of the power.

It is uſual to write ſhillings at the left hand of a ſtroke, and pence at the right; thus, 13/4 is thirteen ſhillings and four pence.

Note, The uſe of theſe characters muſt be perfectly underſtood by the pupil, as he may have occaſion for them.

A

NEW AND *COMPLETE*

SYSTEM OF ARITHMETICK.

ARITHMETICK is the Art or Science of computing by numbers, and conſiſts both in Theory and Practice.—The Theory conſiders the nature and quality of numbers, and demonſtrates the reaſon of practical operations.—The Practice is, that which ſhews the method of working by numbers, ſo as to be moſt uſeful and expeditious for buſineſs, and is compriſed under five principal or fundamental Rules, viz. NOTATION OR NUMERATION, ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION; the knowledge of which is ſo neceſſary, that, ſcarcely any thing in life, and nothing in trade, can be done without it.

NUMERATION

Teaches the different value of figures by their different places, and to read or write any ſum or number by theſe ten characters. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.—0 is called a cypher, and all the reſt are called figures or digits. The names and ſignifications of theſe characters, and the origin or generation of the numbers they ſtand for, are as follow; 0 nothing; 1 one, or a ſingle thing called an unit; $1+1=2$, two; $2+1=3$, three; $3+1=4$, four; $4+1=5$, five; $5+1=6$, ſix; $6+1=7$, ſeven; $7+1=8$, eight; $8+1=9$, nine; $9+1=10$, ten, which has no ſingle character; and thus, by the continual addition of one, all numbers are generated.

2. Beſide the ſimple value of figures, as above noted, they have, each, a local value, according to the following law; viz. In a combination of figures, reckoning from right to left, the figure, in the firſt place, repreſents its primitive ſimple value; that in

the ſecond place, ten times its ſimple value, and ſo on; the value of the figure, in each ſucceeding place, being ten times the value of it, in that immediately preceding it.

3. The values of the places are eſtimated according to their order: The firſt is denominated the place of units; the ſecond, tens; the third, hundreds, and ſo on, as in the table. Thus in the number—5293467: 7, in the firſt place, ſignifies only ſeven; 6, in the ſecond place, ſignifies 6 tens, or ſixty; 4, in the third place, four hundred; 3, in the fourth place, three thouſand; 9, in the fifth place, ninety thouſand; 2, in the ſixth place, two hundred thouſand; 5, in the ſeventh place, is five millions; and the whole, taken together, is read thus; five millions, two hundred and ninety three thouſand, four hundred and ſixty ſeven.

4. A cypher, though it is of no ſignification, itſelf, yet, it poſſeſſes a place, and, when ſet on the right hand of figures, in whole numbers, increaſes their value in the ſame tenfold proportion; thus, 9 ſignifies only nine; but, if a cypher is placed on its right hand, thus, 90, it then becomes ninety; and, if two cyphers be placed on its right, thus, 900, it is nine hundred; &c.

To enumerate any parcel of figures, obſerve the following Rule.

Firſt, commit the words at the head of the Table, viz. units, tens, hundreds, &c. to memory; then, to the ſimple value of each figure, join the name of its place; beginning at the left hand, and reading towards the right.—*More particularly*—1. Place a dot under the right hand figure of the 2d, 4th, 6th, 8th, &c. half periods, and the figure over ſuch dot will, univerſally, have the name of thouſands.—2. Place the figures 1, 2, 3, 4, &c. as indices, over the 2d, 3d, 4th, &c. period: Theſe indices will then ſhew the number of times the millions are involved—the figure under 1, bearing the name of millions, that under 2, the name of billions (or millions of millions) that under 3, trillions (or millions of millions of millions.)

EXAMPLE.

Sextillions.	Quintilli.	Quatrill.	Trillions.	Billions.	Millions.	Units.
th. un.	th. un.	th. un.	th. un.	th. un.	th. un.	c.x.t.c.x.u.
6	5	4	3	2	1	
913,208;	000,341;	620.057;	219,356;	809,379;	120,406;	129,763
Thouſands	*Thouſands*	*Thouſands*	*Thouſands*	*Thouſands*	*Thouſands*	*Thouſands*

NOTE I. Billions is ſubſtituted for millions of millions: Trillions, for millions of millions of millions: Quatrillions, for millions of millions of millions of millions.

Quintillions,

Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, &c. anſwer to millions ſo often involved as their indices reſpectively denote.

Note 2. The right hand figure of each half period has the place of units of that half period; the middle one, that of tens, and the left hand one, that of hundreds.

The Application.

Write down, in proper figures, the following numbers.

Fifteen.
Two hundred and ſeventy nine.
Three thouſand, four hundred and three.
Thirty ſeven thouſand, five hundred and ſixty ſeven.
Four hundred, one thouſand and twenty eight.
Nine millions, ſeventy two thouſand and two hundred.
Fifty five millions, three hundred, nine thouſand and nine.
Eight hundred millions, forty four thouſand, and fifty five.
Two thouſand, five hundred and forty three millions, four hundred and thirty one thouſand, ſeven hundred and two.

Write down, in words at length, the following numbers.

8	437	709040	3476194	7584397647
17	3010	879066	84094007	49163189186
129	76506	4091875	690748591	500098400700

Notation by Roman Letters.

I. One.	XV. Fifteen.	CC. Two hundred.
II. Two.	XVI. Sixteen.	CCC. Three hundred.
III. Three.	XVII. Seventeen.	CCCC. Four hundred.
IV. Four.	XVIII. Eighteen.	D or IↃ. Five hundred.
V. Five.	XIX. Nineteen.	DC. Six hundred.
VI. Six.	XX. Twenty.	DCC. Seven hundred,
VII. Seven.	XXX. Thirty.	DCCC. Eight hundred.
VIII. Eight.	XL. Forty.	DCCCC. Nine hundred.
IX. Nine.	L. Fifty.	M or CIↃ. One Thouſand.
X. Ten.	LX. Sixty.	IↃↃ. Five thouſand.
XI. Eleven.	LXX. Seventy,	IↃↃↃ. Fifty thouſand.
XII. Twelve.	LXXX. Eighty.	IↃↃↃIↃↃↃ. Five hund. tho.
XIII. Thirteen.	XC. Ninety.	MDCCXCVI. One thouſ.
XIV. Fourteen.	C. Hundred.	ſeven hund. and ninety ſix.

A leſs literal number, placed after a greater, always augments the value of the greater; if put before, it diminiſhes it. Thus, VI is 6; IV is 4; XI is 11; IX is 9, &c.

ADDITION

ADDITION

Is the putting together of two or more numbers, or ſums, to make them one total, or whole ſum.

SIMPLE ADDITION

Is the adding of ſeveral integers or whole numbers together, which are all of one kind, or ſort; as 7 pounds, 12 pounds, and 20 pounds, being added together, their aggregate, or ſum total, is 39 pounds.

RULE.

Having placed units under units, tens under tens, &c. draw a line underneath, and begin with the units: After adding up every figure in that column, conſider how many tens are contained in their ſum, and, placing the exceſs under the units, carry ſo many, as you have tens, to the next column, of tens: Proceed in the ſame manner through every column, or row, and ſet down the whole amount of the laſt row.*

PROOF. Begin at the top of the ſum, and reckon the figures downwards, in the ſame manner as they were added upwards, and, if it be right, this aggregate will be equal to the firſt. Or, cut off the upper line of figures, and find the amount of the reſt; then, if the amount and upper line, when added, be equal to the ſum total, the work is ſuppoſed to be right.

ADDITION

* This Rule, as well as the method of Proof, is founded on the known Axiom, "The whole is equal to the ſum of all its parts." The method of placing the numbers, and carrying for the tens, is evident from the nature of notation; for any other diſpoſition of the numbers would alter their value; and carrying one, for every ten, from an inferior to a ſuperior column, is, evidently, right, becauſe one unit in the latter caſe is equal to the value of ten units in the former.

Beſide the method of proof, here given, there is another, by caſting out the nines; thus:

1. Add the figures in the upper row together, and find how many nines are contained in their ſum.
2. Reject the nines, and ſet down the remainder, directly even with the figures, in the row.
3. Do the ſame with each of the given numbers, and ſet all the exceſſes of nines in a column, and find their ſum; then, if the exceſs of nines in this ſum, found as before, is equal to the exceſs of nines in the ſum total; the queſtion is ſuppoſed to be right.

EXAMPLE.

	Exceſs of nines.
5738	5
9156	3
8471	2
5324	5
28689	6

This method depends upon a property of the number 9, which, except 3, belongs to no other digit whatever; viz. that any number, divided by 9, will leave the ſame remainder, as the ſum of its figures, or digits, divided by 9: Which may be thus demonſtrated.

Demonſtration. Let there be any number, as 5432; this, ſeparated into its ſeveral parts, becomes $5000+400+30+2$; but $5000=5\times1000=5\times999+1=5\times999+5$. In like manner $400=4\times99+4$, and $30=3\times9+3$. Therefore, $5432=5\times999+5$, $+4\times99+4$, $+3\times9+3+2=5\times999+4\times99+3\times9+5+4+3+2$. And $\frac{5432}{9}=\frac{5\times999+4\times99+3\times9+5+4+3+2}{9}$; but $5\times999+4\times99$ $+3$

ADDITION and SUBTRACTION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	5	6	7	8	9	10	11	12	13	14
3	5	6	7	8	9	10	11	12	13	14	15
4	6	7	8	9	10	11	12	13	14	15	16
5	7	8	9	10	11	12	13	14	15	16	17
6	8	9	10	11	12	13	14	15	16	17	18
7	9	10	11	12	13	14	15	16	17	18	19
8	10	11	12	13	14	15	16	17	18	19	20
9	11	12	13	14	15	16	17	18	19	20	21
10	12	13	14	15	16	17	18	19	20	21	22

When you would add two numbers, look one of them in the left hand column, and the other atop, and in the common angle of meeting, or, at the right hand of the first, and under the second, you will find the sum—as, 5 and 8 is 13.

When you would subtract: Find the number to be subtracted in the left hand column, run your eye along to the right hand till you find the number from which it is taken, and right over it, atop, you will find the difference—as, 8, taken from 13, leaves 5.

1.	2.	3.	4.	5.	6.
£.	℔	Cwt.	Miles.	Yards.	£.
1	12	123	1234	12345	987654321
2	34	456	5678	67890	123456789
3	56	789	9098	98765	234567891
4	78	12	7654	43210	345678910
5	90	345	3210	12345	456789123
6	1	678	69	67890	567879287
7	23	901	4713	74100	678900028
8	45	234	131	64786	789400690
9	67	567	9128	19876	548769138

7.	8.	9.	10.
1234567	1234567	67	1234567
2345678	723456	123	9876543
3456789	34565	4567	2102865
4567890	4566	89093	4321234
5678209	333	654321	5682098
6789098	90	1234567	6543218

SUBTRACTION

+3×9 is divisible by 9; therefore, 5432, divided by 9, will leave the same remainder as 5+4+3+2, divided by 9; and the same will hold good of any other number whatever. The same property belongs to the number 3: However, this inconveniency attends this method, that, although the work will always prove right, when it is so; it will not, always, be right, when it proves so; I have, therefore, given this demonstration more for the sake of the curious, than for any real advantage.

SUBTRACTION

Teaches to take a leſs number from a greater, to find a third, ſhewing the inequality, exceſs or difference between the given numbers; and it is both ſimple and compound.

SIMPLE SUBTRACTION

Teaches to find the difference between any two numbers, which are of a like kind.

Rule.

Place the larger number uppermoſt, and the leſs underneath, ſo that units may ſtand under units, tens under tens, &c. then, drawing a line underneath, begin with the units, and ſubtract the lower from the upper figure, and ſet down the remainder; but if the lower figure be greater than the upper, borrow ten, and ſubtract the lower figure therefrom: To this difference, add the upper figure, which, being ſet down, you muſt add one to the ten's place of the lower line, for that which you borrowed; and thus proceed through the whole.*

Proof.

In either ſimple or compound Subtraction, add the remainder and the leſs line together, whoſe ſum, if the work be right, will be equal to the greater line: Or, ſubtract the remainder from the greater line, and the difference will be equal to the leſs.

Examples.

	1.	2.	3.	4.	5.	6.
	£.	£.	*Miles.*	*Yards.*	*Feet.*	*Cwt.*
From	25	305	4670	58934	879647	9187641
Take	12	103	4020	6182	164348	91843
Rem.						
Proof.						

7.	8.	9.
100200300400500600700800900	10000	1000000
98076054032011023045067089	9999	1

MULTIPLICATION

* *Dem.* When all the figures of the leſs number are leſs than their correſpondent figures in the greater, the difference of the figures, in the ſeveral like places, muſt, all taken together, make the true difference ſought; becauſe, as the ſum of the parts is equal to the whole; ſo muſt the ſum of the differences, of all the ſimilar parts, be equal to the difference of the whole.

2. When any figure in the greateſt number is leſs than its correſpondent figure in the leſs, the ten, which is added by the Rule, is the value of an unit in the next higher place, by the nature of notation; and the one which is added to the next place of the leſs number, is to diminiſh the correſpondent place of the

MULTIPLICATION

May be accounted the moſt ſerviceable Rule in Arithmetick. It teaches how to increaſe the greater of two numbers given, as often as there are units in the leſs; performs the work of many additions in the moſt compendious manner; brings numbers of great denominations into ſmall, as pounds into ſhillings, pence or farthings, &c. and, by knowing the value of one thing, we find the value of many.

It conſiſts of three parts.

1. The Multiplicand, or number given to be multiplied, and, commonly, the largeſt number.

2. The multiplier, or number to multiply by, commonly, the leaſt number.

3. The Product is the reſult of the work, or the anſwer to the queſtion.

SIMPLE MULTIPLICATION

Is the multiplying any two numbers together, without having regard to their ſignification; as 7 times 8 is 56, &c.

MULTIPLICATION and DIVISION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

To learn this Table, for Multiplication: Find your multiplier in the left hand column, and your multiplicand atop, and in the common angle of meeting, or againſt your multiplier, along at the

the greater, accordingly; which is only taking from one place, and adding as much to another, whereby the total is never changed: And, by this mean, the greater is reſolved into ſuch parts, as are, each, greater than, or equal to, the ſimilar parts of the leſs; and the difference of the correſpondent figures, taken together, will, evidently, make up the difference of the whole.

The truth of the method of proof is evident; for the difference of two numbers, added to the leſs, is, manifeſtly, equal to the greater.

the right hand, and under your multiplicand, you will find the product, or anſwer.

To learn it, for Diviſion : Find the diviſor in the left hand column, and run your eye along the row to the right hand until you find the dividend; then, directly over the dividend, atop, you will find the quotient, ſhewing how often the diviſor is contained in the dividend.

CASE I.

When the multiplier is not more than 12, always placing the greateſt number uppermoſt, ſet the multiplier underneath, units under units, &c. and begin as the Table directs, ſetting down the unit figure under units, and carrying the tens to the next place, in all reſpects as in ſimple addition.*

PROOF.

Multiply the multiplier by the multiplicand.

EXAMPLES.

	1.	2.	3.	4.
	37934	769308	4980076	763896
	2	3	4	5
Prod.				

5.	6.	7.	8.
67589	503764	3918295	9164785
6	7	8	9

9.	10.	11.
4879567	5864734	8583478649
10	11	12

CASE II.

When the multiplier conſiſts of more places than one, multiply each figure in the multiplicand by every figure in the multiplier, beginning with the units, and placing the firſt figure of each product exactly under its multiplier: Laſtly, add theſe ſeveral products together,

* *Dem.* When the multiplier is a ſingle digit, it is plain that we find the Product; for, by multiplying every figure, that is, every part of the multiplicand, we multiply the whole; and, the writing down the products, which are leſs than ten, or the exceſs of tens, in the places of the figures multiplied, and carrying the number of tens, to the product of the next place, is only gathering together the ſimilar parts of the reſpective products, and is, therefore, the ſame, in effect, as though we wrote down the multiplicand as often as the multiplier expreſſes, and added them together; for the ſum of every column is the product of the figures in the place of that column; and the products, collected together, are evidently equal to the whole required product.

ther, in the ſame order as they ſtand, and their ſum will be their total product.*

EXAMPLES.

	1.	2.	3.
	6357534	8324629	46293845
	47	59	76
	44502738		
	25430136		
Prod.	298804098		

4.	5.	6.
647906	760483	91867584
4873	9152	6875
3157245938	6959940416	631589640000

CASE

* If the multiplier be a number, made up of more than one figure; after we have found the product of the multiplicand by the firſt figure of the multiplier, as above, we ſuppoſe the multiplier divided into parts, and, after the ſame manner, find the product of the multiplicand by the ſecond figure of the multiplier; but, as the figure, by which we are multiplying, ſtands in the place of tens, the product muſt be ten times its ſimple value; and, therefore, the firſt figure in this product muſt be noted in the place of tens, or, which is the ſame, directly under the figure we are multiplying by. And, proceeding in the ſame manner with all the figures of the multiplier, ſeparately, it is evident we ſhall multiply all the parts of the multiplicand by all the parts of the multiplier; therefore, theſe ſeveral products, being added together, will be equal to the whole required product.

The reaſon of the method of proof, depends upon this propoſition, that if two numbers are to be multiplied together, either of them may be made the multiplier or multiplicand; and the product will be the ſame.

A ſmall attention to the nature of numbers will make this truth evident; for $5\times9=45=9\times5$; and, in general, $2\times3\times4\times5\times6$, &c. $=3\times2\times6\times5\times4$, &c. without any regard to the order of the terms; and this is true of any number of factors whatever.

N. B. By factors are meant the multiplier and multiplicand.

The following examples are ſubjoined, to make the reaſon of the rule appear as clearly as poſſible.

64753		237956	
5		3728	
15	$=3\times5$	1903648	= 8 times the multiplicand.
25	$=50\times5$	475912	= 20 times ditto.
35	$=700\times5$	1665692	= 700 times ditto.
20	$=4000\times5$	713868	= 3000 times ditto.
30	$=60000\times5$		
323765	$=64753\times5$	887099968	=3728 times ditto.

Multiplication may alſo be proved, by caſting out the nines; but is liable to the inconvenience before mentioned.

 It

CASE III.

When the multiplier is a composite number, that is, when it is produced by the multiplication of any two numbers in the Table, multiply the multiplicand by one of those figures, first, and that product by the other: And the last product will be the total required.*

EXAMPLES.

	1.	2.	3.
Mult.	59375 by 35.	39187 by 48.	91632 by 56.

$$
\begin{array}{lr}
 & 59375 \\
7\times 5=35 & 7 \\
\hline
 & 415625 \\
 & 5 \\
\hline
 & 2078125 \\
\hline
\end{array}
$$

	4.	5.	6.
Mult.	91738 by 81.	35963 by 72.	847396 by 99.

	7.	8.	9.
Mult.	38462 by 108.	749357 by 121.	9043278 by 144.

CASE IV.

When there are cyphers on the right hand of, either the multiplicand, or multiplier, or both, neglect those cyphers; then place the significant figures under one another, and multiply by them only; add them together, as before directed, and place to the right hand as many cyphers as there are in both the factors.

EXAMPLES.

It may likewise be, very naturally, proved by division; for the product, being divided by either of the factors, will, evidently, give the other; and it might not be amiss for the pupil to be taught division, at the same time with multiplication; as it not only serves for proof; but also gives him a readier knowledge of them both. But it would have been contrary to good method to have given this rule in the text, because the pupil is supposed, as yet, to be unacquainted with division.

* The reason of this method is obvious: For any number, multiplied by the component parts of another number, must give the same product, as though it were multiplied by that number at once: Thus, in example first, 5 times the product of 7, multiplied into the given number, makes 35 times that given number, as plainly, as 5 times 7 makes 35.

EXAMPLES.

	1.	2.	3.
	67910	956700	930137000
	5600	320	9500
Prod.	380296000	306144000	8836301500000

	4.	5.	6.
	359260	8196000	623000
	7364	59180	589000
Prod.	2645590640	485039280000	366947000000

CASE V.

When there are cyphers between the significant figures of the multiplier, omit multiplying by them, and place the first figure of each product of the significant figures, exactly under that figure by which you multiply; lastly, add them together, and their sum will be the total product.

EXAMPLES.

	1.	2.	3.
	5397	85630	48976850
	8009	7005	400030
	48573		
	43176		
Prod.	43224573	599838150	19592209305500

CASE VI.

To multiply by 10, 100, 1000, &c. set down the multiplicand underneath, and join the cyphers in your multiplier to the right hand of them.

EXAMPLES.

	1.	2.	3.	4.
	57935	84935	613975	8473965
	10	100	1000	10000
Prod.	579350			

CASE

CASE VII.

To multiply by 99, 999, &c. in one line; place as many dots at the right hand of the multiplicand, as there are figures of 9 in your multiplier, which dots ſuppoſe to be cyphers; then, beginning with the right hand dot, ſubtract the *given* multiplicand from the *new* one, and the remainder will be the total product.*

EXAMPLES.

1.	2.	3.
6473..	857389...	5384976....
99	999	9999
640827	856531611	53844375024

That theſe examples may appear as clear as poſſible, I will illuſtrate them by giving another.

Mult. 371967... by 999	According to the rule, it will ſtand thus,	371967... Minuend. 371967 Subtrah.
371595033		371595033 Rem. or total Prod.

CASE VIII.

To multiply by 13, 14, 15, &c. to 19 *incluſively*, at one multiplication.

RULE I.

Multiply the multiplicand by the unit figure of the multiplier, and add to the product of each multiplication that figure which ſtands next on the right hand of that which you multiplied, and, to the laſt figure in the multiplicand, add what you carry.

EXAMPLES.

* Here it may eaſily be ſeen that, if you multiply any ſum by 9, the product will be but 9 tenths of the product of the ſame ſum, multiplied by 10; and as the annexing of a dot or cypher, to the right hand of the multiplicand, ſuppoſes it to be increaſed tenfold; therefore, ſubtracting the given multiplicand from the tenfold multiplicand, it is evident that the remainder will be ninefold the ſaid given multiplicand, equal to the product of the ſame by 9; and the ſame will hold true of any number of nines.

Note, When the multiplicand has a fraction added to it, as one fourth, one half, &c. add ſuch a part of the multiplier as the fraction makes, to the laſt product: But when ſuch fraction belongs to the multiplier, add to the laſt product ſuch a part of the multiplicand as the fraction denotes.

EXAMPLES.

1.	2.	3.	4.	5.	6.	7.
65497	84916	19345	7398	9108	6173	54937
13	14	15	16	17	18	19
Prod. 851461						

In example firſt, I ſay, 3 times 7 is 21; I put down 1 and carry 2; ſaying, 3 times 9 is 27, and 2 that I carried, makes 29, and 7, the right hand-figure to 9, makes 36; I put down 6, and carry 3; then 3 times 4 is 12, and 3, which I carried, makes 15, and 9, its right hand figure, makes 24; therefore, I put down 4, and carry 2; ſaying, 3 times 5 is fifteen, and 2 which I carried, makes 17, and 4, its right hand figure, makes 21; I, therefore, put down 1 and carry 2; ſaying, 3 times 6 is 18, and 2 which I carried, makes 20, and 5, its right hand figure, makes 25; I, therefore, ſet down 5 and carry 2; laſtly, the 2 which I carry, and 6, the laſt figure in the multiplicand, make 8, which gives the total product.

RULE II.

To multiply by 13, 14, 15, &c. to 19: Place your multiplier at the right of the multiplicand, with the ſign of multiplication between them, and multiply with the unit figure, only, of the multiplier, removing the product one figure to the right hand of the multiplicand; then add all together, and their ſum will be the total product.

EXAMPLES.

1.	2.	3.	4.	5.
75964×13	7598×14	76013×15	8196×16	3179×17
227892				
Prod. 987532				

CASE IX.

To multiply by 111, 112, 113, &c. to 119, ſo as to have the product in one line: Multiply the multiplicand by the unit figure, only, of the multiplier, and add to each multiplication the *two* figures which ſtand next on the right hand to that which you multiplied, and to the *two* laſt figures, ſeparately, add what you carry.

EXAMPLES.

EXAMPLES.

1.	2.	3.	4.	5.
52976	58975	89193	76435	781572
111	112	113	114	115
Prod.				

6.	7.	8.	9.
43958	647358	499789	9417
116	117	118	119
Prod.			1120623

In the laſt example I ſay, 9 times 7 is 63; I ſet down 3 and carry 6; then, 9 times 1 is 9, and 6 I carried makes 15, and 7, its right hand figure, makes 22; I ſet down 2, and carry 2; then, 9 times 4 is 36, and 2 I carry is 38, and its right hand figures, 1 and 7, make 46; I ſet down 6 and carry 4; then, 9 times 9 is 81, and 4 I carried, is 85, and 4 and 1, its right hand figures, make 90; I put down 0, and carry 9, which I add to 9 and 4, the 2 laſt figures, and they make 22; I then put down 2 and carry 2; laſtly, this 2 and 9 make 11, which I ſet down, and the product is complete.

CASE X.

To multiply by 101, 102, 103, &c. to 109, ſo as to have the product in one line:

RULE I.

Multiply the multiplicand by the unit figure of the multiplier, and add to it the next right hand figure, but one, to that which you multiplied, remembering to add to the two laſt figures in your multiplicand, ſeparately, what you carry.

EXAMPLES.

1.	2.	3.	4.	5.
57691	89726	75964	84975	635918
101	102	103	104	105
Prod.				

6.	7.	8.	9.
64791	58493	8493294	6479
106	107	108	109
Prod.			706211

In

In the laſt example I ſay, 9 times 9 is 81; I ſet down 1 and carry 8; then, 9 times 7 is 63, and 8 I carry is 71; I ſet down 1 and carry 7; then, 9 times 4 is 36, and 7 I carry is 43, and 9, its right hand figure but one, makes 52; I ſet down 2 and carry 5; ſaying, 9 times 6 is 54, and 5 I carry is 59, and 7, its next right hand figure but one, makes 66; I ſet down 6 and carry 6, which I add to 4, the laſt figure but one in the multiplicand, and it makes 10; I ſet down 0, and carry 1, which I add to 6, the laſt figure, and it makes 7, which I ſet down, and I have the whole product.

Rule II.

To multiply by 101, 102, 103, &c. to 109: Multiply by the right hand figure, only, of the multiplier, removing the product two figures to the right hand of the multiplicand: Add all together, and the ſum will be the total product.

Examples.

	1.	2.	3.
	64795 × 101	79164 × 102	37598 × 103
	64795		
Product,	6544295		

	4.	5.	6.
	73967 × 104	84973 × 105	748794 × 106
Prod.			

7.	8.	9.
87958 × 107	395867 × 108	5916379 × 109

CASE XI.

To multiply by 21, 31, 41, &c. to 91, in one line:

Rule.

Firſt, bring down the unit figure of the multiplicand, which will, always, be the unit figure of the product, then multiply every figure of the multiplicand by the ten's figure of the multiplier, and to each product add the figure, which ſtands next on the left hand to that which you multiplied.

Examples.

EXAMPLES.

	1.	2.	3.	4.
	6493587	935846	3584956	716298543
	21	31	41	51
Prod.				

	5.	6.	7.	8.
	537984	326478	938467	4793
	61	71	81	91
Prod.				436163

In the last example, I first bring down the unit figure 3, of the multiplicand, for the unit figure of the product; and, then, I say, 9 times 3 is 27, and 9, its left hand figure, makes 36; I set down 6 and carry 3; then, 9 times 9 is 81, and 3 I carry is 84, and 7, its left hand figure, is 91; I set down 1 and carry 9; saying, 9 times 7 is 63, and 9 I carry is 72, and 4, its left hand figure, is 76; I set down 6 and carry 7; lastly, 9 times 4 is 36, and 7 I carry makes 43, which I set down, and have the product complete.

RULE II.

To multiply by 21, 31, 41, &c. to 91: Multiply by the ten's figure, only, of the multiplier, and set the unit figure of the product under the place of tens; add them all together, and their sum will be the total product.

EXAMPLES.

1.	2.	3.	4.
73918 ×21	56934 ×31	86789 ×41	759846 ×51
147836			
1552278			

5.	6.	7.	8.
37954 ×61	73958 ×71	84937 ×81	54937 ×91

CASE XII.

To multiply by 22, 23, 24, &c. to 29; so as to have the product in one line: Multiply every figure of the multiplicand by the unit figure

figure of the multiplier, and add to each product *twice* that figure which ſtands next on the right hand of that figure you multiplied; and to *twice* the laſt figure of the multiplicand, add what you carry.

EXAMPLES.

	1.	2.	3.	4.
	649378	46795	64839	83964723
	22	23	24	25
Prod.				

	5.	6.	7.	8.
	73758	91357	849358	7657
	26	27	28	29
Prod.				222053

In the laſt example, I ſay, 9 times 7 is 63; I ſet down 3 and carry 6; then, 9 times 5 is 45, and 6 I carry is 51, and 7, its right hand figure, added *twice*, makes 65; I, therefore, ſet down 5 and carry 6, ſaying 9 times 6 is 54, and 6 I carry is 60, and 5, its right hand figure, added *twice*, makes 70; I ſet down 0 and carry 7; then, 9 times 7 is 63, and 7 I carry is 70, and 6, its right hand figure, added twice, makes 82; I ſet down 2 and carry 8; laſtly, I add this 8 to *twice* the laſt multiplicand figure, and they make 22, and the whole product ſtands as above.

To multiply any number, viz. whole or decimal, by any number, giving only the Product.

Put down the Product figure of the firſt figure of the multiplicand by the *firſt* of the multiplier. To the product of the *ſecond* of the multiplicand by the *firſt* of the multiplier, add the number to be carried, and the product of the *firſt* of the multiplicand by the *ſecond* of the multiplier; then, carrying for the tens in the ſum, put down the reſt. To the product of the *third* of the multiplicand by the *firſt* of the multiplier add the number to be carried, and the product of the *ſecond* of the multiplicand by the *ſecond* of the multiplier, alſo the product of the *firſt* of the multiplicand by the *third* of the multiplier, carry the tens, and put down the reſt, and ſo proceed till you have multiplied the *higheſt* of the multiplicand by the *loweſt* of the multiplier. Then multiply the *higheſt* of the multiplicand by the *ſecond* of the multiplier: Add the number to be carried, and the product of the *laſt but one* of the multiplicand by the *third* of the multiplier, and the product of the *laſt but two* of the multiplicand by the *fourth* of the multiplier, &c. Then to the product of the *laſt but one* of the multiplicand by the *fourth* of the multiplier; and ſo proceed till you have multiplied the *laſt* of the multiplicand by the *laſt* of the multiplier, which finiſhes the work.

 Example.

Example.

Mult. 5321415
By 2354
Prod. 12526610910

Explanation.

$5 \times 4 = 20$
$1 \times 4 + 2 + 5 \times 5 = 31$
$4 \times 4 + 3 + 1 \times 5 + 5 \times 3 = 39$
$1 \times 4 + 3 + 4 \times 5 + 1 \times 3 + 5 \times 2 = 40$
$2 \times 4 + 4 + 1 \times 5 + 4 \times 3 + 1 \times 2 = 31$
$3 \times 4 + 3 + 2 \times 5 + 1 \times 3 + 4 \times 2 = 36$
$5 \times 4 + 3 + 3 \times 5 + 2 \times 3 + 1 \times 2 = 46$
$5 \times 5 + 4 + 3 \times 3 + 2 \times 2 = 42$
$5 \times 3 + 4 + 3 \times 2 = 25$
$5 \times 2 + 2 = 12$

DIVISION

Teaches to ſeparate any number, or quantity given, into any number of parts aſſigned ; or to find how often one number is contained in another ; or from any two numbers given, to find a third, which ſhall conſiſt of ſo many units, as the one of thoſe given numbers is comprehended in the other ; and is a conciſe way of performing ſeveral Subtractions.

There are four principal parts to be noticed in Diviſion, viz.

1. The Dividend, or number given to be divided.
2. The Diviſor, or number given to divide by.
3. The Quotient, or anſwer to the queſtion, which ſhews how often the diviſor is contained in the dividend.
4. The Remainder (which is always leſs than the Diviſor, and of the ſame name with the Dividend) is very uncertain, as there is ſometimes a Remainder, and ſometimes none.

Diviſion is both ſimple and compound.

PROOF.

Multiply the diviſor and quotient together, and add the remainder, if there be any, to the product ; if the work be right, that ſum will be equal to the dividend.

SIMPLE DIVISION

Is the dividing of one number by another, without regard to their values : As, 56, divided by 8, produces 7 in the quotient : That is, 8 is contained 7 times in 56.*

CASE

* According to the rule, we reſolve the dividend into parts, and find, by trial, the number of times the diviſor is contained in each of thoſe parts ; and the only thing which remains to be proved, is, that the ſeveral figures of the quotient, taken as one number, according to the order, in which they are placed, is the true quotient of the whole dividend by the diviſor ; which may be thus demonſtrated.

Dem.

CASE I.

RULE. Firſt, ſeek how many times the diviſor is contained in a competent number of the firſt figures of the dividend; when found, place the figure in the quotient; multiply the diviſor by this quotient figure, place the product under the left hand figures of the dividend; then ſubtract it therefrom, and bring down the next

Dem. The complete value of the firſt part of the dividend, is, by the nature of notation, 10, 100, 1000, &c. times the ſimple value of what is taken in the operation; accordingly, as there are 1, 2, or 3, &c. figures ſtanding before it; and, conſequently, the true value of the quotient figure, belonging to that part of the dividend, is alſo 10, 100, 1000, &c. times its ſimple value; but the true value of the quotient figure, belonging to that part of the dividend, found by the rule, is alſo 10, 100, 1000, &c. times its ſimple value; for there are as many figures ſet before it, as the number of remaining figures in the dividend; therefore, this firſt quotient figure, taken in its complete value from the place it ſtands in, is the true quotient of the diviſor in the complete value of the firſt part of the dividend. For the ſame reaſon, all the reſt of the figures of the quotient, taken according to their places, are, each, the true quotient of the diviſor, in the complete value of the ſeveral parts of the dividend belonging to each; becauſe, as the firſt figure, on the right hand of each ſucceeding part of the dividend, has a leſs number of figures ſtanding before it, ſo ought their quotients to have; and ſo they are actually ordered; conſequently, taking all the quotient figures in order, as they are placed by the rule, they make one number, which is equal to the ſum of the true quotients of all the ſeveral parts of the dividend; and is, therefore, the true quotient of the whole dividend by the diviſor.

That no obſcurity may remain, in this demonſtration, it is illuſtrated by the following example.

Diviſor 25)74503 Dividend
1ſt. part of the dividend is = 74000
25×2000 = 50000 — — 2000 the 1ſt. quotient.
1ſt. remainder = 24000
add 500
2d. part of the dividend = 24500
25×900 = 22500 — — 900 the 2d quotient.
2d. remainder = 2000
add 00
3d. part of the dividend = 2000
25×80 = 2000 — — 80 the 3d quotient.
00
add 3
4th. part of the dividend = 3
25×0 0 — — 0 the 4th quotient.
Laſt remainder = 3 — 2980 = Sum of all the quotients, or, the Anſwer.

Explan. It is evident the dividend is reſolved into theſe parts, 74000+500+00+3; for the firſt part of the dividend is conſidered only as 74; but yet it is, truly, 74000; and therefore its quotient, inſtead of 2, is 2000, and the remainder 24000; and ſo of the reſt; as may be ſeen in the operation.

When there is no remainder to a diviſion, the quotient is the abſolute and perfect anſwer to the queſtion; but where there is a remainder, it may be obſerved, that

next figure of the dividend to the right hand of the remainder: If when you have brought down a figure to the remainder, it is ſtill leſs than the diviſor, a cypher muſt be placed in the quotient, and

that it goes ſo much towards another time as it approaches the diviſor; thus, if the remainder be half the diviſor, it will go half of a time more, and ſo on; in order, therefore, to complete the quotient, put the laſt remainder to the end of it, above a line, and the diviſor below it.

It is ſometimes difficult to find how often the diviſor may be had in the numbers of the ſeveral ſteps of the operation: The beſt way will be to find how often the firſt figure of the diviſor may be had in the firſt, or two firſt figures of the dividend, and the anſwer, made leſs by one or two, is, generally, the figure wanted; but if, after ſubtracting the product of the diviſor and quotient from the dividend, the remainder be equal to, or exceed the diviſor, the quotient figure muſt be increaſed accordingly.

The reaſon of the method of proof is plain; for, ſince the quotient is the number of times the dividend contains the diviſor, the product of the quotient and diviſor, muſt, evidently, be equal to the dividend.

There are ſeveral other methods made uſe of to prove diviſion; as follow, viz.

RULE I.

Subtract the remainder from the dividend; divide this number by the quotient, and the quotient, found by this diviſion, will be equal to the former diviſor, when the work is right.

RULE II.

Add the remainder and all the products of the ſeveral quotient figures multiplied by the diviſor together, according to the order in which they ſtand in the work, and the ſum, when the work is right, will be equal to the dividend.

Here, the numbers to be added are the products of the diviſor by every figure of the quotient, ſeparately; and each, by its place, poſſeſſes its complete value; therefore, the ſum of the parts, together with the remainder, muſt be equal to the whole. I will illuſtrate the whole by an example proved according to the ſeveral different methods.

79)987654321(12501953
79*
197
158*
396
395*
154
79*
753
711*
422
395*
271
237*
34*
987654321 Proof by addition.

12501953
79 + 34 remainder.
112517577
87513671
+34
987654321 Proof by multiplication.

987654321
−34
12501953)987654287)79 Proof by diviſion.
87513671
112517577
112517577

We need only to refer to the example; except for the proof of addition; where, it may be remarked, that the Aſteriſms ſhew the numbers to be added, and the dotted lines their order.

and another figure be brought down; after which, you muſt ſeek, multiply and ſubtract, till you have brought down every figure of the dividend.

EXAMPLES.

1.

```
Diviſor. Dividend. Quotient.
   3)175817(58605
     15
     --
      25
      24
      --
       18
       18
       --
        17
        15
        --
         2 Rem.
Proof
58605 Quotient
  ×3 Diviſor + 2
------
175817
```

In this example, I find that 3, the diviſor, cannot be contained in the firſt figure of the dividend; therefore, I take two figures, viz. 17, and inquire how often 3 is contained therein, which finding to be 5 times, I place the 5 in the quotient, and multiply the diviſor by it, ſetting the firſt figure of the multiplication under the 7 in the dividend, &c. I then ſubtract 15 from 17, and find a remainder of 2, to the right hand of which I bring down the next figure of the dividend, viz. 5; then, I inquire how often the diviſor 3, is contained in 25, and, finding it to be 8 times, I multiply by 8, and proceed as before, till I bring down the 1, when, finding I cannot have the diviſor in 1, I place 0 in the quotient, and bring down 7 to the 1, and proceed as at the firſt.

Obſerve, that, in multiplying by 3, I add in the 2.

2.

```
29)153598(5296
   145
   ---
     85
     58
     --
     279
     261
     ---
      188
      174
      ---
       14
```

3.

```
6493)91876375(14150
     6493
     -----
     26946
     25972
     -----
       9743
       6493
       ----
       32507
       32465
       -----
         425
```

4. 28)503775(

5. 35)197184(

6. 85)994466(

7. 236)3798567(

8. 3479)483956795(

9. 5679)19647394(

EXAMPLES,

EXAMPLES.

10.
38473)119184693(

11.
641976)9187642959(

12.
5823789)791822376496(

13.
123456789)121932631112635269(

CASE II.

When there is one cypher, or more, at the right hand of the divifor, it or they muft be cut off; alfo, cut off the fame number of figures from the dividend, and then proceed as in Cafe firft: But the figures which were cut off from the dividend muft be placed at the right hand of the remainder.*

EXAMPLES.

1.
```
65|00)3794326|75(58374
      325
      ---
       544
       520
       ---
        243
        195
        ---
         482
         455
         ---
          276
          260
          ---
          1675 Rem.
```

2.
5193|000)8937643|893(

3.
917|0)47658|3(

4.
875|000)91764789430|000(

5.	6.	7.
Quot. Rem.	Quot. Rem.	Quot. Rem.
1|0)9584|6	1|00)76495|80	1|000)9375183 9|462

Note. In dividing by 10, 100, 1000, &c. when you cut off as many figures from the dividend, as there are cyphers in the divifor, your work is done; thofe figures, cut off at the right hand, are the remainder, and thofe on the left, the quotient, as above.

CASE

* The reafon of this contraction is eafy to conceive; for the cutting off the fame figures from each, is the fame as dividing each of them by 10, 100, 1000, &c. and it is evident, that as often as the whole divifor is contained in the whole dividend, fo often muft any part of the divifor be contained in the like part of the dividend; this method is only to avoid a needlefs repetition of cyphers, which would happen in the common way.

CASE III.

To perform divifion without fetting down the multiplication: Firft, feek how often the divifor may be contained, as before directed; place the figure in the quotient, and multiply it by the divifor; fubtract the unit figure of the multiplication from the dividend, and, if you are obliged to borrow in fubtracting, you muft add one extraordinary to the next multiplication, and proceed as before.*

EXAMPLES.

1.

```
756)647395(856
    4259
     4795
      259
```

2.

```
8197)9167846(
```

3.

```
293)5017846(
```

4.

```
5840)8976439(
```

5.

```
75000)9840000(
```

In the firft example I find the firft quotient figure to be 8; then, I fay, 8 times 6 is 48; fubtract the unit figure 8 from the 3 in the dividend, and 5 remains; then, as I was obliged to borrow 1 in fubtracting, I carry 5, that is, 4 tens in 48, and one ten I borrowed, make 5 tens; faying, 8 times 5 is 40, and 5 I carried, makes 45; then, 5 from 7 and 2 remains; I now carry only 4, as I borrowed none in fubtracting; next, I fay 8 times 7 is 56, and 4 I carry, is 60, which, fubtracted from 64, leaves 4; I now bring down the next figure, and proceed, in the fame manner, through the whole.

CASE IV.

Short divifion is, when the divifor does not exceed 12.

RULE.

Firft, feek how often the divifor can be had in the firft figure, or figures, of the dividend; which, when found, place in the quotient;

* The reafon of this rule is the fame as that of the general rule, page 34.

quotient; then, *mentally*, multiply your divisor by the figure placed in the quotient, and subtract the product from the like number of the left hand figures of your dividend, and the units, which remain, must be accounted so many tens, which you must suppose to stand at the left hand of the next figure in the dividend, and to be reckoned with it; then, seek how often you can have your divisor in those two figures; but, if nothing remain, you must then seek how often your divisor is contained in the next figure, or figures, and thus proceed till you have done.

EXAMPLES.

Divisor. Dividend.	2.	3.	4.	5.
2)71935	3)51903	5(633795	6)8471937	7)193847
Quot. 35967—1				

6.	7.	8.	9.
8)5437846	9)45963784	11)91843756	12)1196437847536

CASE V.

When the divisor is such a number, that any two, or more, figures in the Table, being multiplied together, will produce it, divide the given dividend by one of those figures; the quotient, thence arising, by the other, and so on; and the last quotient will be the answer.*

EXAMPLES.

* This follows from the contraction in case 3d. of simple multiplication, of which it is only the reverse; for the fourth part of the half of any thing is evidently the same as the eighth part of the whole; and so of any other number.

As the learner, at present, is supposed to be unacquainted with the nature of fractions, and as the quotient is incomplete without the remainder; I shall here give a rule for finding the true remainder, without having recourse to fractions.

RULE.

Multiply the quotient by the divisor: Subtract the product from the dividend, and the result will be the true remainder.

The Rule, which is most commonly made use of, when the divisor is a composite number, is

RULE II.

Multiply the last remainder by the preceding divisor, or last but one, and to the product add the preceding remainder; multiply this sum by the next preceding divisor, and to the product add the next preceding remainder; and so on, till you have gone through all the divisors and remainders, to the first.

EXAMPLES.

EXAMPLES.

1st. method.	*2d. method.*	*3d. method.*
9)196473	8)196473	72)196473(2728 Quot.
8)21830	9)24559	144
Quot. 2728—57	Quot. 2728—57	524
		504
		207
		144
		633
		576
		57 Remainder.

I have wrought the above queſtion three ways, that the learner may underſtand the method of finding the true remainder, according to this caſe. In the *firſt*, in dividing by 9, 3 remains, and, by 8, 6 remains; which, being the laſt remainder, I multiply it by the firſt diviſor 9, and add in the firſt remainder 3, and they make 57, the true remainder. In the *ſecond* method, dividing by 8, 1 remains, and by 9, 7 remains; I, therefore, multiply 7, the laſt remainder, by 8, adding in the 1, and they make 57, as before. The *third* method is ſelfevident, and ſhews that the other remainders are true.

2. 36)79638 3. 25)197835 4. 84)93975 5. 54)93738764

6. 221)75323939 7. 132)38473692 8. 144)891376429732

Supplement

EXAMPLE.

6)85397 divided by 150

*5)14232—5

5)2846—2

569—1

Anſ. $569\frac{47}{150}$

1 the laſt remainder.
multiply by *5 the laſt diviſor but one.

5
add 2 the ſecond remainder.

7
multiply by 6 the firſt diviſor.

42
add 5 the firſt remainder.

47 the true remainder.

To explain this rule from the example, we may obſerve, that every unit in the firſt quotient may be looked upon as containing 6 of the units in the given dividend; conſequently, every unit, which remains, will contain the ſame; therefore, this remainder muſt be multiplied by 6, to find the units it contains of the given dividend. Again, each unit in the next quotient will contain 5 of the preceding ones, or 30 of the firſt, that is, 6 times 5; therefore, what remains muſt be multiplied by 30, or,

which

Supplement to Contractions in Multiplication.

1. The shortest method of multiplication, when the multiplier is in any even part of 100, 1000, &c. is by division: For if the multiplicand be increased by a number of cyphers equal to the places in the multiplier, and a part of that product taken for the same proportion, which the multiplier bears to 1, and the same number of cyphers annexed to it, the quotient will be the true product.

1. Multiply 39756 into 125.	2. Multiply 57638 by $33\frac{1}{3}$.
$125 = \frac{1}{8}$ of 1000, wherefore,	$33\frac{1}{3} = \frac{1}{3}$ of 100, therefore,
8)39756000	3)5763800
4969500 Product.	$1921266\frac{2}{3}$ Product.

3. Multiply 91378 by $333\frac{1}{3}$.
$333\frac{1}{3} = \frac{1}{3}$ of 1000, therefore,
3)91378000

$30459333\frac{1}{3}$ Product.

2. From the aforementioned property peculiar to the digit 9, it follows, that whatever other digit, with any number of cyphers annexed, is divided by it, the quotient will consist, wholly, of such digits, and so many 9ths of an unit over; hence the following method of multiplying by repetends of any of the digits.

	1.		2.		3.
	645 by 8888.		5394 by 66666.		3798 by 444
	80000		600000		4000
	9)51600000		9)3236400000		9)15192000
	5733333		359600000		1688000
Subtract	573	Subt.	3596	Subt.	1688
Product.	5732760	Prod.	359596404	Prod.	1686312

TABLES *in* COMPOUND ADDITION.

1. Money.*

				marked,
Note,	4 Farthings	make one	Penny,	*qrs. d.*
	12 Pence		Shilling,	*s.*
	20 Shillings		Pound,	£.

Farthings

which is the same thing, by 6 and 5 continually: Now, this is the same as the Rule; for, instead of finding the remainders, separately, they are reduced from the bottom, upwards, step by step, to one another, and the remaining units, of the same class, taken as they occur.

* As the Federal Money depends on the principles of decimal fractions, no tables will be inserted here. The pupil is referred to that head for a knowledge of it.

Farthings,

4 = 1 Penny.

48 = 12 = 1 Shilling.

960 = 240 = 20 = 1 Pound.

PENCE TABLES.

d.	*s.*	*d.*	*d.*	*s.*	*d.*	*s.*	*d.*	*s.*	*d.*
20 =	1	8	120 =	10	0	2 =	24	12 =	144
30 =	2	6	130 =	10	10	3 =	36	13 =	156
40 =	3	4	140 =	11	8	4 =	48	14 =	168
50 =	4	2	150 =	12	6	5 =	60	15 =	180
60 =	5	0	160 =	13	4	6 =	72	16 =	192
70 =	5	10	170 =	14	2	7 =	84	17 =	204
80 =	6	8	180 =	15	0	8 =	96	18 =	216
90 =	7	6	190 =	15	10	9 =	108	19 =	228
100 =	8	4	200 =	16	8	10 =	120	20 =	240
110 =	9	2				11 =	132		

2. TROY WEIGHT.*

24 Grains	make 1	Pennyweight,	*marked* grs. pwt.
20 Pennyweights	make 1	Ounce,	oz.
12 Ounces	make 1	Pound,	℔

Grains,

24 = 1 Pennyweight.

480 = 20 = 1 Ounce.

5760 = 240 = 12 = 1 Pound.

3. AVOIRDUPOIS WEIGHT.†

16 Drams	make 1	Ounce,	*marked* dr. oz.
16 Ounces	make 1	Pound,	℔
28 Pounds	make 1	Quarter of a hundred wt.	qr.
4 Quarters	make 1	Hundred wt. or 112 pounds,	Cwt.
20 Hundred wt.	make 1	Ton,	T.

Drams.

* By this weight are weighed Gold, Silver, Jewels, Electuaries, and all Liquors. An ounce of gold is divided into 24 parts, called carats, and an ounce of silver, into 20 parts, called pennyweights; therefore, to distinguish fineness of metals, such gold as will abide the fire without loss, is accounted 24 carats fine: If it lose 2 carats in trial, it is called 22 carats fine, &c.

A pound of silver, which loses nothing in trial, is 12 ounces fine; but, if it lose 3 pennyweights, it is 11oz. 17pwts, fine, &c.

Alloy is some base metal with which gold or silver is mixed to abate its fineness. 22 carats of gold, and 2 carats of copper, are esteemed the true standard for gold coin in England, the alloy being one eleventh of the fine gold: And 11oz. 2pwts. of fine silver, melted with 18pwts. of copper, make the true standard for silver coin.

NOTE. 175 Troy ounces are precisely equal to 192 Avoirdupois ounces, and 175 Troy pounds are equal to 144 Avoirdupois. 1 ℔ Troy = 5760 grains, and 1 ℔ Avoirdupois = 7000 grains.

† By Avoirdupois are weighed all coarse and drossy goods, grocery and chandlery wares; bread, and all metals, except gold and silver.

A barrel of pork weighs 220℔. A barrel of beef, 220℔. A quintal of fish, 1 Cwt. Avoirdupois. 12 particular things make 1 dozen; 12 dozen 1 gross, and 144 dozen 1 great gross. 20 particular things make 1 score.

A Firkin

Drams,					
16 =	1 Ounce.				
256 =	16 =	1 Pound.			
7168 =	448 =	28 =	1 Quarter.		
28672 =	1792 =	112 =	4 =	1 Hund. wt.	
573440 =	35840 =	2240 =	80 =	20 =	1 Ton.

4. Apothecaries' Weight.*

20 Grains	make 1	Scruple,	*marked* gr. ℈
3 Scruples		Dram,	ʒ
8 Drams		Ounce,	℥
12 Ounces		Pound,	℔

Grains,				
20 =	1 Scruple.			
60 =	3 =	1 Dram.		
480 =	24 =	8 =	1 Ounce.	
5760 =	288 =	96 =	12 =	1 Pound.

5. Cloth Measure.†

2 Inches, and one fourth	make 1	Nail, *marked* in.	na.
4 Nails, or 9 Inches		Quarter of a yd.	qr.
4 Quarters of a yard, or 36 Inches		Yard,	yd.
3 Quarters of a yard, or 27 Inches		Ell Flemish,	E. Fl.
5 Quarters of a yard, or 45 Inches		Ell English,	E. E.
6 Quarters of a yard, or 54 Inches		Ell French,	E. Fr.
4 Quarters, 1 Inch & one 5th, or 37 Inches and one fifth		Ell Scotch,	E. Sc.
3 Quarters and two thirds		Spanish Var.	

Nails,			
4 =	1 Quarter.		
16 =	4 =	1 Yard.	
12 =	3 =	1 Flemish Ell.	
20 =	5 =	1 English Ell.	
24 =	6 =	1 French Ell.	

6. Long

	℔
A Firkin of Foreign Butter	56
——— ——— Soap	94
A Barrel of ——— Anchoives	30
——— ——— Soap	256
——— ——— Raisins	112
A Punch. of ——— Prunes	1120
A Fother of ——— Lead	19½ Cwt.
A Stone of Iron, Shot, or horseman's weight,	14
——— Butcher's Meat,	8
A gallon of Train Oil	7½
A Tod is - - -	28
A Weigh - - -	182
A Sack - - -	364
A Last - - -	4368

* All the weights now used by Apothecaries, above grains, are Avoirdupois.

The Apothecaries' pound and ounce, and the pound and ounce Troy are the same, only differently divided and subdivided.

† All Scotch and Irish linens are bought by the English or American yard, which is the same, and all Dutch linens by the Ell Flemish: But are all sold in America by the American yard; though the Dutch linens are sold in England by the Ell English, and the Scotch and Irish linens, as in America.

The Scotch allow one English yard in every score yards.

6. LONG MEASURE.*

3 Barley corns	make 1	Inch,	*marked* bar. in.
12 Inches	make 1	Foot,	feet.
3 Feet	make 1	Yard,	yd.
$5\frac{1}{2}$ Yards, or $16\frac{1}{2}$ feet	make 1	Rod, Perch, or Pole,	pol.
40 Poles	make 1	Furlong,	fur.
8 Furlongs	make 1	Mile,	mile.
$69\frac{1}{2}$ Statute miles, *nearly*,	make 1	Degree of a great Circle,	deg.
360 Degrees	make 1	A great Circle of the Earth.	

Or in Measuring Distances.

$7\frac{92}{100}$ Inches	make 1	Link.
25 Links	make 1	Pole.
100 Links	make 1	Chain.
10 Chains	make 1	Furlong.
8 Furlongs	make 1	Mile.

Bar. corns, 3 =	1 Inch.					
36 =	12 =	1 Foot.				
108 =	36 =	3 =	1 Yard.			
594 =	198 =	$16\frac{1}{2}$ =	$5\frac{1}{2}$ =	1 Pole.		
23760 =	7960 =	660 =	220 =	40 =	1 Furlong.	
190080 =	63360 =	5280 =	1760 =	320 =	8 =	1 M.

Inches, $7\frac{92}{100}$ =	1 Link,				
198 =	25 =	1 Pole or Perch.			
792 =	100 =	4 =	1 Chain.		
7920 =	1000 =	40 =	10 =	1 Furlong.	
63360 =	8000 =	320 =	80 =	8 =	1 Mile.

7. TIME.†

60 Seconds	make 1	Minute,	*marked* s. m.
60 Minutes	make 1	Hour,	h.
24 Hours	make 1	Day,	d.
7 Days	make 1	Week,	w.
4 Weeks	make 1	Month,	mo.
13 Months, 1 day & 6 hours	make 1	Julian year,	yr.

Seconds

* The use of Long Measure is to measure the distance of places, or any other thing, where length is considered without regard to breadth.

NOTE. 60 geometrical miles make a degree. 4 inches a hand. 5 feet a geometrical pace. 6 points make 1 line, 12 lines an inch, 12 inches a foot, and 6 feet one French toise, or fathom, equal to 6 feet 4 inches, 8,812,875 lines, English measure. 1 English foot equal to 11 inches, 31154 lines French. 66 feet, or 4 poles, make a Gunter's chain. 3 miles make a league.

† By the Calendar, the year is divided in the following manner.

Thirty days hath September, April, June and November;
February twenty eight alone, and all the rest have thirty one.

When you can divide the year of our Lord by 4, without any remainder, it is then Bissextile, or Leap Year, in which February has 29 days.

Seconds, 60 = 1 Minute.
3600 = 60 = 1 Hour.
86400 = 1440 = 24 = 1 Day.
604800 = 10080 = 168 = 7 = 1 Week.
2419200 = 40320 = 672 = 28 = 4 = 1 Month.
31557600 = 525960 = 8766 = 365 *d.* 6 *h.* = 52 *w.* 1 *d.* 6 *h.* = 1 Julian year.*
31558154 = 525969 = 8766 = 365 *d.* 6 *h.* 9 *m.* 14 *s.* = 1 Periodical year.†
31556937 = 525948 = 8765 = 365 *d.* 5 *h.* 48 *m.* 57 *s.* = 1 Tropical year.‡

8. Motion.

60 Seconds		Prime minute,	*marked* ″ ′
60 Minutes		Degree,	°
30 Degrees	make 1	Sign,	s
12 Signs, or 360 degrees		The whole great circle of the Zodiac.‖	

Seconds, 60 = 1 Minute.
3600 = 60 = 1 Degree.
108000 = 1800 = 30 = 1 Sign.
1296000 = 21600 = 360 = 12 = Zodiac.

9. Land *or* Square Measure.

144 Inches		Square foot.
9 Feet		—— Yard.
30¼ Yards, or 272¼ Feet		—— Pole.
40 Poles	make 1	—— Rood.
4 Roods, or 160 Rods, or 4840 yards		—— Acre.
640 Acres		—— Mile.

Inches, 144 = 1 Foot.
1296 = 9 = 1 Yard.
39204 = 272¼ = 30¼ = 1 Pole.
1568160 = 10890 = 1210 = 40 = 1 Rood.
6272640 = 43560 = 4840 = 160 = 4 = 1 Acre.
4014489600 = 27878400 = 3097600 = 102400 = 2560 = 640 = 1 Mile.

10. Solid

* The civil ſolar year of 365 days being ſhort of the true by 5h. 48m. 57ſ. occaſioned the beginning of the year to run forwards through the ſeaſons nearly one day in four years. On this account, Julius Cæſar ordained that one day ſhould be added to February, every fourth year, by cauſing the 24th day to be reckoned twice; and becauſe this 24th day was the ſixth (ſextilis) before the kalends of March, there were, in this year, two of theſe ſextiles, which gave the name of Biſſextile to this year, which, being thus corrected, was from thence called the Julian year.

† A juſt and equal meaſure of the year is called the periodical year, as being the time of the earth's period about the ſun; in departing from any fixed point in the heavens, and returning to the ſame again.

‡ The ſeveral points of the Ecliptic having a retrograde, or backward motion, the Equinox will, as it were, meet the ſun; by which mean the ſun will arrive at the Equinox, or firſt point of Aries, before his revolution is completed, and this ſpace of time is called the tropical year.

‖ The Zodiac is a great circle of the ſphere, containing the 12 ſigns, through which the ſun paſſes.

10. Solid Measure.*

1728 Inches	make 1	Foot.
27 Feet	make 1	Yard.
40 Feet of round Timber, or 50 feet of hewn Timber	make 1	Ton or Load.
128 Solid Feet, i. e. 8 in length, 4 in breadth, and 4 in height,	make 1	Cord of Wood.

11. Wine Measure.†

2 Pints	make one	Quart,	*marked* pts. qts.
4 Quarts	make one	Gallon,	gal.
10 Gallons	make one	Anchor of Brandy,	anc.
18 Gallons	make one	Runlet,	run.
$31\frac{1}{2}$ Gallons	make one	Half an Hogſhead,	$\frac{1}{2}$hhd.
42 Gallons	make one	Tierce,	tier.
63 Gallons	make one	Hogſhead,	hhd.
2 Hogſheads	make one	Pipe or Butt,	P. or B.
2 Pipes	make one	Tun,	Tun.

Cubic Inches.

$28\frac{7}{8}$ = 1 Pint.

$57\frac{3}{4}$ = 2 = 1 Quart.

231 = 8 = 4 = 1 Gallon.

9702 = 336 = 168 = 42 = 1 Tierce.

14553 = 504 = 252 = 63 = $1\frac{1}{2}$ = 1 Hogſhead.

19404 = 672 = 336 = 84 = 2 = $1\frac{1}{3}$ = 1 Puncheon.

29106 = 1008 = 504 = 126 = 3 = 2 = $1\frac{1}{2}$ = 1 Pipe.

58212 = 2016 = 1008 = 252 = 6 = 4 = 3 = 2 = 1 Tun.

12. Ale or Beer Measure.‡

2 Pints	make 1	Quart,	*marked* pts. qts.
4 Quarts	make 1	Gallon,	gal.
8 Gallons	make 1	Firkin of Ale in Lond.	A. fir.
$8\frac{1}{2}$ Gallons	make 1	Firkin of Ale or Beer,	
9 Gallons	make 1	Firkin of Beer in Lond.	B. fir.
2 Firkins	make 1	Kilderkin,	kil.
2 Kilderkins	make 1	Barrel,	bar.
$1\frac{1}{2}$ Barrel, or 54 Gallons	make 1	Hogſhead of Beer,	hhd.
2 Barrels	make 1	Puncheon,	pun.
3 Barrels, or 2 Hogſheads	make 1	Butt,	butt.

Beer.

* By Solid Meaſure are meaſured all things that have length, breadth and depth.

† All Brandies, Spirits, Perry, Cyder, Mead, Vinegar, Honey, and Oil, are meaſured by Wine Meaſure: Honey is, commonly, ſold by the pound Avoirdupois.

‡ Milk is ſold by the Beer quart.

A barrel of Mackarel, and other barrelled fiſh, by an act of this Commonwealth, is to contain not leſs than 30 gallons.

In England, a barrel of Salmon or Eels is 42 gallons, and a barrel of Herrings 32 gallons. The gallon appointed to be uſed for meaſuring all kinds of Liquids in Ireland, is two hundred and ſeventeen Cubic inches, and ſix tenths.

BEER.

Cubic Inches.								
$35\frac{1}{4}$ =	1 Pint.							
$70\frac{1}{2}$ =	2 =	1 Quart.						
282 =	8 =	4 =	1 Gallon.					
2538 =	72 =	36 =	9 =	1 Firkin.				
5076 =	144 =	72 =	18 =	2 =	1 Kilderkin.			
10152 =	288 =	144 =	36 =	4 =	2 =	1 Barrel.		
15228 =	432 =	216 =	54 =	6 =	3 =	$1\frac{1}{2}$ =	1 Hogshead.	
20304 =	576 =	288 =	72 =	8 =	4 =	2 =	1 Puncheon.	
30456 =	864 =	432 =	108 =	12 =	6 =	3 =	2 =	1 Butt.

ALE.

Cubic Inches.							
$35\frac{1}{4}$ =	1 Pint.						
$70\frac{1}{2}$ =	2 =	1 Quart.					
282 =	8 =	4 =	1 Gallon.				
2256 =	64 =	32 =	8 =	1 Firkin.			
4512 =	128 =	64 =	16 =	2 =	1 Kilderkin.		
9024 =	256 =	128 =	32 =	4 =	2 =	1 Barrel.	
13536 =	384 =	192 =	48 =	6 =	3 =	$1\frac{1}{2}$ =	1 Hogshead.

13. DRY MEASURE.*

2 Pints	make one	Quart,	*marked* pts. qts.
2 Quarts		Pottle,	pot.
2 Pottles		Gallon,	gal.
2 Gallons		Peck,	pk.
4 Pecks		Bushel,	bu.
2 Bushels		Strike,	str.
2 Strikes		Coom,	co.
2 Cooms		Quarter,	qr.
4 Quarters		Chaldron,	ch.
$4\frac{1}{2}$ Quarters		Chaldron in London.	
5 Quarters		Wey,	wey.
2 Weys		Last,	last.

Cubic Inches.								
$268\frac{4}{5}$ =	1 Gallon.							
$537\frac{3}{5}$ =	2 =	1 Peck.						
$2150\frac{2}{5}$ =	8 =	4 =	1 Bushel.					
$4300\frac{4}{5}$ =	16 =	8 =	2 =	1 Strike.				
$8601\frac{3}{5}$ =	32 =	16 =	4 =	2 =	1 Coom.			
$17203\frac{1}{5}$ =	64 =	32 =	8 =	4 =	2 =	1 Quarter.		
86016 =	320 =	160 =	40 =	20 =	10 =	5 =	1 Wey.	
172032 =	640 =	320 =	80 =	40 =	20 =	10 =	2 =	1 Last.

COMPOUND

* This measure is applied to all dry goods, as Corn, Seed, Fruit, Roots, Salt, Sand, Oysters and Coals.

A Winchester bushel is $18\frac{1}{2}$ inches diameter, and 8 inches deep.

COMPOUND ADDITION

Is the adding of ſeveral numbers together, having different denominations, as Pounds, Shillings, Pence, &c. Tons, Hundreds, Quarters, &c.

Rule.*

1. Place the numbers ſo that thoſe of the ſame denomination may ſtand directly under each other.

2. Add the firſt Column or denomination together as in whole numbers; then divide the ſum by as many of the ſame denomination as make one of the next greater, ſetting down the remainder under the column added, and carry the quotient to the next ſuperior denomination, continuing the ſame to the laſt, which add as in ſimple addition.

1. Money.

Examples.

1.			2.				3.				4.			
£.	s.	d.	£.	s.	d.	qr.	£.	s.	d.	qr.	£.	s.	d.	qr.
9	16	10	47	17	6	2	847	11	11	3	915	10	10	2
7	10	9	3	9	10	3	491	19	6	1	64	8	9	1
0	18	6	75	13	9	1	59	6	10	0	5	16	11	3
5	11	11	4	11	11	0	747	16	1	2	419	2	10	2
6	0	8	0	16	8	2	849	12	11	3	491	19	11	3
5	9	10	17	6	2	1	741	17	8	2	762	17	6	1

5.				6.				7.				8.			
£.	s.	d.	qr.	£.	s.	d.	qr.	£.	s.	d.	qr.	£.	s.	d.	qr.
479	11	11	2	764	13	10	2	7	17	10	3	584	19	10	3
64	17	8	3	43	9	8	1	60	6	8	0	765	14	8	1
912	16	10	0	59	17	11	2	7	15	11	2	91	17	10	2
497	5	4	2	817	16	9	3	18	19	9	3	18	19	6	3
69	10	11	3	762	19	10	1	91	16	8	2	847	13	8	2
917	6	9	2	419	17	6	2	918	17	10	3	918	17	11	0

French Money.

Note, 12 Deniers, or Pence } make 1 { Sol or Shilling.
20 Sols, or Shillings } make 1 { Livre, or Pound.
3 Livres, or Pounds } make 1 { Crown of exchange.
6 Livres, or Pounds } make 1 { Real Crown, or ecu d'argent.

Cr.

* The reaſon of this Rule is evident from what has been ſaid in Simple Addition: For, in addition of money, as 1, in the pence, is equal to 4 in the farthings; 1, in the ſhillings, to 12 in the pence; and 1, in the pounds, to 20 in the ſhillings; therefore, carrying as directed, is the arranging the money, ariſing from each column, properly, in the ſcale of denominations; and this reaſoning will hold good in the addition of compound numbers, of any denomination whatever.

Cr. of exc.	liv.	sols.	den.
976	2	17	10
379	1	12	6
491	1	0	11
592	2	15	8

Ecu d'ar.	liv.	sols.	den.
567	5	13	11
389	1	19	6
548	4	17	10
632	3	11	9

Dutch Money.

Note, 8 Phennings, 2 Groats, or 16 Phennings, 20 Stivers } make 1 { Groat. Stiver. Guilder or Florin.

Also,

12 Groats, or 6 Stivers, 20 Schillings, or 6 Guilders } make 1 { Schilling. Pound.

Guild.	stiv.	gr.	ph.
197	17	1	7
348	12	0	6
491	13	1	3
749	19	1	4

Guild.	stiv.	ph.
549	19	14
317	16	12
859	13	8
467	10	15

£.	sch.	gro.
357	18	11
508	12	6
497	13	10
618	17	8

2. Troy Weight.

1.

lb	oz.	pwts.	gr.
767	10	17	22
39	6	9	17
417	11	16	18
935	9	17	19
478	10	17	22
387	9	16	15

2.

lb	oz.	pwts.	gr.
649	11	19	20
32	9	6	5
841	10	11	19
473	9	17	23
764	11	8	9
165	6	10	19

3.

lb	oz.	pwts.	gr.
859	9	15	20
437	10	17	22
640	11	6	0
738	9	12	18
49	0	16	17
584	10	0	9

3. Avoirdupois Weight.

1.

lb	oz.	dr.
19	13	12
21	9	6
4	15	15
22	10	5
18	13	12
6	11	10

2.

Cwt.	qrs.	lb
17	3	19
18	1	27
9	2	9
14	3	16
12	0	6
15	2	0

3.

T.	Cwt.	qrs.	lb
59	13	2	17
6	17	1	21
45	11	3	25
57	16	2	19
75	17	3	17
6	19	0	26

4.

T.	Cwt.	qrs.	lb	oz.	dr.
91	17	2	25	13	15
19	9	0	17	10	12
14	13	2	0	9	11
47	11	3	19	14	0
69	0	1	0	0	12
77	19	3	27	15	11

4. Apothecaries' Weight.

1.

ʒ	℈	gr.
9	1	17
3	2	19
6	1	17
4	0	6
5	2	12
8	1	10

2.

℥	ʒ	℈	gr.
10	7	2	19
6	3	0	12
7	6	1	17
9	5	2	12
6	1	0	16
9	3	2	19

3.

℔	℥	ʒ	℈	gr.
12	11	6	1	15
4	9	1	0	12
91	10	7	2	16
4	8	1	2	19
6	0	0	1	10
4	9	2	1	6

4.

℔	℥	ʒ	℈	gr.
5	9	3	2	13
4	8	6	0	19
9	10	5	2	12
6	5	6	1	17
8	9	4	0	0
7	1	0	1	17

5. Cloth Measure.

1.

Yd.	qr.	n.
76	2	3
3	3	1
42	3	3
57	2	2
16	3	3
49	2	2

2.

E.E.	qr.	n.
91	3	2
49	4	3
6	2	3
84	4	1
7	0	0
61	2	1

3.

E.Fl.	qr.	n.
75	2	1
7	1	3
84	0	2
76	2	3
48	2	2
9	2	3

4.

E.Fr.	qr.	n.
49	3	3
39	5	2
24	2	1
67	4	3
48	2	2
6	3	3

5.

Yds.	qr.	n.
914	2	3
49	2	1
561	3	0
84	0	2
549	3	1
617	1	3

6. Long Measure.

1.

Ft.	in.	bar.
9	11	2
6	9	1
7	0	2
8	10	0
9	6	2
7	10	2

2.

Yd.	ft.	in.
7	2	11
4	1	6
6	0	10
7	2	9
8	1	10
9	2	11

3.

Pol.	ft.	in.
12	11	10
9	16	9
8	12	11
7	15	6
4	14	9
5	11	11

4.

Mil.	fur.	pol.
9	7	36
7	3	19
4	1	24
6	5	12
4	6	9
5	1	10

5.

Deg.	mi.	fur.	pol.	ft.	in.	br.
759	56	6	29	15	10	2
317	39	1	36	11	6	1
497	63	7	24	9	8	1
562	17	0	11	13	11	0
64	48	5	17	9	4	2
764	52	4	19	15	11	1

7. Time.

1.

W.	d.	h.	m.	s.
3	6	22	57	42
1	5	19	31	28
2	3	17	9	15
3	0	9	17	58
1	1	16	19	10
2	2	20	53	48

2.

Mo.	d.	h.	m.
5	24	19	45
4	27	21	35
9	18	0	12
4	19	23	19
8	11	12	13
9	19	8	29

3.

Y.	mo.	d.
19	10	19
7	9	27
4	8	16
1	11	14
17	6	9
12	5	20

4.

Y.	mo.	w.	d.	h.	m.	s.
57	11	3	6	23	29	55
4	8	1	1	19	45	38
29	9	2	3	17	18	19
46	10	2	5	11	50	13
19	9	2	1	16	18	17
45	9	3	5	18	17	59

8. Motion.

1.

17°	55′	48″
1	37	51
28	19	45
19	19	37

2.

25°	49′	51″
4	21	36
19	47	18
25	25	39

3.

9s	29°	35′	53″
10	0	18	31
4	17	13	42
6	19	50	0

9. Land or Square Measure.

1.

Pol.	*feet.*	*in.*
36	179	137
19	248	119
12	96	75
18	110	122
9	269	24
25	221	143

2.

Yds.	*ft.*	*in.*
28	7	119
9	3	75
29	6	120
4	8	12
9	1	119
8	3	43

3.

Acres.	*rood.*	*pol.*	*feet.*	*in.*
756	3	37	245	128
29	1	28	93	25
416	2	31	128	119
37	1	19	218	20
61	0	0	92	103
191	1	25	129	136

10. Solid Measure.

1.

Ton.	*feet.*	*in.*
29	36	1229
12	19	64
8	11	917
19	8	1001
5	0	523
17	39	1119

2.

Yds.	*feet.*	*in.*
75	22	1412
9	26	195
3	19	1091
28	15	1110
49	24	218
18	17	1225

3.

Cord.	*feet.*	*in.*
37	119	1015
9	110	159
48	127	1071
8	111	956
21	9	27
9	28	1091

11. Wine Measure.

1.

Tier.	*gal.*	*qts.*	*pts.*
37	39	3	1
9	17	2	1
4	28	0	0
32	19	1	1
9	0	3	1
12	40	1	1

2.

Hhd.	*gal.*	*qts.*	*pts.*
51	53	1	1
27	39	3	0
9	18	0	1
0	9	2	1
16	24	1	1
5	0	3	0

3.

Ton.	*hhd.*	*gal.*	*qts.*
37	2	37	2
19	1	59	1
28	2	0	0
19	0	47	1
37	1	17	3
14	2	48	2

12. Ale *and* Beer Measure.

1.			2.			3.		
A. B.	*fir.*	*gal.*	*B. B.*	*fir.*	*gal.*	*Hhd.*	*gal.*	*qts.*
49	3	7	29	1	8	379	53	3
26	2	3	19	3	5	19	0	1
9	0	4	16	0	3	121	37	2
27	3	0	9	1	8	467	19	1
27	1	6	14	2	0	591	16	0
19	3	7	17	1	5	75	0	2

13. Dry Measure.

1.				2.				3.			
Qrs.	*bu.*	*p.*	*qt.*	*Bus.*	*p.*	*qt.*	*pt.*	*Ch.*	*bu.*	*p.*	*qts.*
64	7	3	7	37	2	5	1	37	27	3	5
9	4	1	5	19	3	7	0	6	29	1	7
19	6	2	1	16	2	0	1	15	30	0	0
4	0	2	0	5	1	6	1	4	11	3	0
17	3	0	6	9	0	3	0	5	0	1	0
9	5	3	4	19	3	0	1	2	0	2	1

COMPOUND SUBTRACTION

Teaches to find the difference, inequality, or excefs, between any two fums of divers denominations.

Rule.*

Place thofe numbers under each other, which are of the fame denomination, the lefs being below the greater; begin with the leaft denomination, and, if it exceed the figure over it, borrow as many units as make one of the next greater; fubtract it therefrom; and to the difference add the upper figure, remembering, always, to add one to the next fuperior denomination, for that which you borrowed.

1. Money.

1.	*£.*	*s.*	*d.*	*qr.*	2.	*£.*	*s.*	*d.*	*qr.*
Borrowed	349	15	6	1	Lent	791	9	8	1
Paid	195	11	8	1	Received	197	16	4	2
Remains to pay	154	3	10	0	Due to me				
Proof									

3.

* The reafon of this Rule will readily appear, from what was faid in Simple Subtraction; for the borrowing depends upon the fame principle, and is only different, as the numbers to be fubtracted are of different denominations.

	3.				4.				5.			
	£.	s.	d.	qr.	£.	s.	d.	qr.	£.	s.	d.	qr.
From	439	9	10	1	843	12	1	3	569	7	5	2
Take	190	0	10	3	746	15	0	2	508	16	4	0
Rem.												

	6.					7.			
	£.	s.	d.			£.	s.	d.	qr.
Borrowed	19372	12	6		Lent	27109	5	8	3
Paid at	293	16	8	0	Received	5196	15	10	0
	74	9	7	2		384	17	6	2
fundry	9113	11	0	1	at	4187	18	11	1
	1994	0	10	3	feveral	1649	16	8	0
	3914	19	0	0		917	9	10	3
times.	1064	17	9	1	times.	3196	0	2	1
Paid in all					Rec. in all				
Remains to be paid					Remains due				

2. Troy Weight.

	1.				2.				3.			
	℔	oz.	pwt.	gr.	℔	oz.	pwt.	gr.	℔	oz.	pwt.	gr.
Bought	749	5	13	16	379	8	12	10	543	3	9	13
Sold	96	9	19	13	148	4	16	19	179	1	15	18
Rem.												

3. Avoirdupois Weight.

	1.			2.			3.				4.					
	℔	oz.	dr.	C.	qr.	℔	T.	cwt.	qr.	℔	T.	cwt.	qr.	℔	oz.	dr.
Bought	7	9	12	8	2	13	5	13	1	12	9	11	3	17	5	12
Sold	3	12	9	4	1	15	1	12	2	17	3	12	1	19	10	9
Rem.																

4. Apothecaries' Weight.

1.					2.					3.				
℔	℥	ʒ	℈	gr.	℔	℥	ʒ	℈	gr.	℔	℥	ʒ	℈	gr.
71	9	3	1	13	65	10	6	2	10	84	1	1	1	1
37	8	4	1	16	31	8	4	2	9	65	9	3	1	17

5. Cloth

5. Cloth Measure.

1.			2.			3.			4.		
Yds.	*qr.*	*n.*	*E. E.*	*qr.*	*n.*	*E. Fl.*	*qr.*	*n.*	*E. Fr.*	*qr.*	*n.*
35	1	2	467	3	1	765	1	3	549	4	2
19	1	3	291	3	2	149	2	1	197	4	3

6. Long Measure.

1.			2.			3.			4.							
Yds.	*ft.*	*in.*	*Pol.*	*ft.*	*in.*	*Mil.*	*fur.*	*pol.*	*Deg.*	*m.*	*fur.*	*p.*	*yds.*	*ft.*	*in.*	*bar.*
28	2	10	21	11	9	76	3	11	38	41	3	29	2	1	7	2
17	2	11	9	13	8	27	3	21	19	35	5	31	3	1	9	1

7. Time.

1.					2.				3.			4.						
Mo.	*d.*	*h.*	*m.*	*s.*	*Mo.*	*w.*	*d.*	*h.*	*Y.*	*mo.*	*d.*	*Y.*	*mo.*	*w.*	*d.*	*h.*	*m.*	*s.*
6	17	18	27	19	9	2	5	15	7	3	13	48	9	2	5	19	27	31
1	21	16	41	35	4	3	5	15	4	2	19	19	9	3	4	20	19	49

8 Motion.

1.			2.				3.			
79°	21′	31″	6s	11°	12′	48″	4s	19°	41′	22″
41	41	52	3	8	39	29	1	22	19	45

9. Land or Square Measure.

1.			2.			3.				
A.	*R.*	*Pol.*	*A.*	*R.*	*Pol.*	*A.*	*R.*	*Pol.*	*ft.*	*in.*
29	1	10	29	2	17	56	3	19	27	110
24	1	25	17	1	36	29	0	21	210	129

10. Solid Measure.

1.			2.			3.		
Tons.	*ft.*	*in.*	*Yds.*	*ft.*	*in.*	*Cords.*	*ft.*	*in.*
49	19	1100	79	11	917	349	97	1250
38	36	1296	17	25	1095	192	127	1349

11. Wine

11. Wine Measure.

1.				2.			3.			4.		
Tun.	*gal.*	*qt.*	*pt.*	*Tier.*	*gal.*	*qt.*	*Hhd.*	*gal.*	*qt.*	*Tun.*	*hhd.*	*gal.*
79	1	2	1	19	17	1	375	41	2	532	1	19
38	1	3	1	12	29	2	197	36	3	197	1	47

12. Ale and Beer Measure.

1.				2.					3.		
A. B.	*fir.*	*gal.*	*qt.*	*B. B.*	*fir.*	*gal.*	*qts.*	*pts.*	*Hhds.*	*gal.*	*qts.*
39	1	2	1	21	3	5	2	0	769	17	1
24	3	6	2	19	1	7	2	1	391	42	3

13. Dry Measure.

1.				2.				3.			
Qu.	*bu.*	*pk.*	*qt.*	*Bu.*	*pk.*	*qts.*	*pts.*	*Chal.*	*bu.*	*pk.*	*qts.*
56	2	2	1	91	1	3	2	39	12	2	1
39	3	1	2	29	2	1	1	24	25	3	2

PROBLEMS

Refulting from a Comparifon *of the preceding Rules.*

PROB. 1. Having the fum of two numbers, and one of them given, to find the other.

Rule. Subtract the given number from the given fum, and the remainder will be the number required.

Let 288 be the fum of two numbers; one of which is 115, the other is required?

From 288 the Sum,
Take 115 the given number.
Remains 173 the other.

PROB. 2. Having the greater of two numbers, and the difference between that and the lefs given, to find the lefs.

Rule. Subtract the one from the other.

Let the greater number be 325, and the difference between that and the other, 198: What is the other?

From 325 the greater,
Take 198 the difference.
Rem. 127 the lefs.

PROB. 3. Having the leaft of two numbers given, and the difference between that and a greater, to find the greater.

Rule. Add them together.

Given { 127 the lefs number.
198 the difference.

Sum 325 the greater number required.

PROB. 4. Having the fum and difference of two numbers given, to find thofe numbers.

Rule.

Rule. To half the ſum add half the difference, and the ſum is the greater, and from half the ſum take half the difference, and the remainder is the leſs. Or, from the ſum take the difference, and half the remainder is the leaſt: To the leaſt add the given difference, and the ſum is the greateſt.

What are thoſe two numbers, whoſe ſum is 48, and difference 14?

2)48 2)14 $24+7=31$ the greater, & $24-7=17$ the leſs.
$\frac{1}{2}$ ſum $=24$ $\frac{1}{2}$ diff. $=7$ Or $\overline{48-14}\div2=17$, & $17+14=31$.

PROB. 5. Having the ſum of two numbers and the difference of their ſquares given, to find thoſe numbers.

Rule. Divide the difference of their ſquares by the ſum of the numbers, and the quotient will be their difference: You will then have their ſum and difference to find the numbers by Prob. 4.

What two numbers are thoſe, whoſe ſum is 32, and the difference of whoſe ſquares is 256?

Half ſum	16
Half diff.	4
Greater	20
Leſs	12

32)256(8 difference.
256

PROB. 6. Having the difference of two numbers and the difference of their ſquares given, to find thoſe numbers.

Rule. Divide the difference of their ſquares by the difference of the numbers, and the quotient will be their ſum; then proceed by Prob. 4.

What are thoſe two numbers, whoſe difference is 20, and the difference of whoſe ſquares is 2000?

20)2000(100 ſum. $50+10=60$, the greater, & $50-10=40$, the leſs.

For more Queſtions of this nature, ſee Miſcell. Queſ. Problems 46, 47, 48 and 49; but, as the extraction of the ſquare root is there concerned, they could not be admitted here.

PROB. 7. Having the product of two numbers, and one of them given, to find the other.

Rule. Divide the product by the given number, and the quotient will be the number required.

Let the product of two numbers be 288, and one of them 8; I demand the other?

8)288
Anſw. 36

PROB. 8. Having the dividend and quotient, to find the diviſor.

Rule. Divide the dividend by the quotient.

COR. Hence we get another method of proving *Diviſion.*

Given $\begin{cases}\text{288 the Dividend.}\\ \text{36 the Quotient.}\end{cases}$
Required the Diviſor.

36)288(8 Diviſor.
288

PROB. 9. Having the Diviſor and Quotient given, to find the Dividend.

Rule. Multiply them together.

Given { 8 the Divisor. / 36 the Quotient. } 36 × 8

Required the Dividend. 288 the Dividend.

By a due consideration and application of these Problems only, many questions (of which kind are some of the following) may be resolved in a short and elegant manner, although some of them are generally supposed to belong to higher rules.

Application *of the preceding Rules.*

1. The least of two numbers is 19418, and the difference between them is 2384: What is the greater, and sum of both?
19418+2384=21802 greater, and 19418+21802=41220 sum.
2. Suppose a man born in the year 1743; when will he be 57 years of age? 1743+57= 1800 *Ans.*
3. What number is that, which, being added to 19418, will make 21802? 21802—19418=2384 *Ans.*
4. Gen. *Washington* was born in 1732; what is his age in 1787?
1787—1732= 55 *Ans.*
5. America was discovered by Columbus in 1492 and its Independence declared in 1776: How many years have elapsed between those two Æras? 1776—1492= 284 *Ans.*
6. The Massacre at Boston, by the British Troops, happened, March 5th, 1770, and the Battle at Lexington, April 19th, 1775: How long between?
April 19th, 1775—March 5th, 1770=5y. 1m. 14d. *Ans.*
7. Gen. Burgoyne and his army were captured October 17th, 1777, and Earl Cornwallis and his army, October 19th, 1781: What space of time between?
Oct. 19th, 1781—Oct. 17th, 1777=4 years and 2 days, *Ans.*
8. The war between America and England commenced April 19th, 1775, and a general peace took place January 20th, 1783; How long did the war continue?
January 20th, 1783—April 19th, 1775=7y. 9m. 1d. *Ans.*
9. A, B, C and D purchased a quantity of goods in partnership; A paid £.12 10*s.* a dollar and a crown piece; B, 35*s.* C 29*s.* 10*d.* and D, 79*d.*: What did the goods cost? *Ans.* £.16 14 1.
10. A man borrowed, at different times, these several sums, viz. £.29 5*s.* £.18 17*s.* 6*d.* £.45 12*s.* £.98, 3 dollars, one crown piece and an half; pray how much was he in debt? *Ans.* £.193 2 6.
11. There are 4 numbers; the first 317, the second 912, the third 1229, and the fourth as much as the other three, abating 97: What is the sum of them all? *Ans.* 4819.
12. Bought a quantity of goods for £.125 10*s.* paid for truckage 45*s.* for freight 79*s.* 6*d.* for duties 35*s.* 10*d.* and my expenses were 53*s.* 9*d.*: What did the goods stand me in? *Ans.* £.136 4 1.
13. A Gentleman left his son £.1725 more than his daughter, whose fortune was 15 thousand, 15 hundred and 15 pounds: What

What was the ſon's portion, and what did the whole eſtate amount to?

Anſ. The ſon's fortune, £.18240, and the whol eeſtate £.34755.

14. A merchant had 6 debtors, who together owed him £.2917 10*s.* 6*d.* A, B, C, D and E, owed him £.1675 13*s.* 9*d.* of it: What was F's debt? *Anſ.* 1241 16 9.

15. What is the difference between £.1309 7*s.* 1*d.* and the amount of £.345 13*s.* 4*d.* and £.571 4*s.* 8*d.*? *Anſ.* £.392 9 1.

16. A Merchant, at his firſt engaging in trade, owed £.937 15*s.* he had in caſh £.1755 3*s.* 6*d.* in goods £.459 12*s.* 3*d.* in good debts £.197 16*s.* and he cleared the firſt year £.249 19 10. What was the neat balance at the year's end? *Anſ.* £.1724 16 7.

17. What ſum of money muſt be divided between 12 men, ſo as that each may receive £.155? £.155 × 12 = 1860 *Ans.*

18. What number muſt I multiply by 9, that the product may be 675? 675 ÷ 9 = 75 *Anſ.*

19. A Privateer of 175 men took a prize, which amounted to £.59 per man, beſide the owner's half: What was the value of the prize? 175 × 59 × 2 = £.20650 *Anſ.*

20. What is the difference between thrice five and thirty: and thrice thirty five? $35 \times 3 - \overline{5 \times 3 + 30} = 60$ *Ans.*

21. The ſum of two numbers is 750; the leſs 248: What is their difference, product and the ſquare of their difference?*

750 — 248 = 502 the greater number, 502 — 248 = 254 difference, 502 × 248 = 124496 product, and 254 × 254 = 64516 ſquare of the difference.

22. What is the difference between ſix dozen dozen, and half a dozen dozen; and what is their product, and the quotient of the greater by the leſs? *Anſ.* 6 × 12 × 12 — 6 × 12 = 792 diff. 6 × 12 × 12 × 6 × 12 = 62208 product, and 6 × 12 × 12 ÷ 6 × 12 = 12 quotient.

23. There are two numbers; the greater of them is 25 times 78, and their difference is 9 times 15; their ſum and product are required.

Anſ. 78 × 25 = 1950 the greater, $1950 - \overline{15 \times 9} = 1815$ the leſs. 1950 + 1815 = 3765 the ſum, and 1950 × 1815 = 3539250 the prod.

24. A Merchant began trade with £.25327; for 6 years together, he cleared £.1253 per annum; the next 5 years, he cleared £.1729 per annum; but, the laſt 4 years, had the misfortune to loſe £.3019 per annum: What was he worth at the 15 years' end? *Anſ.* £.29414.

25. If a man ſpends £.192 in a year: What is that per calendar month? 192 ÷ 12 = £.16 *Anſ.*

26. If the Federal Debt, which is 42 million dollars, be equally divided between the 13 States: What will be the ſhare of each? *Anſ.* $3230769\frac{3}{13}$ dollars.

27. If

* A Number is ſaid to be ſquared, when it is multiplied into itſelf.

27. If 9000 men march in a column of 750 deep: How many march abreaſt? $9000 \div 750 = 12$ *Anſ.*

28. What number, deducted from the 32d part of 3072, will leave the 96th part of the ſame? $\overline{3072 \div 32} - 32 = 64$ *Anſ.*

29. What number is that, which, multiplied by 3589, will produce 92050672? $92050672 \div 3589 = 25648$ *Anſ.*

30. Suppoſe the quotient ariſing from the diviſion of two numbers to be 5379, the diviſor 37625: What is the dividend, if the remainder came out 9357?

$37625 \times 5379 + 9357 = 202394232$ *Anſ.*

31. There is a certain number, which being divided by 7, the quotient reſulting multiplied by 3, that product divided by 5, from the quotient 20 being ſubtracted; and 30 added to the remainder, the half ſum ſhall make 35: Can you tell me the number? $\overline{\overline{35 \times 2 - 30} + 20 \times 5 \times 7} \div 3 = 700$ *Anſ.*

32. A ſheepfold was robbed three nights ſucceſſively; the firſt night, half the ſheep were ſtolen, and half a ſheep more; the ſecond, half the remainder were loſt, and half a ſheep more; the laſt night they took half what were left and half a ſheep more; by which time they were reduced to 30: How many were there at firſt?

Begin with 30, and, reckoning back from the laſt night to the firſt, you will find that 31 were ſtolen the 3d night, 62 the 2d, and 124 the firſt. *Anſ.* 247.

33. Two boys, A and B, had 850 cheſnuts between them; but A had 150 more than B: How many had each?

$850 \div 2 = 425$ half ſum, and $150 \div 2 = 75$ half diff. then $425 + 75 = 500$ A's, and $425 - 75 = 350$ B's.

34. A and B played at marbles, having 14 apiece at the firſt; but after playing ſeveral games, B, having loſt ſome of his, would play no longer, and it was found that the difference of the ſquares of the numbers, which each then had, was 336: Pray, how many did B loſe?

$14 + 14 = 28$ ſum, $336 \div 28 = 12$ diff. $28 \div 2 = 14$ half ſum, and $12 \div 2 = 6$ half diff. then $14 + 6 = 20$ A retired with, and $14 - 6 = 8$ B had left, therefore B loſt $14 - 8 = 6$.

35. Said Harry to Charles, My father gave me 12 more apples than he gave my brother Jack, and the difference of the ſquares of our ſeparate parcels was 288: Now, if you are Arithmetician enough to tell how many he gave us, each, you ſhall have half of mine?

$288 \div 12 = 24$ the whole: $24 \div 2 = 12$ and $12 \div 2 = 6$, then $12 + 6 = 18 =$ Harry's ſhare, and $12 - 6 = 6 =$ Jack's ſhare.

36. What number added to the 27th part of 6615, will make 570? $570 - \overline{6615 \div 27} = 325$ *Anſ.*

REDUCTION

REDUCTION

Teaches to bring, or exchange, numbers of one denomination to others of different denominations, retaining the ſame value.

It is of *two* ſorts, viz. Deſcending and Aſcending; the former of which is performed by multiplication, and the latter, by diviſion.

Reduction Descending.

Rule.*

Multiply the higheſt denomination, given, by ſo many of the next leſs as make one of that greater, and thus continue until you have brought it down as low as your queſtion requires.

Proof. Change the order of the queſtion, and divide your laſt product by the laſt multiplier, and ſo on.

Examples.

1. In £.27 15*s.* 9*d.* 2*qrs.* how many farthings?

£. *s.* *d.* *qr.*
27 15 9 2
multiplied by 20 = *ſhillings in a pound.*

555 = *ſhillings.*
—— *by* 12 = *pence in a ſhilling.*

6669 = *pence.*
—— *by* 4 = *farthings in a penny.*

Anſ. = 26678 *farthings.*

Note. In multiplying by 20, I added in the 15*s.* by 12, the 9*d.* and by 4, the 2*qrs.* which muſt alway be done in like caſes.

To prove the above queſtion, change the order of it, and it will ſtand thus: In 26678 farthings how many pounds?

4)26678

12)6669 2*qrs.*

2|0)55|5 9*d.*

Anſwer, £.27 15 9 2

2. In £.36 12*s.* 10*d.* 1*qr.* how many farthings? *Anſ.* 35177.
3. In £.95 11*s.* 5*d.* 3*qr.* how many farthings? *Anſ.* 91751.
4. In £.719 9*s.* 11*d.* how many half pence? *Anſ.* 345358.
5. In 29 guineas, at 28*s.* how many pence? *Anſ.* 9744.

6. In

* The reaſon of this Rule is exceedingly obvious; for pounds are brought into ſhillings by multiplying them by 20; ſhillings into pence by multiplying them by 12; and pence into farthings by multiplying them by 4; and the contrary by diviſion; and this will be true in the reduction of numbers conſiſting of any denomination whatever.

6. In 37 piſtoles, at 22*s.* how many ſhillings, pence and farthings. *Anſ.* 814*s.* 9768*d.* 39072*qrs.*

7. In 49 half johannes, at 48*s.* how many ſixpences? *Anſ.* 4704.

8. In 473 French crowns, at 6*s.* 8*d.* how many threepences? *Anſ.* 12613$\frac{1}{3}$.

9. In 53 moidores, at 36*s.* how many ſhillings, pence, and farthings? *Anſ.* 1908*s.* 22896*d.* 91584*qrs.*

10. In £.29 how many groats, threepences, pence and farthings? *Anſ.* 1740 *groats*, 2320 *threepences*, 6960*d.* 27840*qrs.*

11. Reduce 47 guineas and one fourth of a guinea into ſhillings, ſixpences, groats, threepences, twopences, pence and farthings. *Anſ.* 1323 *ſhill.* 2646 *ſixpences*, 3969 *groats*, 5292 *threepences*, 7938 *twopences*, 15876 *pence*, *and* 63504 *qrs.*

REDUCTION ASCENDING.

RULE.

Divide the loweſt denomination given, by ſo many of that name, as make one of the next higher, and thus continue till you have brought it into that denomination which your queſtion requires.

EXAMPLES.

1. In 547325 farthings how many pence, ſhillings and pounds?

Farthings in a penny = 4)547325

Pence in a ſhilling = 12)136831 1 *qr.*

Shillings in a pound = 2|0)1140|2 7*d.*

£.570 2*s.* 7*d.* 1*qr.*

Anſ. 136831*d.* 11402*s.* *and* £.570

Note. The remainder is always of the ſame name as the dividend.

2. Bring 35177 farthings into pounds.
3. Bring 91751 farthings into pence, &c.
4. Bring 345358 half pence into pence, ſhillings and pounds.
5. Reduce 9744 pence to guineas, at 28*s.* per guinea.
6. In 39072 farthings how many piſtoles, at 22*s.*?
7. In 4704 ſixpences how many half johannes?
8. In 12613$\frac{1}{3}$ threepences how many French crowns, at 6*s.* 8*d*?
9. In 91584 farthings how many moidores, at 36*s.*?
10. In 27840 farthings how many pence, threepences, groats, ſhillings and pounds?
11. In 63504 farthings how many pence, twopences, threepences, groats, ſixpences, ſhillings and guineas?

Note. The preceding queſtions may ſerve as proofs to thoſe in Reduction deſcending.

Reduction Descending and Ascending.

1. Money.

1. In £.97 how many pence and English or French crowns, at 6s. 8d.? *Anſ.* 23280d. *and* 291 *crowns.*

2. In 947 English crowns, at 6s. 8d. how many ſhillings, and English guineas? *Anſ.* 6313s. 4d. *and* 225 *guin.* 13s. 4d.

3. In 519 English half crowns how many pence and pounds? *Anſ.* 20760d. *and* £.86 10s.

4. In 1259 groats how many farthings, pence, ſhillings and guineas? *Anſ.* 20144qrs. 5036d. 419s. 8d. *and* 14 *guin.* 27s. 8d.

5. In 75 piſtoles how many pounds? *Anſ.* £.82 10s.

6. In 735 French crowns how many ſhillings and French guineas, at 26s. 8d. *Anſ.* 4900s. *and* 183 *guin.* 20s.

7. In 5793 pence how many farthings, pounds and piſtoles? *Anſ.* 23172qrs. £.24 2s. 9d. *and* 21 *piſtoles,* 20s. 9d.

8. In £.99 how many ſhillings, and half johannes, at 48s.? *Anſ.* 1980s. *and* 41 *half joes.* 12s.

9. In £179 how many guineas? *Anſ.* 127 *guin.* 24s.

10. In £.345 how many moidores? *Anſ.* 191 *moid.* 24s.

11. In 59 half joes, 37 moidores, 45 guineas, 63 piſtoles, 24 English crowns, and 19 dollars; how many pounds, half joes, moidores, guineas, piſtoles, English crowns, dollars, ſhillings, pence and farthings?

Anſ. £.354 4s. 147 *half joes,* 28s. 196 *moidores.* 28s. 253 *guineas,* 322 *piſtoles,* 1062 *Eng. crowns,* 4s. 1180 *dollars,* 4s. 7084 *ſhillings,* 85008d. *and* 340032qrs.

When it is required to know how many ſorts of coin, of different values, and of equal number, are contained in any number of another kind; reduce the ſeveral ſorts of coin into the loweſt denomination mentioned, and add them together for a diviſor; then reduce the money given, into the ſame denomination for a dividend, and the quotient, ariſing from the diviſion, will be the number required.

Note. Obſerve the ſame direction in weights and meaſures.

1. In 275 half johannes how many moidores, guineas, piſtoles, dollars, ſhillings and ſixpences, of each the like number?

A moidore is 36s. that is	72 *ſixpences.*	275 *half joes.*
		48 *ſhills. in a half johan.*
A guinea is 28s. that is	56 *ditto.*	2200
		1100
A piſtole is 22s. that is	44 *ditto.*	13200 *ſhillings.*
		2 *ſixp. in a ſhill.*
A dollar is 6s. that is	12 *ditto.*	*Dividend* = 26400 *ſixpences.*
One ſhilling has	2 *do.*	187)26400(141 *of each, and* 33 *ſixp. or* 16s. 6d. *over, the anſwer.*
	1 *do.*	
Diviſor =	187 *ſixpences.*	

2. A

2. A Gentleman distributed £.37. 10s. between 4 poor persons, in the following manner, viz. that as often as the first had 20s. the second should have 15s. the third, 10s. and the fourth, 5s. What did each person receive? *Ans. The first man £.15.*

Troy Weight.

1. How many grs. in a silver bowl, that weighs 3 lb 10oz. 17 *pwts.?*

```
     lb  oz.  pwt.
      3   10   17
     12 ounces in a pound.
     ---
     46 ounces.
     20 pennyweights in an ounce.
    ----
    932 pennyweights.
     24 grains in one pwt.
   -----
   3728
   1864
  ------
Proof. 24)22368 grains, answer.
  ------
  2|0)93|2
   12) 46—12 pwt.
     lb 3—10 oz.
```

2. In 487 *ozs.* how many *pwts. & grs.?* *Ans.* 9740*pwt.* & 233760*gr.*

3. In 13 ingots of gold, each weighing 9*oz.* 5*pwt.* how many grains? *Ans.* 57720 *gr.*

4. In 97397 *grs.* how many pounds? *Ans.* 16lb. 10*oz.* 18*pwt.* 5*gr.*

5. How many rings, each weighing 5*pwt.* 7*gr.* may be made of 3lb. 5*oz.* 16*pwt.* 2*gr.* of gold? *Ans.* 158.

3. Avoirdupois.

```
        Cwt. qrs. lb   oz.
1. In 91    3    17   14 how many ounces?
       4
     ----
     367 quarters.                 Proof.
      28                         16)164702
     ----                         --------
    2943                         28)10293  14oz.
    735                            -----
   ------                            367  17lb.
   10293 pounds.                     ---
      16                         Cwt. 91  3qrs.
   ------                            ---
   61762
  10294
  ------
  164702 ounces.
```

2. In

2. In 12 tons, 15*cwt.* 1*qr.* 19lb 6*oz.* 12*dr.* how many drams? *Ans.* 7323500*dr.*

3. In 24lb 11*oz.* 9*dr.* how many drams? *Ans.* 6329*dr.*

4. In 44800 pounds, how many drams and tons? *Ans.* 11468800*dr.* *and* 20 *tons.*

5. In 28lb Avoirdupois, how many pounds Troy?

```
                28
              7000 grains in 1lb Avoird.
            ------
grs. in  }
1lb tr.  } =576|0)19600|0(34lb
               1728
               ----
                2320
                2304
                ----
                 160
                  12
                 ---
     576|0)192|0(0 oz.
                20
           -------
     576|0)3840|0)6 pwt.
           3456
           ----
            3840
              24
            ----
            1536
            768
            ----
     576|0)9216|0(16 gr.
           576
           ----
           3456
           3456
           ----
```

6. In 47 lb 9 oz. 13 pwt. 17 gr. Troy, how many pounds Avoirdupois?

```
          lb  oz. pwt. gr.
          47   9   13   17
          12
         ----
          573
           20
        -----
        11473
           24
        -----
        45899
       22947
       ------
7|000)275|369(39lb
       21
       --
       65
       63
       ---
        2369
          16
        -----
        14214
        2369
        -----
7|000)37|904(5 oz.
       35
       ----
        2904
          16
        -----
        17424
        2904
        -----
7|000)46|464(6 4464/7000 dr.
       42
       ----
        4464
```


4. APOTHECARIES'

4. Apothecaries' Weight.

1. How many grains are there in 37℔ 6℥ ?

℔ ℥
37 6
12

450 *ounces.*
8

3600 *drams.*
3

10800 *scruples.*
20

216000 *grains.*

Proof.
2|0)21600|0

3)10800

8)3600

12)450

37℔ 6℥

2. In 9℔ 8℥ 1ʒ 2℈ 19*gr.* how many grains ?

Ans. 55799*gr.*

3. In 55799 grains how many pounds, &c.

Ans. 9℔ 8℥ 1ʒ 2℈ 19*gr.*

5. Cloth Measure.

1. In 127 yards how many quarters and nails ?

4

508 *qrs.*
4

Ans. 2032 *nails.*

Proof.
4)2032

4)508

127 *yards.*

2. In 9173 nails how many yards ? *Ans.* 573*yds.* 1*qr.* 1*n.*
3. In 75 ells English how many quarters and nails ?
Ans. 375*qrs.* 1500*n.*
4. In 56 ells Flemish, how many quarters and nails ?
Ans. 168*qrs.* 672*n.*
5. In 39 ells French how many quarters and nails ?
Ans. 234*qrs.* 936*n.*
6. In 7248 nails how many yards, ells Flemish, ells English and ells French ?
Ans. 453*yds.* 604 *ells Flem.* 362 *ells Eng.* 2*qrs.* 302 *ells French.*
7. In 19 pieces of cloth, each 15 yards, 2 quarters, how many yards, quarters and nails ?
Ans. 294*yds.* 2*qrs.* 1178*qrs. and* 4712*n.*

6. Long Measure.

1. How many barley corns will reach from Newburyport to Boston, it being 43 miles ?

43 *miles.*
8
344 *furlongs.*
40
13760 *rods.*
$5\frac{1}{2}$
68800
6880
75680 *yards.*
3
227040 *feet.*
12
2724480 *inches.*
3
8173440 *Answer.*

Proof.
3)8173440
12)2724480
3)227040
11)75680
6880
2
4,0)1376,0
8)344
43

Here I divide by 11, and multiply the quotient by 2, because twice $5\frac{1}{2}$ is 11; or I might first have multiplied by 2, and, then, have divided the product by 11.

2. How many barley corns will reach round the globe, it being 360 degrees? *Ans.* 4755801600.

3. How many inches from Newburyport to London, it being 2700 miles? *Ans.* 171072000.

4. How often will a wheel, of 16 feet and 6 inches circumference, turn round in the distance from Newburyport to Cambridge, it being 42 miles? *Ans.* 13440 *times.*

5. In 190080 inches, how many yards and leagues? *Ans.* 5280 *yds. and* 1 *league.*

7. TIME.

1. In 20 years how many seconds?

d. *h.*
365 6 *in a year*
24
1466
730
8766 *hours in* 1 *year.*
20
175320 *hours in* 20 *years.*
60
10519200 *minutes in ditto.*
60
631152000 *seconds in ditto.*

Proof.
6|0)63115200|0
6|0)1051920|0
2|0)17532|0
4×6)8766
4)1461
365 *d.* 6*h.*

2. Suppose your age to be 15*y.* 19*d.* 21*h.* 37*m.* 45*s.* how many seconds are there in it, allowing 365 days and 6 hours to the year? *Ans.* 475047465.

3. In

3. In 31558154 seconds how many solar years? *Ans.* 1 *year.*
4. How many minutes from the first day of January to the 14th day of August, inclusively? *Ans.* 325440.
5. How many days since the commencement of the christian Æra?
6. How many minutes since the commencement of the American war, which happened on the 19th day of April, 1775?
7. How many seconds between the commencement of the war, April 19th, 1775, and the independence of the United States of America, which took place the 4th day of July, 1776?
Ans. 38102400.

8. MOTION.

1. In 9 signs, 13° 25′, how many seconds?

```
  9s  13°  25′
  30
  ---
  283 degrees.
   60
  ---
 17005 minutes.
    60
 -----
1020300 seconds.
```

Proof.

```
6|0)102030|0
   ---------
6|0)1700|5
   -------
3|0)28|3—25
   ---
     9s 13° 25′
```

9. LAND or SQUARE MEASURE.

1. In 29 acres, 3 roods, 19 poles, how many roods and perches?

```
Acres. R. Poles.
  29   3   19
   4
  ---
 119 roods.
  40
  ---
Answer 4779 perches.
```

Proof.

```
4|0)477|9
    ----
4)119—19p.
  ---
29ac. 3 roods.
```

2. In 1997 poles how many acres? *Ans.* 12*a.* 1*r.* 37*p.*
3. In 89763 square yards how many acres, &c.?
Ans. 18*a.* 2*r.* 7*p.* 101*ft.* 36*in.*
4. How many square feet, square yards, and square poles, in a square mile?
Ans. 27878400 *feet,* 3097600 *yards, and* 102400 *poles.*

10. SOLID MEASURE.

1. In 15 tons of hewn timber how many solid inches?
15 *tons.*

15 *tons.*	*Proof.*
50	1728)1296000(75|0
750 *feet.*	12096
1728	15 *tons.*
6000	8640
1500	8640
5250	
750	

Ans. 1296000 *inches.*

2. In 9 tons of round timber how many inches?

Ans. 622080

3. In 25 cords of wood how many inches? *Ans.* 5529600

11. WINE MEASURE.

1. In 9*hhds.* 15*gals.* 3*qts.* of wine how many quarts?

hhds. gal. qts.	*Proof.*
9 15 3	4)2331
63	63)582—3*qts.*
32	9*hhds.*—15*gals.*
55	
582 *gallons.*	
4	

Ans. 2331 *quarts.*

2. In 12 pipes of wine how many pints? *Ans.* 12096.

3. In 9758 pints of brandy how many pipes?

Ans. 9*p.* 1*hhd.* 22*gal.* 3*qts.*

5. In 1008 quarts of cyder how many tons? *Ans.* 1 *ton.*

12. ALE *or* BEER MEASURE.

1. In 29*hhds.* of beer how many pints?

hhds.	*Proof.*
29	2)12528
54	4)6264
116	54(1566
145	29 *hhds.*
1566 *gallons.*	
4	
6264 *quarts.*	
2	

Ans. 12528 *pints.*

2. In

2. In 47*bar.* 18*gal.* of ale how many pints? *Anſ.* 13680.
3. In 36 puncheons of beer how many butts? *Anſ.* 24.

13. Dry Measure.

1. In 42 chaldrons of coals how many pecks?

```
Chaldrons.                    Proof.
   42                       4)5376
   32                      32)1344(42
  ----                        128
   84                         ---
  126                          64
  ----                         64
 1344 buſhels.                 --
    4
  ----
Anſ. 5376 pecks.
```

2. In 75 buſhels of corn how many pints? *Anſ.* 4800.
3. In 9376 quarts how many buſhels? *Anſ.* 293.

VULGAR FRACTIONS.

Fractions, or broken numbers, are expreſſions for any aſſignable parts of an unit, or whole number; and are repreſented by two numbers, placed one above another, with a line drawn between them, thus; $\frac{5}{8}$, $\frac{4}{3}$, &c. ſignifying five eighths, four thirds, that is, one and one third, &c.

The figure above the line, is called the *numerator*, and that below it, the *denominator*.

The denominator (which is the diviſor in diviſion) ſhews how many parts the integer is divided into; and the numerator (which is the remainder after diviſion) ſhews how many of thoſe parts are meant by the fraction.

Fractions are either proper, improper, ſingle, compound, or mixed. Any whole number may be made an improper fraction by drawing a line under it, and putting unity, or 1 for a denominator, as 9 may be expreſſed fractionwiſe, thus $\frac{9}{1}$, and 12 thus $\frac{12}{1}$, &c.

1. A *ſingle*, *ſimple*, or *proper fraction* is, when the numerator is leſs than the denominator, as, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{19}{35}$, &c. and is a *ſimple* expreſſion for any number of parts of the integer.

2. An improper fraction is, when the numerator exceeds the denominator, as, $\frac{5}{3}$, $\frac{8}{5}$; $\frac{12}{4}$, &c.

3. A *compound fraction* is the fraction of a fraction, coupled by the word *of*, thus, $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{7}{8}$, &c. which are read thus, two thirds of three fourths; one half of three fifths of ſeven eighths.

4. A *mixed number* is compoſed of a whole number and a fraction, as $7\frac{3}{5}$, $35\frac{4}{13}$, &c. that is, ſeven and three fifths, &c.

5. A

5. A fraction is ſaid to be in its leaſt, or loweſt terms, when it is expreſſed by the leaſt numbers poſſible.

6. The *common meaſure* of two, or more numbers, is that number which will divide each of them without a remainder: Thus, 5 is the common meaſure of 10, 20 and 30; and the *greateſt* number, which will do this, is called the *greateſt common meaſure.*

7. A number, which can be meaſured by two, or more, numbers, is called their *common multiple*: And, if it be the leaſt number, which can be ſo meaſured, it is called the *leaſt common multiple*; thus, 40, 60, 80, 100, are multiples of 4 and 5; but their leaſt common multiple is 20.

8. A *prime number* is that, which can only be meaſured by itſelf, or an unit.

9. That number, which is produced by multiplying ſeveral numbers together, is called a *compoſite number.*

10. A *perfect number* is equal to the ſum of all its aliquot parts.*

PROBLEM I.†

To find the greateſt common meaſure of two, or more, numbers.

RULE.

1. If there be two numbers only, divide the greater by the leſs, and this diviſor by the remainder, and ſo on, always dividing the laſt diviſor by the laſt remainder, till nothing remain, then will the laſt diviſor be the greateſt common meaſure required.

2. When there are more than two numbers, find the greateſt common meaſure of two of them, as before; then, of *that* common meaſure and one of the other numbers, and ſo on, through all the numbers, to the laſt; then will the greateſt common meaſure, laſt found, be the anſwer.

3. If 1 happens to be the common meaſure, the given numbers are prime to each other, and found to be incommenſurable, or in their loweſt terms.

EXAMPLES.

* The following perfect numbers are all which are, at preſent, known.

6	8589869056
28	137438691328
496	2305843008139952128
8128	2417851639228158837784576
33550336	9903520314282971830448816128

† This and the following problem will be found very uſeful in the doctrine of fractions, and ſeveral other parts of Arithmetic.

The truth of the rule may be ſhewn from the firſt example: For, ſince 108 meaſures 216, it alſo meaſures 216+108, or 324.

Again, ſince 108 meaſures 216 and 324, it alſo meaſures $\overline{5\times324}+216$, or 1836. In the ſame manner it will be found to meaſure $\overline{2\times1836}+324$, or 3996, and ſo on.

It is alſo the greateſt common meaſure; for, ſuppoſe there be a greater, then, ſince the greater meaſures 1836 and 3996, it alſo meaſures the remainder 324; and ſince it meaſures 324 and 1836, it alſo meaſures the remainder 216; in the ſame manner it will be found to meaſure the remainder 108; that is, the greater meaſures the leſs, which is abſurd; therefore, 108 is the greateſt common meaſure.

In the ſame manner, the demonſtration may be applied to one or more numbers.

EXAMPLES.

1. What is the greateſt common meaſure of 1836, 3996, and 1044?

```
1836)3996(2
     3672
     ----
      324)1836(5
          1620
          ----
           216)324(1
               216
               ---
Common meaſ.=108)216(2
                 216
```

So 108 is the greateſt common meaſure of 3996 and 1836.

```
Hence 108)1044(9
           972
           ---
            72)108(1
                72
                --
Laſt greateſt com. meaſ.=36)72(2
                            72
                            --
```

Therefore, 36 is the anſwer required.

2. What is the greateſt common meaſure of 1224 and 1080? *Anſ.* 72.

3. What is the greateſt common meaſure of 1440, 672 and 3472? *Anſ.* 16.

PROBLEM II.*

To find the leaſt common multiple of two, or more, numbers.

RULE.

1. Divide by any number that will divide two, or more, of the given numbers without a remainder, and ſet the quotients, together with the undivided numbers, in a line beneath.

2. Divide the ſecond line, as before, and ſo on, till there are no two numbers, that can be divided; then, the continued product of the diviſors and quotients will give the multiple required.

EXAMPLES.

1. What is the leaſt common multiple of 6, 10, 16 and 20?

```
*5)6   10   16   20
   ----------------
*2)6    2   16    4
   ----------------
*2)3    1    8    2
   ----------------
  *3    1   *4    1
   ----------------
*  *  *  *  *
5×2×2×3×4=240 Anſ.
```

I ſurvey my given numbers, and find that five will divide two of them, viz. 10 and 20, which I divide by 5, bringing into a line with the quotients the numbers, which 5 will not meaſure: Again, I view the numbers in the ſecond line, and find 2 will meaſure them all, and I get 3, 1, 8, 2 in the third line, and find that two will meaſure 8 and 2, and in the fourth line get 3, 1, 4, 1, all prime; I then multiply the prime numbers and the diviſors continually into each other, for the number ſought, and find it to be 240.

2. What

* The reaſon of this rule may alſo be ſhewn from the firſt example: Thus, it is evident that $6\times10\times16\times20$ (=19200) may be divided by 6, 10, 16 and 20, without a remainder; but 20 is a multiple of 5; therefore, $6\times10\times16\times4$, or 3840, is alſo diviſible by 6, 10, 16 and 20. Alſo, 16 is a multiple of 4; therefore, $6\times10\times4\times4=960$, is alſo diviſible by 6, 10, 16 and 20. Alſo, 10 is a multiple of 2; therefore, $6\times5\times4\times4=480$, is alſo diviſible by 6, 10, 16 and 20. Alſo, 6 is a multiple of 2; therefore, $3\times5\times4\times4=240$, is alſo diviſible by 6, 10, 16 and 20; and is evidently the leaſt number that can be ſo divided.

2. What is the leaſt common multiple of 6 and 8? *Anſ.* 24.

3. What is the leaſt number that 3, 5, 8 and 10 will meaſure? *Anſ.* 120.

4. What is the leaſt number which can be divided by the 9 digits, ſeparately, without a remainder? *Anſ.* 2520

REDUCTION *of* VULGAR FRACTIONS

Is the bringing of them out of one form into another, in order to prepare them for the operations of Addition, Subtraction, &c.

CASE I.*

To abbreviate, or reduce fractions to their loweſt terms.

RULE.

Divide the terms of the given fraction, by any number, which will divide them without a remainder, and the quotients, again, in the ſame manner; and ſo on, till it appears that there is no number greater than 1, which will divide them, and the fraction will be in its loweſt terms. *Or,*

Divide both the terms of the fraction by their greateſt common meaſure, and the quotients will be the terms of the fraction required.

EXAMPLES.

1. Reduce $\frac{288}{480}$ to its loweſt terms.

$8)\frac{288}{480} = \overset{(4)}{\frac{36}{60}} = \overset{(3)}{\frac{9}{15}} = \frac{3}{5}$ *the anſwer.*

Or

* That dividing both the terms, that is, both numerator and denominator of the fraction equally by any number whatever, will give another fraction, equal to the former, is evident: And if thoſe diviſions be performed as often as can be done, or the common diviſor be the greateſt poſſible, the terms of the reſulting fraction muſt be the leaſt poſſible.

Note 1. Any number, ending with an even number or cypher, is diviſible by 2.

2. Any number, ending with 5 or 0, is diviſible by 5.

3. If the right hand place of any number be 0, the whole is diviſible by 10.

4. If the two right hand figures of any number be diviſible by 4, the whole is diviſible by 4.

5. If the three right hand figures of any number be diviſible by 8, the whole is diviſible by 8.

6. If the ſum of the digits, conſtituting any number, be diviſible by 3 or 9, the whole is diviſible by 3 or 9.

7. If a number cannot be divided by ſome number leſs than the ſquare root thereof, that number is a *prime.*

8. All *prime* numbers, except 2 and 5, have 1, 3, 7 or 9 in the place of units: And all other numbers are *compoſite.*

9. When numbers, with the ſign of Addition or Subtraction between them, are to be divided by any numbers, each of the numbers muſt be divided: Thus, $\frac{6+9+12}{3} = 2+3+4 = 9$.

10. But if the numbers have the ſign of Multiplication between them; then only one of them muſt be divided: Thus, $\frac{4\times6\times10}{2\times5} = \frac{2\times6\times10}{1\times5} = \frac{2\times6\times2}{1\times1} = \frac{24}{1} = 24$.

Or thus:

288)480(1
288
———
192)288(1
192
———
Com. meaf. 96)192(2
192

Therefore 96 *is the greateft common meafure.*

and 96 $\left\{ \frac{288}{480} = \frac{3}{5} \right.$ *the fame as before.*

2. Reduce $\frac{96}{544}$ to its loweft terms. *Anf.* $\frac{3}{17}$.
3. Reduce $\frac{384}{1152}$ to its loweft terms. *Anf.* $\frac{1}{3}$.
4. Reduce $\frac{57}{456}$ to its loweft terms. *Anf.* $\frac{1}{8}$.
5. Reduce $\frac{46}{184}$ to its loweft terms. *Anf.* $\frac{1}{4}$.
6. Reduce $\frac{1429}{2858}$ to its loweft terms. *Anf.* $\frac{1}{2}$.

CASE II.

To reduce a mixed number to its equivalent improper fraction.

RULE.*

Multiply the whole number by the denominator of the fraction, and add the numerator of the fraction to the product; under which, fubjoin the denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce $36\frac{5}{8}$ to its equivalent improper fraction.

$$\begin{array}{r} 36 \\ 8+5 \\ \hline \textit{Anf.}\ 293 \\ \hline 8 \end{array}$$

I multiply 36 by 8, and adding the numerator 5 to the product, as I multiply, the fum 293 is the numerator of the fraction fought, and 8 the denominator: So that $\frac{293}{8}$ is the improper fraction, equal to $36\frac{5}{8}$.

Or, $\frac{36\times8+5}{8}=\frac{293}{8}$ *Anf. as before.*

2. Reduce $127\frac{4}{17}$ to its equivalent improper fraction. *Anf.* $\frac{2163}{17}$

3. Reduce $653\frac{3}{19}$ to its equivalent improper fraction. *Anf.* $\frac{12410}{19}$

CASE III.†

To reduce a whole number to an equivalent fraction, having a given denominator.

RULE.

* All fractions reprefent a divifion of the numerator by the denominator, and are taken altogether as proper and adequate expreffions of the quotient. Thus the quotient of 3 divided by 4 is $\frac{3}{4}$; from whence the rule is manifeft; for if any number is multiplied and divided by the fame number, it is evident the quotient muft be the fame as the quantity firft given.

† Multiplication and Divifion are here equally ufed, and confequently the refult is the fame as the quantity firft propofed.

RULE.

Multiply the whole number by the given denominator : Place the product over the said denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce 6 to a fraction, whose denomiator shall be 8.

$6\times 8=48$, *and* $\frac{48}{8}$ *the answer.—Proof* $\frac{48}{8}=48\div 8=6$.

2. Reduce 15 to a fraction, whose denominator shall be 12.

Ans. $\frac{180}{12}$.

3. Reduce 100 to a fraction, whose denominator shall be 70.

Ans. $\frac{7000}{70}=\frac{100}{1}=100$.

CASE IV.*

To reduce an improper fraction to its equivalent whole, or mixed number.

RULE

Divide the numerator by the denominator ; the quotient will be the whole number, and the remainder, if any, will be the numerator to the given denominator.

EXAMPLES.

1. Reduce $\frac{293}{8}$ to its equivalent whole, or mixed number.

```
8)293(36 5/8 Answer.
  24
  --
   53
   48
   --
    5
```

Or, $\frac{293}{8}=293\div 8=36\frac{5}{8}$ *as before.*

2. Reduce $\frac{2163}{17}$ to its equivalent whole, or mixed number.

Ans. $127\frac{4}{17}$.

3. Reduce $\frac{12410}{19}$ to its eqvivalent whole, or mixed number.

Ans. $653\frac{3}{19}$.

4. Reduce $\frac{45}{5}$ to its equivalent whole number. *Ans.* 9.

CASE V.†

To reduce a compound fraction to an equivalent simple one.

RULE.

Multiply all the numerators continually together for a new numerator,

* This rule is evidently the reverse of case 2d, and has its reason in the nature of common division.

† That a compound fraction may be represented by a simple one is very evident ; since a part of a part must be equal to some part of the whole. The truth of the rule for this reduction may be shewn as follows.

Let the compound fraction to be reduced, be $\frac{3}{4}$ of $\frac{6}{10}$. Then $\frac{1}{4}$ of $\frac{6}{10}=\frac{6}{10}\div 4=\frac{6}{40}$, and consequently $\frac{3}{4}$ of $\frac{6}{10}=\frac{6}{40}\times 3=\frac{18}{40}$ the same as by the rule.

If the compound fraction consists of more numbers than two, the two first may be reduced to one, and that one and the third will be the same as a fraction of two numbers, and so on.

numerator, and all the denominators, for a new denominator, and they will form the ſimple fraction required.

If part of the compound fraction be a whole or mixed number, it muſt be reduced to an improper fraction, by caſe 2d, or 3d.

If the denominator of any member of a compound fraction be equal to the numerator of another member thereof, theſe equal numerators and denominators may be expunged, and the other members continually multiplied, (as by the rule) will produce the fraction required in lower terms.

EXAMPLES.

1. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a ſimple fraction.

$\frac{1\times2\times3\times4}{2\times3\times4\times5}=\frac{24}{120}=\frac{1}{5}$ *the anſwer.*

Or, by expunging the equal numerators and denominators, it will give $\frac{1}{5}$ as before.

2. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{11}{12}$ to a ſimple fraction.

$\frac{3\times4\times5\times11}{4\times5\times6\times12}=\frac{660}{1440}=\frac{11}{24}$ *Anſ.* Or, by expunging the equal numerators and denominators it will be $\frac{3\times11}{6\times12}=\frac{33}{72}=\frac{11}{24}$ *as before.*

3. Reduce $\frac{5}{8}$ of $\frac{6}{7}$ of $\frac{15}{19}$ to a ſimple fraction. *Anſ.* $\frac{225}{532}$.

4. Reduce $\frac{3}{12}$ of $\frac{13}{15}$ of $\frac{8}{17}$ of 20 to a ſimple (or improper) fraction. *Anſ.* $\frac{624}{306}=2\frac{2}{51}$.

5. Reduce $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $12\frac{1}{2}$ to a ſimple (or improper) fraction. *Anſ.* $\frac{75}{64}=1\frac{11}{64}$.

CASE VI.

To reduce fractions of different denominators to equivalent fractions, having a common denominator.

RULE I.*

Multiply each numerator into all the denominators, except its own, for a new numerator, and all the denominators into each other, continually, for a common denominator.

EXAMPLES.

1. Reduce $\frac{1}{4}$, $\frac{2}{5}$ and $\frac{5}{8}$ to equivalent fractions, having a common denominator.

$1\times5\times8=40$ *the new numerator for* $\frac{1}{4}$.

$2\times4\times8$

* By placing the numbers multiplied properly under one another, it will be ſeen that the numerator and denominator of every fraction are multiplied by the very ſame number, and conſequently their values are not altered. Thus, in the firſt example.

$\frac{1}{4}\left|\frac{\times5\times8}{\times5\times8}\right.\quad \frac{2}{5}\left|\frac{\times4\times8}{\times4\times8}\right.\quad \frac{5}{8}\left|\frac{\times4\times5}{\times4\times5}\right.$

In the ſecond rule, the common denominator is a multiple of all the denominators, and conſequently will divide by any of them; Therefore, proper parts may be taken for all the numerators as required.

$2\times4\times8 = 64$ *the new numerator for* $\frac{2}{5}$.
$5\times4\times5 = 100$ *ditto for* $\frac{5}{8}$.
$4\times5\times8 = 160$ *the common denominator.*

Therefore, the new equivalent fractions are $\frac{40}{160}$, $\frac{64}{160}$, *and* $\frac{100}{160}$, *the anſwer.*

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to fractions having a common denominator. *Anſ.* $\frac{576}{1152}$, $\frac{768}{1152}$, $\frac{864}{1152}$, $\frac{960}{1152}$, $\frac{1008}{1152}$.

3. Reduce $\frac{1}{2}$, $\frac{2}{3}$ of $\frac{5}{6}$, $7\frac{3}{4}$ and $\frac{3}{13}$ to a common denominator. *Anſ.* $\frac{935}{1872}$, $\frac{1040}{1872}$, $\frac{14508}{1872}$, $\frac{432}{1872}$.

4. Reduce $\frac{11}{15}$, $\frac{3}{4}$ of $1\frac{1}{2}$, $\frac{7}{12}$, and $\frac{5}{8}$ to a common denominator. *Anſ.* $\frac{8448}{11520}$, $\frac{21600}{11520}$, $\frac{6720}{11520}$, $\frac{7200}{11520}$.

RULE II.

To reduce any given fractions to others, which ſhall have the leaſt common denominator.

1. By Problem 2, page 79, find the leaſt common multiple of all the denominators of the given fractions, and it will be the common denominator required.

2. Divide the common denominator by the denominator of each fraction, and multiply the quotient by the numerator, and the product will be the numerator of the fraction required.

EXAMPLES.

1. Reduce $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{7}{8}$ to fractions having the leaſt common denominator poſſible.

$$\begin{array}{r|ccc} 4) & 3 & 4 & 8 \\ \hline & 3 & 1 & 2 \end{array}$$

$4\times3\times2 = 24 =$ *leaſt common denominator.*

$\overline{24\div3}\times1=8$ *the 1ſt numerator;* $\overline{24\div4}\times3=18$ *the 2d numerator;* $\overline{24\div8}\times7=21$ *the 3d. numerator.*

Whence, the required fractions, are $\frac{8}{24}$, $\frac{18}{24}$, $\frac{21}{24}$.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to fractions having the leaſt common denominator. *Anſ.* $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$ *and* $\frac{48}{60}$.

CASE VII.

To reduce a fraction of one denomination to the fraction of another, but greater, retaining the ſame value.

RULE.*

Reduce the given fraction to a compound one, by comparing it with all the denominations between *it* and *that* denomination you would reduce it to; laſtly, reduce this compound fraction to a ſingle one, by caſe 5th, and you will have a fraction of the required denomination, equal in value to the given fraction.

EXAMPLES.

* The reaſon of this and the next rule is explained in the rule for reducing compound fractions to ſimple ones.

EXAMPLES.

1. Reduce $\frac{3}{5}$ of a penny to the fraction of a pound.

By comparing it, it becomes $\frac{3}{5}$ of $\frac{1}{12}$ of $\frac{1}{20}$; which reduced by case 5th, will be $3\times1\times1=3$

And $5\times12\times20=1200=\frac{1}{400}$

2. Reduce $\frac{3}{4}$ of a farthing to the fraction of a pound.

Ans. £ $\frac{1}{1280}$.

3. Reduce $\frac{5}{8}$ of a penny to the fraction of a guinea.

Ans. $\frac{5}{2688}$ *guinea.*

4. Reduce $\frac{12}{19}$ of a shilling to the fraction of a moidore.

Ans. $\frac{1}{57}$ *moidore.*

5. Reduce $\frac{4}{7}$ of an ounce to the fraction of a ℔ Avoirdupois.

Ans. $\frac{1}{28}$ ℔.

*6. Reduce 3*s.* 6*d.* to the fraction of a pound. *Ans.* $\frac{7}{40}$£.

7. Reduce 13*s.* 6*d.* to the fraction of a pistole.

Ans. $\frac{27}{44}$ *pistole.*

†8. Reduce $\frac{4}{5}$ of a pound to the fraction of a guinea.

Ans. $\frac{4}{7}$ *guinea.*

9. Reduce $\frac{7}{8}$ of a *pwt.* to the fraction of a pound Troy.

Ans. $\frac{7}{1920}$ ℔.

10. Reduce $\frac{8}{9}$ of a ℔ Avoirdupois to the fraction of 1 *Cwt.*

Ans. $\frac{1}{126}$ *Cwt.*

11. Express $5\frac{1}{2}$ furlongs in the fraction of a mile.

Ans. $\frac{11}{16}$ *mile.*

CASE VIII.

To reduce a fraction of one denomination to the fraction of another, but less, retaining the same value.

RULE.

Multiply the given numerator by the parts of the denominations between *it* and *that* denomination you would reduce it to, for a new numerator, which place over the given denominator: *Or*, only invert the parts contained in the integer, and make of them a compound fraction as before, then, reduce it to a simple one.

EXAMPLES.

1. Reduce $\frac{1}{400}$ of a pound to the fraction of a penny.

By comparing it, the fraction will be $\frac{1}{400}$ of $\frac{20}{1}$ of $\frac{12}{1}$, then $\frac{1\times20\times12}{400\times1\times1}=\frac{240}{400}=\frac{3}{5}$ *answer.*

2. Reduce

* 3*s.* 6*d.*=42*d.* and 1£.=240*d.* therefore, $\frac{42}{240}=\frac{7}{40}$£.

† $\frac{4}{5}$£.= $\frac{4}{5}$ of $\frac{20}{1}=\frac{4\times20}{5\times1}=\frac{80}{5}$*s.* and $\frac{80}{5}$ of $\frac{1}{28}=\frac{80\times1}{5\times28}=\frac{80}{140}=\frac{4}{7}$ guinea

2. Reduce $\frac{1}{1280}$ of a pound to the fraction of a farthing.
Anſ. $\frac{3}{4}$ *qr.*

3. Reduce $\frac{5}{2688}$ of a guinea to the fraction of a penny.
Anſ. $\frac{5}{8}$*d.*

4. Reduce $\frac{1}{57}$ of a moidore to the fraction of a ſhilling.
Anſ. $1\frac{2}{9}$*s.*

5. Reduce $\frac{1}{28}$ of a ℔ Avoirdupois to the fraction of an ounce.
Anſ. $\frac{4}{7}$*oz.*

*6. Reduce $\frac{4}{7}$ of a guinea to the fraction of a pound.
Anſ. $\frac{4}{5}$£.

7. Reduce $\frac{7}{1920}$ of a ℔ Troy to the fraction of a *pwt.*
Anſ $\frac{7}{8}$*pwt.*

8. Reduce $\frac{1}{126}$ of *cwt.* to the fraction of a ℔ Avoirdupois.
Anſ. $\frac{8}{9}$℔.

CASE IX.

To find the value of a fraction in the known parts of the integer, as of coin, weight, meaſure, &c.

RULE.†

Multiply the numerator by the parts in the next inferior denomination, and divide the product by the denominator; and if any thing remain, multiply it by the next inferior denomination, and divide by the denominator as before, and ſo on, as far as neceſſary; and the quotients placed after one another, in their order, will be the anſwer required.

EXAMPLES.

1. What is the value of $\frac{5}{7}$ of a pound?

```
      5
     20
    ---
  7)100
    ---
    14s.—2
         12
         --
       7)24
         --
         3d.—3
              4
             --
           7)12
             --
             1 5/7 qr.
```

I multiply 5 by 20, the number of ſhillings in £.1, and the product 100 I divide by the denominator 7, and get the quotient 14*s.* and 2 remaining, I multiply it by 12, and again dividing the product by 7, find the quotient 3*d.* and 3 remains, which I multiply by 4, and dividing as before, the quotient is 1*qr.* and 5 remaining, I place it over the numerator, and find the *anſwer* 14*s.* 3*d.* $1\frac{5}{7}$*qr.*

2. What

* $\frac{4}{7}$ *Guin.* $= \frac{4}{7}$ of $\frac{28}{1} = \frac{4\times 28}{7\times 1} = \frac{112}{7}$*s.* & $\frac{112}{7}$ of $\frac{1}{20} = \frac{112}{140} = \frac{4}{5}$£.

† As the numerator of a fraction may be conſidered as a remainder, and the denominator as a diviſor: This rule therefore has its reaſon in the nature of diviſion.

2. What is the value of $\frac{9}{24}$ of a shilling? *Ans.* $4\frac{1}{2}d.$

3. What is the value of $\frac{17}{29}$ of a *cwt.*? *Ans.* 2*qr.* 9lb 10*oz.* $7\frac{21}{29}dr.$

4. What is the value of $\frac{4}{5}$ of a lb Avoirdupois? *Ans.* 12*oz.* $12\frac{4}{5}dr.$

5. What is the value of $\frac{3}{5}$ of a lb Troy? *Ans.* 7*oz.* 4*pwt.*

6. What is the value of $\frac{3}{13}$ of a ton? *Ans.* 4*cwt.* 2*qrs.* 12lb 14*oz.* $12\frac{4}{13}dr.$

7. What is the value of $\frac{6}{9}$ of a yard. *Ans.* 2*qrs.* $2\frac{2}{3}n.$

8. What is the value of $\frac{7}{8}$ of an ell English? *Ans.* 4*qrs.* $1\frac{1}{2}n.$

9. What is the value of $\frac{5}{6}$ of a mile? *Ans.* 6*fur.* 26*p.* 11*ft.*

10 What is the value of $\frac{9}{13}$ of a day? *Ans.* 16*h.* 36*m.* $55\frac{5}{13}s.$

11. The value of $\frac{12}{17}$ of a Julian year is required? *Ans.* 257*d.* 19*h.* 45*m.* $52\frac{16}{17}s.$

12. The value of $\frac{9}{14}$ of a guinea is demanded? *Ans.* 18*s.*

13. What is the value of $\frac{15}{16}$ of a dollar? *Ans.* 5*s.* $7\frac{1}{2}d.$

14. What is the value of $\frac{3}{5}$ of a moidore? *Ans.* 21*s.* $7\frac{1}{5}d.$

15. What is the value of $\frac{6}{7}$ of an acre? *Ans.* 3*r.* $17\frac{1}{7}p.$

CASE X.

To reduce any given quantity to the fraction of any greater denomination of the same kind.

RULE.*

Reduce the given quantity to the lowest term mentioned, for a numerator; then reduce the integral part to the same term, for a denominator; which will be the fraction required.

EXAMPLES.

1. Reduce 14*s.* $3\frac{1}{4}d.$ $\frac{5}{7}$ to the fraction of a pound.

20 *Integral part.*	*s.* *d.*
12	14 $3\frac{1}{4}$, $\frac{5}{7}$
——	12
240	——
4	171
——	4
960	——
7	685
——	7+5
6720 *denominator.*	——
	4800 *num.*

Ans. $\frac{4800}{6720}=\frac{5}{7}£.$

2. Reduce $4\frac{1}{2}d.$ to the fraction of a shilling. *Ans.* $\frac{3}{8}s.$

3. Reduce

* This case is the reverse of the former, therefore proves it.

Note. If there be a fraction given with the said quantity, it must be further reduced to the denominative parts thereof, adding thereto the numerator.

3. Reduce 2*qrs.* 9lb 10*oz.* $7\frac{13}{29}$*dr.* to the fraction of a *Cwt.*
Ans. $\frac{17}{29}$ *Cwt.*

4. Reduce 12*oz.* $12\frac{4}{5}$*dr.* to the fraction of a lb Avoirdupois.
Ans. $\frac{4}{5}$lb.

5. Reduce 7*oz.* 4*pwt.* to the fraction of a lb Troy.
Ans. $\frac{3}{5}$ lb.

6. Reduce 4*cwt.* 2*qrs.* 12lb 14*oz.* $12\frac{4}{13}$*dr.* to the fraction of a ton. *Ans.* $\frac{3}{13}$ *ton.*

7. Reduce 2*qrs.* $2\frac{2}{3}$*n.* to the fraction of a yard. *Ans.* $\frac{2}{3}$*yd.*

8. Reduce 4*qrs.* $1\frac{1}{2}$*n.* to the fraction of an ell English.
Ans. $\frac{7}{8}$ *E. E.*

9. Reduce 6*fur.* 26*po.* 11*ft.* to the fraction of a mile.
Ans. $\frac{5}{6}$ *mile,*

10. Reduce 16*h.* 36*m.* $55\frac{5}{13}$*s.* to the fraction of a day.
Ans. $\frac{9}{13}$ *day.*

11. Reduce 257*d.* 19*h.* 45*m.* $52\frac{16}{17}$*s.* to the fraction of a Julian year. *Ans.* $\frac{12}{17}$ *J. Y.*

12. Reduce 18*s.* to the fraction of a guinea. *Ans.* $\frac{9}{14}$ *guin.*

13. Reduce 5*s.* $7\frac{1}{2}$*d.* to the fraction of a dollar. *Ans.* $\frac{15}{16}$ *dol.*

14. Reduce 21*s.* $7\frac{1}{5}$*d.* to the fraction of a moidore.
Ans. $\frac{3}{5}$ *moid.*

15. Reduce 3*r.* $17\frac{1}{7}$*p.* to the fraction of an acre.
Ans. $\frac{6}{7}$ *acre.*

ADDITION *of* VULGAR FRACTIONS.

RULE.*

Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different integers to those of the same; and all of them to a common denominator; then the sum of the numerators written over the common denominator will be the sum of the fractions required.

EXAMPLES.

1. Add $7\frac{4}{5}$, $\frac{5}{7}$ of $\frac{3}{8}$ and 7 together.
First. $7\frac{4}{5} = \frac{39}{5}$, $\frac{5}{7}$ of $\frac{3}{8} = \frac{15}{56}$, and $7 = \frac{7}{1}$.
Then the fractions are $\frac{39}{5}$, $\frac{15}{56}$, and $\frac{7}{1}$; *therefore,*

$$39 \times 56 \times 1 = 2184$$
$$15 \times 5 \times 1 = 75$$
$$7 \times 5 \times 56 = 1960$$

$$\frac{4219}{5 \times 56 \times 1 = 280} = 15\frac{19}{280}$$

Or, thus, $\frac{2184+75+1960}{280} = 15\frac{19}{280}$.

2. Add

* Fractions, before they are reduced to a common denominator, are entirely dissimilar, and therefore cannot be incorporated with one another; but when they are reduced to a common denominator, and made parts of the same thing; their sum, or difference, may then be as properly expressed by the sum or difference of the numerators, as the sum or difference of any two quantities whatever, by the sum or difference of their individuals; whence the reason of the rules, both for Addition and Subtraction, is manifest.

2. Add $\frac{3}{7}$, $9\frac{1}{5}$, and $\frac{2}{3}$ of $\frac{1}{2}$ together. *Anſ.* $9\frac{101}{105}$.

3. What is the ſum of $\frac{4}{5}$, $\frac{5}{6}$ of $\frac{3}{8}$ of $\frac{1}{4}$, and $8\frac{4}{13}$? *Anſ.* $9\frac{2319}{12480}$.

4. What is the ſum of $\frac{7}{10}$ of $4\frac{5}{8}$, $\frac{3}{4}$ of $\frac{1}{3}$, and $9\frac{1}{4}$? *Anſ.* $12\frac{59}{80}$.

5. Add $\frac{1}{9}$£. $\frac{3}{7}$*s.* and $\frac{4}{5}$*d.* together. *Anſ.* 2*s.* 8*d.* $\frac{64}{105}$.

6. What is the ſum of $\frac{2}{5}$ of 17£. $9\frac{5}{8}$£. and $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{4}{7}$£.? *Anſ.* £.16 12*s.* $3\frac{5}{7}$*d.*

7. Add $\frac{3}{4}$ of a yard, $\frac{1}{3}$ of a foot, and $\frac{5}{8}$ of a mile together. *Anſ.* 1100*yds.* 2*ft.* 7*inches.*

8. Add $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, $\frac{1}{2}$ of an hour, and $\frac{3}{4}$ of a minute together. *Anſ.* 2 *days,* 2 *hours,* 30 *minutes,* 45 *ſeconds.*

Subtraction *of* Vulgar Fractions.

Rule.*

Prepare the fractions as in Addition, and the difference of the numerators, written above the common denominator, will give the difference of the fractions required. *Note,* a fraction is ſubtracted from a whole number, by taking the numerator of the fraction from its denominator, and placing the remainder over the denominator, then taking one from the whole number.

Examples.

1. From $\frac{3}{4}$ take $\frac{2}{7}$ of $\frac{5}{8}$.

$\frac{2}{7}$ of $\frac{5}{8} = \frac{10}{56} = \frac{5}{28}$. *Then the fractions are* $\frac{3}{4}$ *and* $\frac{5}{28}$.

$$\left.\begin{array}{l} 3 \times 28 = 84 \\ 5 \times 4 = 20 \\ 4 \times 28 = 112 \text{ com. den.} \end{array}\right\} \begin{array}{l} \frac{3}{4} = \frac{84}{112} \textit{ and } \frac{5}{28} = \frac{20}{112}, \textit{ therefore,} \\ \frac{84}{112} - \frac{20}{112} = \frac{64}{112} = \frac{4}{7} \textit{ Remainder.} \end{array}$$

2. From $\frac{49}{50}$ take $\frac{5}{9}$. *Anſ.* $\frac{191}{450}$.
3. From $37\frac{1}{4}$ take $19\frac{4}{7}$. *Anſ.* $17\frac{19}{28}$.
4. From $13\frac{1}{3}$ take $\frac{3}{4}$ of 15. *Anſ.* $2\frac{1}{12}$.
5. From $\frac{1}{4}$£. take $\frac{9}{10}$*s.* *Anſ.* 4*s.* $1\frac{1}{5}$.
6. From $\frac{5}{7}$*oz.* take $\frac{3}{4}$*pwt.* *Anſ.* 13*pwt.* $12\frac{6}{7}$*gr.*
7. From $\frac{1}{2}$ of a league take $\frac{3}{8}$ of a mile. *Anſ.* 1*mi.* 1*fur.*
8. From 5 weeks take $19\frac{4}{5}$ days. *Anſ.* 15*da.* 4*ho.* 48*min.*

Multiplication

* In ſubtracting mixed numbers, when the lower fraction (the ſubtrahend) is greater than the upper one, (the minuend) you may, without reducing them to improper fractions, ſubtract the numerator of the ſubtrahend from the common denominator, and to that difference add the numerator of the minuend, and carry one to the integer of the ſubtrahend.

Example.

From $19\frac{3}{8}$ *take* $12\frac{7}{8}$.——$19\frac{3}{8} - 12\frac{7}{8} = 6\frac{1}{2}$.

MULTIPLICATION *of* VULGAR FRACTIONS.

RULE.*

Reduce compound fractions to simple ones, and mixed numbers to improper fractions; then the product of the numerators will be the numerator, and the product of the denominators, the denominator of the product required. *Note*, where several fractions are to be multiplied, if the numerator of one fraction be equal to the denominator of another, their equal numerators and denominators may be omitted.

EXAMPLES.

1. What is the continued product of $4\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{4}$ of $\frac{7}{8}$, and 6.

$$4\tfrac{1}{3} = \tfrac{13}{3},\ \tfrac{1}{4}\ \textit{of}\ \tfrac{7}{8} = \frac{1 \times 7}{4 \times 8} = \tfrac{7}{32},\ \textit{and}\ 6 = \tfrac{6}{1}.$$

$$\textit{Then}\ \tfrac{13}{3} \times \tfrac{1}{5} \times \tfrac{7}{32} \times \tfrac{6}{1} = \frac{13 \times 1 \times 7 \times 6}{3 \times 5 \times 32 \times 1} = \tfrac{546}{480} = 1\tfrac{11}{80}\ \textit{the answer.}$$

2. Multiply $\frac{4}{17}$ by $\frac{5}{27}$. *Ans.* $\frac{20}{459}$.
3. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$. *Ans.* $\frac{7}{8}$.
4. Multiply $\frac{1}{3}$ of 5 by $\frac{3}{4}$ of $\frac{2}{7}$. *Ans.* $\frac{5}{14}$.
5. Multiply $\frac{3}{7}$ of $\frac{5}{9}$ by $\frac{4}{5}$ of $\frac{1}{5}$ of $11\frac{3}{7}$. *Ans.* $\frac{64}{147}$.
6. Multiply $9\frac{3}{4}$, $\frac{1}{2}$ of $\frac{2}{5}$, and $12\frac{4}{7}$ continually together. *Ans.* $24\frac{18}{35}$.
7. What is the continual product of $\frac{3}{4}$ of $\frac{2}{3}$, $5\frac{1}{2}$, 7 and $\frac{1}{3}$ of $\frac{5}{8}$? *Ans.* $4\frac{1}{96}$.
8. What is the continual product of 7, $\frac{1}{2}$, $\frac{5}{7}$ of $\frac{3}{8}$, and $3\frac{1}{9}$? *Ans.* $2\frac{11}{12}$.

Another method for the Multiplication of mixed Quantities.

Case 1. To multiply a *whole number* by a *fraction*, or a *fraction* by a *whole number*.

Rule. Multiply the whole number by the numerator of the fraction, and divide the product by the denominator: But if the numerator be 1, divide by the denominator only.

	1.	2.	3.	4.	5.	6.	7.
Mult.	8	15	28	36	48	325	259
By	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{7}{12}$
Prod.	2	$7\frac{1}{2}$	$9\frac{1}{3}$	3)72	4)144	8)1625	12)1813
			Prod.	24	36	$203\frac{1}{8}$	$151\frac{1}{12}$

Case 2. To multiply a *whole number* by a *mixed one.*

Rule. Multiply by the fraction as in case 1st; then multiply by the whole number, and add the two products, as in the examples—or,

* Multiplication of a fraction implies the taking some part or parts of the multiplicand, and therefore may truly be expressed by a compound fraction. Thus $\frac{4}{5}$ multiplied by $\frac{3}{8}$ is the same as $\frac{4}{5}$ of $\frac{3}{8}$; and as the directions of the rule agree with the method already given, to reduce these fractions to simple ones, it is shewn to be right.

or, to multiply a mixed number by a whole one, change the place of the factors, and proceed as the rule directs.—*See example* 6.

	1.	2.	3.	4.	5.	6.
Mult.	15	35	68	42	129	$1\frac{7}{15}$
By	$3\frac{1}{2}$	$5\frac{1}{3}$	$7\frac{11}{12}$	$9\frac{3}{7}$	$8\frac{5}{8}$	24
	$7\frac{1}{2}$	$11\frac{2}{3}$	748	126	645	*Mult.* 24
	45	175	$62\frac{4}{12}$	18	$80\frac{5}{8}$	*By* $1\frac{7}{15}$
	$52\frac{1}{2}$	$186\frac{2}{3}$	476	378	1032	15)168
			$538\frac{4}{12}$	396	$1112\frac{5}{8}$	$11\frac{3}{15}$
						24
						Prod. $35\frac{3}{15} = \frac{1}{5}$

Case 3. To multiply a *mixed number* by a *mixed number.*

Rule. Multiply the integral part of the multiplicand by the denominator of its fractional part, and add thereto its numerator: Then multiply by the mixed multiplier, by Case 2d, and divide the product by the denominator of the fractional part of the multiplicand, as in the following example.

Mult. $42\frac{3}{5}$ { *1st.* $42\frac{3}{5} = 213$
By $8\frac{2}{3}$ { *which mult. by* $8\frac{2}{3}$

3)426

142
1704

5)1846

Product $= 369\frac{1}{5}$

After this manner may feet and inches be multiplied, calling 1 inch $\frac{1}{12}$ of a foot, 2 inches $\frac{1}{6}$, 3 inches $\frac{1}{4}$, 4 inches $\frac{1}{3}$, 5 inches $\frac{5}{12}$, 6 inches $\frac{1}{2}$, 7 inches $\frac{7}{12}$, 8 inches $\frac{2}{3}$, 9 inches $\frac{3}{4}$, 10 inches $\frac{5}{6}$, and 11 inches $\frac{11}{12}$ of a foot.

Division *of* Vulgar Fractions.

Rule.*

Prepare the fractions as before: Then, invert the divisor and proceed exactly as in Multiplication: The products will be the quotient required.

Examples.

1. Divide $\frac{1}{3}$ of 17 by $\frac{2}{3}$ of $\frac{6}{8}$.

$\frac{1}{3}$ *of* $17 = \frac{1}{3}$ *of* $\frac{17}{1} = \frac{1 \times 17}{3 \times 1} = \frac{17}{3}$, & $\frac{2}{3}$ *of* $\frac{6}{8} = \frac{12}{24} = \frac{1}{2}$; *therefore,*

$$\frac{17}{3} \div \frac{1}{2} = \frac{17 \times 2}{3 \times 1} = \frac{34}{3} = 11\frac{1}{3} \textit{ the quotient required.}$$

2. Divide $\frac{5}{7}$ by $\frac{3}{5}$. *Ans.* $1\frac{4}{21}$.
3. Divide $12\frac{1}{5}$ by $\frac{1}{3}$ of 7. *Ans.* $5\frac{8}{35}$.

4. Divide

* The reason of the Rule may be shewn thus. Suppose it were required to divide $\frac{4}{5}$ by $\frac{2}{7}$. Now $\frac{4}{5} \div 2$ is manifestly $\frac{1}{2}$ of $\frac{4}{5}$, or $\frac{4}{2 \times 5}$; but $\frac{2}{7} = \frac{1}{7}$ of 2; therefore, $\frac{1}{7}$ of 2, or $\frac{2}{7}$, must be contained 7 times as often in $\frac{4}{5}$ as 2, that is $\frac{4 \times 7}{5 \times 2}$ = the answer, which is according to the rule.

Note. To multiply a fraction by an integer, divide the denominator, or multiply the numerator by it; and to divide by an integer, divide the numerator, or multiply the denominator by it.

4. Divide $5\frac{1}{8}$ by $7\frac{3}{4}$. *Anſ.* $\frac{41}{62}$.
5. Divide $\frac{3}{7}$ by 9. *Anſ.* $\frac{1}{21}$.
6. Divide $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{2}{3}$ by $\frac{1}{8}$ of $\frac{3}{4}$. *Anſ.* $\frac{8}{9}$.
7. Divide 7 by $\frac{3}{8}$. *Anſ.* $18\frac{2}{3}$.
8. Divide $4204\frac{1}{6}$ by $\frac{7}{8}$ of 112. *Anſ.* $42\frac{529}{588}$.

DECIMAL FRACTIONS.

Decimal Fractions are of ſuch a nature, that they vary in the ſame proportion, and are managed by the ſame method of operation, as whole numbers are.

On this account, every proper Fraction is ſuppoſed to be reducible to another, whoſe denominator ſhall be 10, 100, 1000, &c. viz. Unity, with a number of cyphers annexed; and Fractions with ſuch denominators are called *Decimal Fractions*: Such are, $\frac{5}{10}$, $\frac{55}{100}$, $\frac{675}{1000}$, &c.

As the denominator of a decimal fraction is always 10, or 100, or 1000, &c. the denominators need not be expreſſed: For the numerator only may be made to expreſs the true value: For this purpoſe it is only required to write the numerator with a point before it at the left hand, to diſtinguiſh it from a whole number, when it conſiſts of ſo many figures as the denominator hath cyphers annexed to unity, or 1; ſo $\frac{5}{10}$ is written ,5; $\frac{33}{100}$,33; $\frac{735}{1000}$,735, &c.

Note. The point prefixed is called a Separatrix.

But if the numerator has not ſo many places as the denominator has cyphers, put ſo many cyphers before it, viz. at the left hand, as will make up the defect; ſo write $\frac{5}{100}$ thus ,05; and $\frac{6}{1000}$ thus ,006, &c. And thus do theſe fractions receive the form of whole numbers.

The 1ſt, 2d, 3d, 4th, &c. places of decimals, counting from the left hand toward the right, are called primes, ſeconds, thirds, fourths, &c.

We may conſider unity as a fixed point, from whence whole numbers proceed infinitely increaſing toward the left hand, and decimals infinitely decreaſing toward the right hand to 0, as in the following

TABLE.

C Millions	X Millions	Millions	C Thouſands	X Thouſands	Thouſands	Hundreds	Tens	Units		Tenth Parts	Hundredth Parts	Thouſandth Parts	X Thouſandth Parts	C Thouſandth Parts	Millionth Parts	X Millionth Parts	C Millionth Parts
9	8	7	6	5	4	3	2	1	,	2	3	4	5	6	7	8	9

From this table it is evident, that, in decimals, as well as in whole numbers, each figure takes its value by its diſtance from unit's

unit's place: If it be in the firſt place after units (or the ſeparating point) it ſignifies tenths; if in the ſecond, hundredths, &c. decreaſing in each place in a tenfold proportion.

Conſequently, every ſingle figure expreſſing a decimal has for its denominator an unit or 1, with ſo many cyphers as its place is diſtant from unit's place: Thus ,2 in the decimal part of the table $= \frac{2}{10}$; $3 = \frac{3}{100}$; $4 = \frac{4}{1000}$, &c. And if a decimal be expreſſed by ſeveral figures, the denominator is 1, with ſo many cyphers as the loweſt figure is diſtant from unit's place. So ,357 ſignifies $\frac{357}{1000}$, and $,0053 = \frac{53}{10000}$, &c.

Cyphers, placed at the right hand of a decimal fraction, do not alter its value, ſince every ſignificant figure continues to poſſeſs the ſame place: So ,5, ,50 and ,500 are all of the ſame value, and each equal to $\frac{1}{2}$.

But cyphers, placed at the left hand of a decimal, do alter its value, every cypher depreſſing it to $\frac{1}{10}$ of the value it had before, by removing every ſignificant figure one place further from the place of units. So ,5, ,05, ,005, all expreſs different decimals, viz. ,5, $\frac{5}{10}$; ,05, $\frac{5}{100}$; ,005, $\frac{5}{1000}$.

Hence may be obſerved the contrary effects of cyphers being annexed to whole numbers, and decimals.

It is likewiſe evident from the table, that, ſince the places of decimals decreaſe in a tenfold proportion from units downwards, ſo they conſequently increaſe in a tenfold proportion from the right hand toward the left, as the places of whole numbers do: For, ten hundredth parts make one tenth, ten tenths make 1; ten units, ten; ten tens, one hundred, &c. viz. $\frac{10}{100} = \frac{1}{10}$, $\frac{10}{10} = 1$, and $1 \times 10 = 10$, which proves that decimals are ſubject to the ſame law of Notation, and conſequently of operation, as whole numbers are.

Decimal Fractions of unequal denominators are reduced to one common denominator, when there are annexed to the right hand of thoſe, which have fewer places, ſo many cyphers, as make them equal in places with that which has the moſt. So theſe decimals, ,5, ,06, ,455, may be reduced to the decimals ,500, ,060, and ,455, which have, all, 1000 for their denominator.

Of Decimals, that is the greateſt, whoſe higheſt figure is greateſt, whether they conſiſt of an equal or unequal number of places: Thus, ,5 is greater than ,459, for if it be reduced to the ſame denominator with ,459, it will be ,500.

A mixed number, viz. a whole number, with a decimal annexed, is equal to an improper fraction, whoſe numerator is all the figures of the mixed number, taken as one whole number, and the denominator, that of the decimal part. So 45,309 is equal to $\frac{45309}{1000}$, as is evident from the method given to reduce a mixed number to an improper fraction:

Thus, $45 \times 1000 + ,309 = \frac{45309}{1000}$ *as above.*

ADDITION

ADDITION *of* DECIMALS.

RULE.

1. Place the numbers, whether mixed, or pure decimals, under each other, according to the value of their places.

2. Find their ſum as in whole numbers, and point off ſo many places for decimals, as are equal to the greateſt number of decimal places in any of the given numbers.

EXAMPLES.

1. Find the ſum of 19,073+2,3597+223+,0197581+3478,1 +12,358.

```
   19,073
    2,3597
  223
     ,0197581
 3478,1
   12,358
 ------------
 3734,9104581 the sum.
```

2. Required the ſum of 429+21,37+355,003+1,07+1,7? *Anſ.* 808,143.

3. Required the ſum of 5,3+11,973+49+,9+1,7314+34,3? *Anſ.* 103,2044.

4. Required the ſum of 973+19+1,75+93,7164+,9501? *Anſ.* 1088,4165.

SUBTRACTION *of* DECIMALS.

RULE.

Place the numbers according to their value; then ſubtract as in whole numbers and point off the decimals as in Addition.

EXAMPLES.

1. Find the difference of 1793,13 and 817,05693.

From	1793,13
Take	817,05693
Remainder	976,07307

2. From 171,195 take 125,9176. *Anſ.* 45,2774.

3. From 219,1384 take 195,91. *Anſ.* 23,2284.

4. From 480 take 245,0075. *Anſ.* 234,9925.

MULTIPLICATION

MULTIPLICATION *of* DECIMALS.

CASE I.

RULE.

1. Whether they be mixed numbers, or pure decimals, plase the factors and multiply them as in whole numbers.

2. Point off ſo many figures from the product as there are decimal places in both the factors; and if there be not ſo many places in the product, ſupply the defect by prefixing cyphers.

EXAMPLES.

$$\begin{array}{rr}
\text{1. Multiply} & ,02345 \\
\text{by} & ,00163 \\
\hline
 & 7035 \\
 & 14070 \\
 & 2345 \\
\hline
 & ,0000382235 \text{ \textit{the product.}}
\end{array}$$

2. Multiply 25,238 by 12,17. *Anſ.* 307,14646.
3. Multiply ,3759 by ,945. *Anſ.* ,3552255.
4. Multiply ,84179 by ,0385. *Anſ.* ,032408915.

To multiply by 10, 100, 1000, &c. remove the ſeparating point ſo many places to the right hand, as the multiplier has cyphers.

$$\textit{So } ,345 \left\{ \textit{Multip. by} \right\} \left\{ \begin{array}{l} 10 \\ 100 \\ 1000 \end{array} \right\} \textit{makes} \left\{ \begin{array}{l} 3,45 \\ 34,5 \\ 345 \end{array} \right.$$

For ,345 × 10 is 3,450, &c.

CASE II.

To contract the operation, ſo as to retain ſo many decimal places in the Product as may be thought neceſſary.

RULE.

1. Write the unit's place of the multiplier under that figure of the multiplicand, whoſe place you would reſerve in the product; and diſpoſe of the reſt of the figures in a contrary order to what they are uſually placed in.

2. In multiplying, reject all the figures which are to the right hand of the multiplying digit, and ſet down the products, ſo that their right hand figures may fall in a ſtraight line below each other; obſerving to increaſe the firſt figure of every line with what would ariſe by carrying 1 from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, &c. from the preceding figures, when you begin to multiply, and the ſum will be the product required.

EXAMPLES.

Example.

1. It is required to multiply 56,7534916 by 5,376928, and to retain only five places of decimals in the product.

Contracted way.	*Common way.*	
56,7534916	56,7534916	
829673,5	5,376928	
28376746 . .	45	40279328
1702605 . . .	113	5069832
397274	5107	814244
34052	34052	09496
5108	397274	4412
113	1702604	748
45	28376745	80
305,15943	305,15943	80818048

By the operation in the *common way*, it is evident, that all the figures, which are cut off at the right hand by the perpendicular line, are wholly omitted in the *contracted way*, and the last product here is the first there; consequently, the reason of placing the multiplier in a reverse order, must appear very plainly.

Division of Decimals.

Rule.*

1. The places of decimal parts in the divisor and quotient counted together must always be equal to those in the dividend; therefore, divide as in whole numbers, and, from the right hand of the quotient, point off so many places for decimals, as the decimal places in the dividend exceed those in the divisor.

2. If the places of the quotient be not so many as the rule require, supply the defect by prefixing cyphers to the left hand.

3. If at any time there be a remainder, or the decimal places in the divisor be more than those in the dividend, cyphers may be annexed to the dividend, or to the remainder, and the quotient carried on to any degree of exactness.

Examples.

* The reason of pointing off so many decimal places in the quotient, as those in the dividend exceed those in the divisor, will easily appear; for, since the number of decimal places in the dividend is equal to those in the divisor and quotient taken together, by the nature of multiplication: It therefore follows, that the quotient contains so many as the dividend exceeds the divisor.

EXAMPLES.

1.

```
219),117841075(,000538087, &c.
      1095
      ----
       834
       657
       ---
       1771
       1752
       ----
         1907
         1752
         ----
          1555
          1533
          ----
            22
```

2.

```
,3719)38,0000(102,178,&c.
      3719
      ----
       8100
       7438
       ----
        6620
        3719
        ----
        29010
        26033
        -----
         29770
         29752
         -----
            18
```

In Example 1st, the divisor having no decimals, the quotient must have so many as there are in the dividend. In Example 2, the dividend being an integer must have at least so many cyphers annexed, as there are decimals in the divisor, and so far the quotient will be whole numbers; then annexing more cyphers, the remaining figures in the quotient will be decimals, according to the Rule.

3d. 133)5737(43.1353+ (4th.) 23.7)65321(2756.16+
5th. *72)918,217(12753+ (6th.) 25.17)315.6293(1 53+
7th. ,317)29,417(92+ (8th.) 37 9),0059374(15 +
9th. ,375),173948375(463862+

Having a multiplier, to find a divisor which shall give a quotient equal to the product by that multiplier.

RULE.

Divide unity by the given multiplier, and the quotient will be the divisor sought.

What divisor is that, by which dividing 5357, shall give a quotient equal to the product of the same number multiplied by 250?

250)1,000(,004 *the answer.* And ,004)5357,000(1339250.
Proof. 5357 × 250 = 1339250.

Having a divisor, to find a multiplier which shall give a product equal to the quotient by that divisor.

RULE.

Divide unity by the given divisor, and the quotient will be the multiplier sought.

What multiplier is that, by which multiplying 5357, shall give a product equal to the quotient of the same number divided by ,004?

.004)1,000(250 *the answer: Therefore,* 5357 × 250 = 5357 ÷ ,004 = 1339250.

CASE II.

To contract Division, when there are many decimals in the dividend, and the divisor is large.

RULE.

* The following questions are left unpointed in the quotient, to exercise the learner.

RULE.

1. Whatever place of the dividend correſponds with the unit's place of the diviſor, at the firſt ſtep of the Diviſion, the ſame place muſt the firſt figure of the quotient have.

2. In dividing, reject the laſt right hand figure of the diviſor, at every ſtep (inſtead of bringing down a figure, as in common) and make the laſt remainder the dividend for the new diviſor at every ſtep: Thus continue the diviſion until the diviſor ſhall be exhauſted.

```
99,5678)4,6789837568(,0469931 quotient.
         3 982712
         --------
 99,567)  696271
          597402
          ------
  99,56)   98869
           89604
           -----
   99,5)    9265
            8955
            ----
     99)     310
             297
             ---
      9)      13
               9
               -
Remainder      4
```

Here, the unit's place of the diviſor in the firſt ſtep falls under 7 in the place of hundredths in the dividend; therefore, I put 4, the firſt quotient figure, in the place of *hundredths*, by prefixing a cypher.

I have ſet down every diviſor to explain the work; but you need only put a daſh over every figure rejected, as you proceed, to ſhew it is omitted.

When decimals or whole numbers are to be divided by 10, 100, 1000, &c. (viz. unity with cyphers,) it is performed by removing the ſeparatrix, in the dividend, ſo many places toward the left hand as there are cyphers in the diviſor.

EXAMPLES.

10	Dividing	7654	The Quot. is 765,4
100			76,54
1000			7,654
10000			,7654

REDUCTION *of* DECIMALS.

CASE I.

To reduce a Vulgar Fraction to its equivalent Decimal.

RULE.*

Divide the numerator by the denominator, as in diviſion of decimals, and the quotient will be the decimal required: Or, ſo many

* Let the given vulgar fraction, whoſe decimal expreſſion is required, be $\frac{9}{15}$. Now, ſince every decimal fraction has 10, 100, 1000, &c. for its denominator; and if two fractions be equal, it will be, as the denominator of one is to its numerator; ſo is the denominator of the other to its numerator, therefore, as $15 : 9 :: 10$, &c. $: \frac{9 \times 10}{15} = \frac{90}{15} = ,6$ the numerator of the decimal required; and is the ſame as by the rule.

many cyphers as you annex to the given numerator, ſo many places muſt be pointed off in the quotient, and if there be not ſo many places of figures in the quotient, the deficiency muſt be ſupplied by prefixing ſo many cyphers before the quotient figures.

EXAMPLES.

1. Reduce $\frac{1}{8}$ to a decimal. 8)1,000
,125 *Anſwer.*

2. Reduce $\frac{3}{8}$, $\frac{5}{8}$ and $\frac{2}{3}$ to decimals. *Anſwers.* ,375. ,625. ,666+.

3. Reduce $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{7}{8}$ to decimals.
Anſwers. ,25. ,5. ,75. ,333+. ,8. ,833+. ,875.

4. Reduce $\frac{5}{19}$, $\frac{27}{39}$, $\frac{12}{480}$ and $\frac{9}{36}$ to decimals.
Anſwers. ,263+. ,692+. ,025. ,25.

5. Reduce $\frac{7}{375}$, $\frac{9}{1129}$ and $\frac{5}{1875}$ to decimals.
Anſwers. ,0186+. ,00797+. ,00266+.

CASE II.

To reduce numbers of different denominations, as of Money, Weight and Meaſure, to their equivalent decimal values.

RULE.*

1. Write the given numbers perpendicularly under each other for dividends, proceeding orderly from the leaſt to the greateſt.

2. Oppoſite to each dividend, on the left hand, place ſuch a number for a diviſor as will bring it to the next ſuperior denomination, and draw a line perpendicularly between them.

3. Begin with the higheſt, and write the quotient of each diviſion, as decimal parts, on the right hand of the dividend next below it, and ſo on, till they are all uſed, and the laſt quotient will be the decimal ſought.

1. Reduce 17*s.* 8$\frac{3}{4}$*d.* to the decimal of a pound.

4	3,
12	8,75
20	17,729166 &c.
	,886458 &c. the decimal required.

Here, in dividing 3 by 4, I ſuppoſe 2 cyphers to be annexed to the 3, which make it 3,00, and ,75 is the quotient, which I write againſt 8 in the next line; this quotient, viz. 8,75 being pence and decimal parts of a penny, I divide them by 12, which brings them to ſhillings and decimal parts, I therefore divide by 20, and (there being no whole number) the quotient is decimal parts of a pound.

2. Reduce

* The reaſon of the rule may be explained from the firſt example: Thus, three farthings are $\frac{3}{4}$ of a penny, which, reduced to a decimal, is ,75; conſequently, 8$\frac{3}{4}$*d.* may be expreſſed, 8,75*d.* but 8,75 is $\frac{875}{100}$ of a penny $=\frac{875}{1200}$ of a ſhilling, which, reduced to a decimal, is ,729166*s.*+. In like manner, 17,729166*s.*+ are $\frac{17729166}{100000}$ = ,886458+ as by the rule.

2. Reduce 1, 2, 3, 4, and ſo on to 19 ſhillings, to decimals.

Shillings.	1	2	3	4	5	6	7	8	9	10
Anſwers.	,05.	,1.	,15.	,2.	,25.	,3.	,35.	,4.	,45.	,5.
Shil.	11	12	13	14	15	16	17	18	19	
Anſ.	,55.	,6.	,65.	,7.	,75.	,8.	,85.	,9.	,95.	

Here, when the ſhillings are even, half the number, with a point prefixed, is their decimal expreſſion ; but, if the number be odd, annex a cypher to the ſhillings, and then halving them, you will have their decimal expreſſion.

* 3. Reduce 1, 2, 3, and ſo on to 11 pence, to the decimals of a ſhilling.

Pence.	1	2	3	4	5	6
Anſwers.	,083+.	,166.	,25.	,333+.	,416+.	,5.
Pence.	7	8	9	10	11	
Anſwers.	,583+.	,666+.	,75.	,833+.	,916+.	

4. Reduce 1, 2, 3, &c. to 11 pence, to the decimals of a pound.

Pence.	1	2	3	4	5	
Anſwers.	,00416+.	,0083+.	,0125.	,01666+.	,0208+.	
Pence.	6	7	8	9	10	11
Anſ.	,025.	,02916+.	,0333+.	,0375.	,0416+.	,04583+.

5. Reduce 1, 2 and 3 farthings to the decimals of a penny.

1qr.=,25*d.* *2qrs.*=,5*d.* *and 3qrs.*=,75*d.* *Anſwers.*

6. Reduce 1, 2 and 3 farthings to the decimals of a ſhilling.

Anſ. *1qr.* =,02083+*s.* *2qrs.*=,04166+*s.* *3qrs.*=,0625*s.*

7. Reduce 1, 2 and 3 farthings to the decimals of a pound.

Anſ. *1qr.*=,0010416+£. *2qrs.*=,002083+£. *3qrs* =,003125£.

8. Reduce 13*s.* 5½*d.* to the decimal of a pound.

Anſ. ,6729+.

9. Reduce 7*cwt.* 3*qr.* 17lb 10*oz.* 12*dr.* to the decimal of a ton.

Anſ. ,39538+.

10. Reduce 10*oz.* 13*pwt.* 9*gr.* to the decimal of a pound Troy.

Anſ. ,8890625.

11. Reduce 3*qrs.* 3*n.* to the decimal of a yard.

Anſ. ,9375.

12. Reduce 5*fur.* 12*po.* to the decimal of a mile.

Anſ. ,6625.

13. Reduce 55*m.* 37*ſec.* to the decimal of a day.

Anſ. ,03862+.

CASE III.

To find the decimal of any number of ſhillings, pence and farthings, by Inſpection.

RULE.

* The anſwers to this queſtion are the ſame as the decimal parts of a foot.

RULE.*

1. Write *half* the greatest even number of shillings for the first decimal figure.

2. Let the farthings in the given pence and farthings possess the second and third places; observing to increase the *second* place, or place of *hundredths*, by 5, if the shillings be odd, and the third place by 1, when the farthings exceed 12, and by 2, when they exceed 36.

EXAMPLES.

1. Find the decimal of 13*s*. $9\frac{3}{4}$*d*. by Inspection.

	,6 . .	$= \frac{1}{2}$ *of* 12*s*.
	5	*for the odd shilling.*
	39	= *the farthings in* $9\frac{3}{4}$*d*.
Add	2	*for the excess of* 36.
	,691	= *decimal required.*

2. Find, by Inspection, the decimal expressions of 18*s*. $3\frac{1}{4}$*d*. and 17*s*. $8\frac{1}{2}$*d*. *Ans*. £ . ,914 *and* £ . ,885.

3. Value the following sums, by Inspection, and find their total, viz. 15*s*. 3*d*. +8*s*. $11\frac{1}{2}$*d*. +10*s*. $6\frac{1}{4}$*d*. +1*s*, $8\frac{1}{2}$*d*. +$\frac{1}{2}$*d*. +$2\frac{1}{4}$*d*. *Ans*. £ .1,834 *the total.*

CASE IV.

To find the value of any given decimal in the terms of the Integer.

RULE.

1. Multiply the decimal by the number of parts in the next less denomination, and cut off so many places for a remainder, to the right hand, as there are places in the given decimal.

2. Multiply the remainder by the next inferior denomination, and cut off a remainder as before.

3. Proceed

* The invention of the rule is as follows: As shillings are so many 20ths of a pound, half of them must be so many tenths, and consequently take the place of tenths in the decimal; but when they are odd, their half will always consist of two figures, the first of which will be half the even number, next less, and the second a 5: Again, farthings are so many 960ths of a pound, and had it happened that 1000, instead of 960, had made a pound, it is plain any number of farthings would have made so many thousandths, and might have taken their place in the decimal accordingly. But 960 increased by $\frac{1}{24}$ part of itself, is = 1000, consequently, any number of farthings, increased by their $\frac{1}{24}$ part, will be an exact decimal expression for them: Whence, if the number of farthings be more than 12, $\frac{1}{24}$ part is greater than $\frac{1}{2}$*qr*. and, therefore, 1 must be added; and when the number of farthings is more than 36, $\frac{1}{24}$ part is greater than $1\frac{1}{2}$*qr*, for which 2 must be added.

3 Proceed in this manner through all the parts of the integer, and the ſeveral denominations, ſtanding on the left hand, make the anſwer.

EXAMPLES.

1. Find the value of ,73968 of a pound.

$$\begin{array}{r} 20 \\ \hline 14{,}79360 \\ 12 \\ \hline 9{,}52320 \\ 4 \\ \hline 2{,}09280 \end{array}$$ *Anſ.* 14s. $9\frac{1}{2}$d.

2. What is the value of ,679 of a ſhilling? *Anſ.* 8,148.
3. What is the value of ,9999£.? *Anſ.* 19s. $11\frac{3}{4}$d. $\frac{9}{10}$—or £.1.
4. What is the value of ,617 of a *Cwt.*? *Anſ.* 2qrs. 13℔ 1oz. $10\frac{6}{10}$dr.
5. What is the value of ,8593 of a ℔ Troy? *Anſ.* 10oz. 6pwt. 5gr.
6. What is the value of ,397 of a yard? *Anſ.* 1qr. 2n.
7. What is the value of ,8469 of a degree? *Anſ.* 58m. 6fur. 35po. 0ft. 11in.
8. What is the value of ,569 of a year? *Anſ.* 207da. 16h. 26m. 24ſec.
9. What is the value of ,713 of a day? *Anſ.* 17h. 6m. 43ſec.

CASE V.

To find the value of any decimal of a pound by Inſpection.

RULE.

Double the firſt figures or place of tenths, for ſhillings, and if the ſecond figure be 5, or more than 5. reckon another ſhilling; then, after the 5 is deducted, call the figures in the ſecond and third places ſo many farthings, abating 1 when they are above 12, and 2 when above 36, and the reſult will be the anſwer.

Note. When the Decimal has but 2 figures, if any thing remain after the ſhillings are taken out, a cypher muſt be annexed to the right hand, or ſuppoſed to be ſo.

EXAMPLES.

1. Find the value of ,876£. by Inſpection.

16s. = *double of* 8.
1s. *for the* 5 *in the ſecond place, which is to be taken out of* 7.
And $6\frac{1}{2}$d. = 26 *farthings remain, to be added.*
Deduct $\frac{1}{4}$d. *for the exceſs of* 12.

17s. $6\frac{1}{4}$d. *the anſwer.*

2. Find

2. Find, by Inspection, the value of ,495£.

8s. - = *double of* 4.

1s. - *for the* 5 *in the place of hundredths.*

10d. = 40 *farthings, a* 0 *being annexed to the remaining* 4.

Deduct $\frac{1}{2}$*d. for the excess of* 36.

———

9s. 9$\frac{1}{2}$*d. the answer.*

3. Find the value of ,097£. by Inspection. *Ans.* 1s. 11$\frac{1}{4}$*d.*

4. Value the following decimals by Inspection, and find their sum, *viz.* ,785£. + ,537£. + ,916£. + ,74£. + ,5£. + ,25£. + ,09£. + ,008£. *Ans.* £.3 16s. 6d.

FEDERAL MONEY.

The pupil being well acquainted with Decimals, it will be proper to introduce here an account of the Federal Money, as settled by Congress, the 8th of August, 1786, when it was "*Resolved,*

"That the standard of the United States of America, for gold and silver, shall be eleven parts fine and one part alloy.

"That the *Money Unit* of the United States (being by the Resolve of Congress, of the 6th of July 1785, a Dollar) shall contain, of fine silver, 375$\frac{64}{100}$ grains.

"That the money of account, to correspond with the division of coins, agreeably to the above Resolve, proceed in a decimal ratio, agreeably to the forms and manner following, *viz.*

"Mill, the lowest money of account, of which 1000 shall be equal to the federal dollar, or money unit, - 0,001.

"Cent, the highest copper piece, of which 100 shall be equal to the federal dollar, - - - - - 0,010.

"Dime, the lowest silver coin, of which 10 shall be equal to the dollar, - - - - - - - 0,100.

"Dollar, the highest silver coin, - - - - - 1,000

"That, betwixt the dollar and the lowest copper coin, as fixed by the Resolve of Congress of the 6th of July, 1785, there shall be *three* silver coins, and *one* copper coin.

"That the silver coins shall be as follow: One coin containing 187$\frac{82}{100}$ grains of fine silver, to be called a *Half Dollar*: One coin containing 75$\frac{128}{1000}$ grains of fine silver, to be called a *Double Dime*: And one coin containing 37$\frac{64}{1000}$ grains of fine silver, to be called a *Dime.*

"That the two copper coins shall be as follow: One equal to the one hundredth part of the federal dollar, to be called a *Cent*; and one equal to the two hundredth part of the federal dollar, to be called a *Half Cent.*

"That 2$\frac{1}{4}$ ℔ Avoirdupois weight of copper, shall constitute 100 Cents.

"That

"That there ſhall be two gold coins: One containing $246\frac{268}{1000}$ grains of fine gold, equal to 10 dollars, to be ſtamped with the impreſſion of the American Eagle, and to be called an *Eagle*: One containing $123\frac{134}{1000}$ grains of fine gold, equal to 5 dollars, to be ſtamped in like manner, and to be called a *Half Eagle*.

"That the mint price of a pound Troy weight, of uncoined ſilver, eleven parts fine and one part alloy, ſhall be 9 dollars, 9 dimes and 2 cents.

"That the mint price of one pound Troy weight of uncoined gold, eleven parts fine and one part alloy, ſhall be 209 dollars, 7 dimes and 7 cents."

As the money of account proceeds in a decuple, or tenfold proportion, ſo any number of dollars, dimes, cents and mills, is ſimply the expreſſion of dollars and decimal parts of a dollar: Thus 9 dollars and 8 dimes are expreſſed $9,8=9\frac{8}{10}$*doll.*—12 dollars, 4 dimes and 7 cents thus, $12,47=12\frac{47}{100}$*doll.*—20 dollars, 3 dimes, 4 cents and 5 mills, thus $20,345=20\frac{345}{1000}$*doll.*—100 dollars and 9 mills, thus $100,009=100\frac{9}{1000}$*doll.* and 50 dollars, 5 cents, thus, $50,05=50\frac{5}{100}$*doll.* wherefore, it is, in all reſpects, added, ſubtracted, multiplied and divided, the ſame as decimals; and, of all coins, it is the moſt ſimple.

				Marked.	*Mills.*	*Cent.*	*Dime.*	*Dol.*	*Eag.*
Note.	10 Mills,	*make one*	Cent.	*m. c.*	10 =	1			
	10 Cents,		Dime.	*d.*	100 =	10 =	1		
	10 Dimes,		Dollar.	*D.*	1000 =	100 =	10 =	1	
	10 Dollars,		Eagle.	*E.*	10000 =	1000 =	100 =	10 =	1.

ADDITION *of the* FEDERAL MONEY.

Add $25\frac{1}{2}$ eagles; 7 dollars, 8 dimes, 3 cents, 4 mills; 125 dollars, 8 cents; 5 eagles, 9 mills; 18 dollars, 7 cents and 4 mills together.*

	Eagles. Dollars. Dimes. Cents. Mills.
1ſt.	255
2d.	7,834
3d.	125,080
4th.	50,009
5th.	18,074
Sum.	455,997

Note. That the dollars occupy the firſt place at the left hand of the comma, and eagles, all the places at the left of dollars: But eagles and dollars, reckoned together, expreſs the number of dollars contained in the ſum, as 349 is 34 eagles and 9 dollars; equal to 349 dollars, &c.

SUBTRACTION.

* It may be obſerved that the ſum exhibits the particular number of each different piece of money contained in it, viz. 455997 mills $=45599\frac{7}{10}$ cents $=4559\frac{97}{100}$ dimes $=455\frac{997}{1000}$ dollars $=45\frac{5997}{10000}$ eagles $=$ 4 5 5, 9 9 7.
E. D. d. c. m.

Alſo,

SUBTRACTION.

	E. D. d. c. m.	D. d. c. m.	D. d. c. m.
From	13479, 8 1 5	495641, 0 0 1	798,
Take	8985, 9 4 6	218796, 7 9 5	459, 3 7 9
Rem.	4493, 8 6 9	276844, 2 0 6	338, 6 2 1
Proof.	13479, 8 1 5		

MULTIPLICATION.

When you have pointed off the decimals in the product, according to the rule in Multiplication of Decimals, all beyond the mills, or third place of decimals, are decimal parts of a mill.

Bought 37 horfes for fhipping, at 48, 5 7 3 (D. d. c. m.) per head: What came they to?

```
 D. d. c. m.
 48,5 7 3
      3 7
 --------
 340011
145719
 --------
1797,201
 D. d. c. m.
```

```
        D. d. c. m.
Mult.   45,8 4 6
by      23,0 9 4
        --------
         183384
        412614
      1375380
      91692
      ----------
Anf. 1058,767 524/1000
```

DIVISION.

1. If 1000 oranges coft 10 dollars, what is that apiece?

```
     D. d. c. D. d. c.
1000)10,0 0(0, 0 1 Anf.*
     10 0 0
```

2. If 3730 bufhels of corn coft 2025,39 dollars: What is that per bufhel?

```
             D.d. c. m.
3730)2025,39(0,5 4 3 Anf.
     18650
     -----
      16039
      14920
      -----
       11190
       11190
```

3. Divide 12976 dollars between 7 men.

```
7)12976
  --------
  1853,714 2/7 Anf. in dol. dim. &c.
```


Alfo, the names of the coins, lefs than a dollar, are fignificant of their values. For the *mill*, which ftands in the 3d place at the right hand of the comma, or place of thoufandths, is contracted from *mille*, the Latin for *thoufand*: *Cent*, which occupies the fecond place, or place of hundredths, is an abbreviation of *centum*, the Latin for *hundred*: And *dime*, which is in the firft place, or place of tenths, is derived from *difme*, the French for *tenths*.

* And here I would remind the learner, that when he has brought down all his whole numbers, or dollars, in the dividend, he muft place a comma in the quotient; and if, when he has brought down the next figure, he cannot have the divifor once, he muft place a cypher at the right hand of the comma, in the place of dimes.

DECIMAL TABLES OF COIN, WEIGHT *and* MEASURE.

TABLE I. COIN.

£.1 the Integer.

Shil.	dec.	Shil.	dec.
19	,95	9	,45
18	,9	8	,4
17	,85	7	,35
16	,8	6	,3
15	,75	5	,25
14	,7	4	,2
13	,65	3	,15
12	,6	2	,1
11	,55	1	,05
10	,5		

Pence.	Decimals.
11	,045833
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Farthings	Decimals
3	,003125
2	,0020833
1	,0010416

TABLE II.

COIN & Long Meas. 1 Shil. & 1 Foot the Integer.

Pence & Inches.	Decimals.
11	,916666
10	,833333
9	,75
8	,666666
7	,583333
6	,5
5	,416666
4	,333333
3	,25
2	,166666
1	,083333

Farthings	Decimals
3	,0625
2	,041666
1	,020833

TABLE III.

TROY WEIGHT. 1 lb the Integer. Ounces the same as TABLE II.

Pwts.	Decimals.
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Grains.	Decimals.
12	,002083
11	,00191
10	,001736
9	,001562
8	,001389
7	,001215
6	,001042
5	,000868
4	,000694
3	,000521
2	,000347
1	,000173

1 Oz. the Integer. Pennyweights the same as Shillings in the first Table.

Grains.	Decimals.
12	,025
11	,022916
10	,020833
9	,01875
8	,016665
7	,014583
6	,0125
5	,010416
4	,008333
3	,00625
2	,004166
1	,002083

TABLE IV.

AVOIRDUPOIS Wt. 112 lb the Integer.

Qrs.	Decimals.
3	,75
2	,5
1	,25

Pounds.	Decimals.
27	,241071
26	,232143
25	,223214
24	,214286
23	,205357
22	,196428
21	,1875
20	,178571
19	,169643
18	,160714
17	,151786
16	,142857
15	,133928
14	,125
13	,110671
12	,107143
11	,098214
10	,089285
9	,080357
8	,071428
7	,0625
6	,053571
5	,044643
4	,035714
3	,026786
2	,017857
1	,008928

Ounces.	Decimals.
15	,008370
14	,007812
13	,007254
12	,006696
11	,006138
10	,00558
9	,005022
8	,004464
7	,003906
6	,003348
5	,00279
4	,002232
3	,001674
2	,001116
1	,000558

qrs. of ozs.	Decim.
3	,000418
2	,000279
1	,000139

TABLE V.

AVOIRD. WEIGHT. 1 lb the Integer.

Ounces.	Decimals.
15	·9375
14	,875
13	,8125
12	,75
11	,6875
10	,625
9	,5625
8	,5
7	,4375
6	,375
5	,3125
4	,25
3	,1875
2	,125
1	,0625

Drams.	Decimals.
15	,059493
14	,055587
13	,051681
12	,047775
11	,043868
10	,039962
9	,036056
8	,03215
7	,027343
6	,023437
5	,019531
4	,015625
3	,011718
2	,007812
1	,003906

TABLE VI.

CLOTH MEASURE. 1 Yard the Integer.

Quarters.	Decimals.
3	,75
2	,5
1	,25

Nails.	Decimals.
3	,1875
2	,125
1	,0625

TABLE VII.

LONG MEASURE. 1 Mile the Integer.

Yards.	Decimals.
1000	,568182
900	,511364
800	,451545
700	,397727
600	,34
500	,284091
400	,227272
300	,170454
200	,113636
100	,056818
90	,051136
80	,045454
70	,039773
60	,034091
50	,028409
40	,022727
30	,017045
20	,011364
10	,005682
9	,005114
8	,004545
7	,003977
6	,003409
5	,002841
4	,002273
3	,001704
2	,001136
1	,000568

Feet.	Decimals.
2	,0003787
1	,0001892

Inches.	Decimals.
6	,0000947
5	,000079
4	,0000632
3	,0000474
2	,0000316
1	,0000158

TABLE VIII.
LIQUID MEAS.
1 Gallon the Integer.
Quarts the same as qrs. in TABLE VI.

1 Pint.	,125
3 Gills.	,09375
2	,0625
1	,03125

TABLE IX.
TIME.
1 Year the Integer.
Months the same as Pence in TABLE II.

Days.	Decimals.
365	1.000000
300	,821928
200	,547945
100	,273973

Days.	Decimals.
90	,246575
80	,219178
70	,191781
60	,164383
50	,136986
40	,109589
30	,082192
20	,054794
10	,027397
9	,024657
8	,021918
7	,019178
6	,016438
5	,013698
4	,010959
3	,008219
2	,005479
1	,002739

1 Day the Integer.

Hours.	Decimals.
23	,958333
22	,916666
21	,875
20	,833333
19	,791666
18	,75
17	,708333
16	,666666
15	,625
14	,583333
13	,541666
12	,5
11	,458333
10	,416666
9	,375
8	,333333
7	,291666
6	,25

Hours.	Decimals.
5	,208333
4	,166666
3	,125
2	,083333
1	,041666

Minutes.	Decimals.
30	,020833
20	,013888
10	,006944
9	,00625
8	,005555
7	,004861
6	,004166
5	,003472
4	,002777
3	,002083
2	,001388
1	,000694

COMPOUND MULTIPLICATION*

Is extremely useful in finding the value of Goods, &c. And as in Compound Addition we carry from the lowest denomination to the next higher, so we begin and carry in Compound Multiplication: One general rule being to multiply the price by the quantity.

CASE I.

When the quantity does not exceed 12 *yards, pounds, &c.* Set down the price of 1, and place the quantity underneath the least denomination, for the multiplier, and, in multiplying by it, observe the same rules for carrying from one denomination to another as in Compound Addition.

INTRODUCTORY EXAMPLES.

	1.			2.			3.			4.		
	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
Multiply	15	17	1	25	12	8	45	9	10	67	18	$6\frac{1}{2}$
by			2			3			4			5

	5.			6.			7.			8.	
19	6	$8\frac{1}{4}$	13	12	11	31	16	$8\frac{1}{4}$	12	17	$10\frac{1}{2}$
		6			7			8			9

9.

* The product of a number, consisting of several parts or denominations, by any simple number whatever, will be expressed by taking the product of that simple number, and each part by itself, as so many distinct questions: Thus £.33 15*s.* 9*d.* multiplied by 5, will be £.165 75*s.* 45*d.* = (by taking the shillings from the pence, and the pounds from the shillings, and placing them in the shillings and pounds respectively,) £.168 18*s.* 9*d.* and this will be true when the multiplicand is any compound number whatever.

9.			10.			11.			12.		
£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
4	13	$4\frac{3}{4}$	8	15	$11\frac{3}{4}$	6	19	$1\frac{1}{2}$	14	17	$18\frac{1}{4}$
		10			11			12			9
									133	19	$2\frac{1}{4}$

In the laſt example, I ſay, 9 times 1 is 9 farthings = $2\frac{1}{4}d$. I ſet down $\frac{1}{4}$ and carry 2, ſaying, 9 times 8 is 72, and 2 I carry makes 74 pence = 6*s*. 2*d*. I ſet down 2 in the pence, and carry 6; then, 9 times 7 (the unit figure in the ſhillings) is 63, and 6 I carry is 69, I ſet down 9 under the unit figure of the ſhillings, and carry 6, ſaying, 9 times 1 is 9, and 6 I carry is 15, then I halve 15, which is 7 and 1 over, which I ſet in the ten's place in ſhillings, and that, with the 9, makes 19 ſhillings; then I carry the 7 as pounds: Laſtly, 9 times 4 is 36, and 7 I carry, are 43 pounds; I ſet down 3 and carry 4, ſaying, 9 times 1 is 9, and 4 I carry makes 13, which I ſet down, and the product is £.133 19*s*. $2\frac{1}{4}d$.

PRACTICAL QUESTIONS.

1. What will 9 yards of cloth, at 5*s*. 4*d*. per yard, come to?

	£.	s.	d.	
	0	5	4	*price of one yard.*
Multiplied by			9	*yards.*
Anſ.	£.2	8	0	*price of 9 yards.*

Queſtions.		s.	d.	
2d.	3 yards, at	15	4	*per yard.*
3d.	6 —— at	9	10	
4th.	5 —— at	29	6	
5th.	9 —— at	13	$7\frac{1}{2}$	
6th.	7 —— at	39	$10\frac{1}{4}$	

CASE II.

When the multiplier, that is, the quantity, is above 12, you muſt multiply by two ſuch numbers, as, when multiplied together, will produce the given quantity.

EXAMPLES.

1. What will 144 yards of cloth coſt, at 3*s*. $5\frac{1}{2}d$. per yard?

	£.	s.	d.	
	0	3	$5\frac{1}{2}$	*price of one yard.*
Multiplied by			12	
Produces	2	1	6	*price of* 12 *yards.*
Multiplied by			12	
Anſwer,	£.24	18	0	*price of* 144 *yards.*

Queſtions.

Questions.						*Answers.*		
			s.	*d.*		*£.*	*s.*	*d.*
2d.	24 yards	at	6	$3\frac{3}{4}$	per yard. =	7	11	6
3d.	27 ——	at	9	10	—— =	13	5	6
4th.	44 ——	at	12	$4\frac{1}{2}$	—— =	27	4	6
5th.	55 ——	at	8	$3\frac{1}{4}$	—— =	22	14	$10\frac{1}{4}$
6th.	72 ——	at	19	11	—— =	71	14	0

CASE III.

When the quantity is ſuch a number, as that no two numbers in the table will produce it, exactly: Then multiply by two ſuch numbers as come the neareſt to it; and for the number wanting, multiply the given price of one yard by the ſaid number of yards wanting, and add the products together for the anſwer; but if the product of the two numbers exceed the given quantity, then find the value of the overplus, which ſubtract from the laſt product, and the remainder will be the anſwer.

EXAMPLES.

1. What will 47 yards of cloth, at 17*s.* 9*d.* per yard, come to?

	£.	*s.*	*d.*	
	0	17	9	*price of* 1 *yard.*
Multiplied by			5	
Produces	4	8	9	*price of* 5 *yards.*
Multiplied by			9	
Produces	39	18	9	*price of* 45 *yards.*
Add	1	15	6	*price of* 2 *yards.*
Anſ. £.41		14	3	*price of* 47 *yards.*

Note. This may be performed by firſt finding the value of 48 yards, from which if you ſubtract the price of 1, the remainder will be the anſwer as above.

Questions.						*Answers.*		
	Yds.		*s.*	*d.*		*£.*	*s.*	*d.*
2d.	75	at	5	$7\frac{1}{2}$	per yard. =	21	1	$10\frac{1}{2}$
3d.	$67\frac{1}{2}$	—	16	$3\frac{1}{4}$	—— =	54	18	$3\frac{1}{4}$
4th.	59	—	9	7	—— =	28	5	5
5th.	$135\frac{1}{4}$	—	43	4	—— =	293	0	10
6th.	$112\frac{1}{4}$	—	15	$11\frac{3}{4}$	—— =	90	1	$7\frac{3}{4}$

CASE IV.

When the quantity is any number above the Multiplication Table: Multiply the price of 1 yard by 10, which will produce the price of 10 yards: *This* product, multiplied by 10, will give the price of 100 yards; then, if the quantity do not exceed hundreds, you muſt multiply the price of one hundred by the number of hundreds

dreds in your queſtion; the price of ten by the number of tens; and the price of unity, or 1, by the number of units: Laſtly, add theſe ſeveral products together, and the ſum will be the anſwer.

E X A M P L E S.

1. What will 359 yards of cloth, at 4*s.* 7½*d.* per yard, amount to?

	£.	s.	d.	
	0	4	7½	*price of one yard.*
			10	
	2	6	3	*price of 10 yards.*
			10	
	23	2	6	*price of 100 yards.*
			3	
	69	7	6	*price of 300 yards.*
5 times the price of 10 yds. =	11	11	3	*price of 50 yards.*
9 times the price of 1 yd. =	2	1	7½	*price of 9 yards.*
Anſwer £.	83	0	4½	*price of 359 yards.*

Queſtions.		s.	d.			£.	s.	d.
2d.	297 yards, at	17	3½	per yard.	=	256	15	7½
3d.	473 ———	9	11¼	———	=	235	0	5¼
4th.	512 ———	15	10	———	=	405	6	8
5th.	624 ———	12	8	———	=	395	4	0
6th.	765 ———	19	9½	———	=	757	0	7½

C A S E V.

When the quantity does not exceed 200, nor the price 12 pence, then, by the pence table, find what it comes to, at one penny per yard, &c. and, multiplying this ſum by the number of pence in the price, the product will be the anſwer.

E X A M P L E S.

1. What will 129 yards coſt, at 9¾*d.* per yard?

	£.	s.	d.	
129 *pence* =	0	10	9	*the price at 1d. per yd.*
			9	
	4	16	9	*the price at 9d. per yd.*
Half of 10*s.* 9*d.* =		5	4½	*the price at ½d. per yd.*
One fourth of 10*s.* 9*d.* =		2	8¼	*the price at ¼d. per yd.*
Anſwer £.	5	4	9¾	*the price at 9¾d. per yd.*

Queſtions,

Questions.					Answers.		
	Yds.				£.	s.	d.
2d.	$145\frac{1}{2}$	at	$11\frac{1}{4}d.$	per yard =	6	19	$2\frac{1}{2}$
3d.	75	—	7	——— =	2	3	9
4th.	$63\frac{1}{2}$	—	$3\frac{3}{4}$	——— =	0	19	10
5th.	$123\frac{3}{4}$	—	$2\frac{1}{2}$	——— =	1	5	$9\frac{1}{4}$
6th.	137	—	$4\frac{1}{4}$	——— =	2	8	$6\frac{1}{4}$

CASE IV.

To find the value of one hundred weight :—As 112 is the groſs hundred, ſo 112 farthings are = 2*s.* 4*d.* and 112 pence = 9*s.* 4*d.* therefore, if the price be farthings, or not more than 3*d.* multiply 2*s.* 4*d.* by the farthings in the price of 1 ℔, or, if the price be pence, multiply 9*s.* 4*d.* by the pence in the price of 1 ℔, and in either caſe the product will be the anſwer.

EXAMPLES.

1. What will 1 Cwt. of chalk come to, at $1\frac{1}{2}d.$ per ℔?

112 *farthings* =	0	2	4	*price of* 1 *Cwt. at* $\frac{1}{4}d.$ *per* ℔.
$1\frac{1}{2}d.$ =			6	*farthings in the price.*
Anſwer £.	0	14	0	*price of* 1 *Cwt. at* $1\frac{1}{2}d.$ *per* ℔.

2. 1 Cwt. of tin at $2\frac{1}{4}d.$ per ℔?

		s.	*d.*	
		2	4	*price of* 1 *Cwt. at* $\frac{1}{4}d.$ *per* ℔.
			9	*farthings in the price of* 1 ℔.
Anſwer £.	1	1	0	*price of* 1 *Cwt. at* $2\frac{1}{4}d.$ *per* ℔.

3. 1 Cwt. of lead at 7*d.* per ℔?

		s.	*d.*	
		9	4	*price of* 1 *Cwt. at* 1*d. per* ℔.
			7	*pence in the price of* 1 ℔.
£.	3	5	4	*price of* 1 *Cwt. at* 7*d. per* ℔.

Queſtions.					Anſwers.		
	Cwt.				£.	s.	d.
4th.	1 of	copper at	$0\frac{3}{4}d.$	per ℔ =	0	7	0
5th.	1 of	———	$2\frac{1}{2}d.\ \frac{1}{2}q.$	=	1	4	6
6th.	1 of	———	$4\frac{1}{2}d.$	=	2	2	0

CASE VII.

To find the value of two, or more, hundreds, by having the price of one pound :—Firſt, find the price of 1 Cwt. by the laſt Caſe, and then

then proceed to find the value of the whole by Caſe 1ſt. or 2d, as the queſtion may require.

EXAMPLES.

1. What is the value of $5\frac{1}{4}$*cwt.* of ſugar, at 6*d.* per ℔?

	£.	s.	d.	
	0	9	4	*price of* 1 *cwt. at* 1*d. per* ℔.
			6	
	2	16	0	*price of ditto at* 6*d. per* ℔.
			5	
	14	0	0	*price of* 5*cwt.*
1 *qr. of a cwt.* =	0	14	0	*price of* $\frac{1}{4}$*cwt.*
Anſ. £.	14	14	0	*price of* $5\frac{1}{4}$*cwt.*

Queſtions.	*Cwt.*			*Anſwers.* £.	s.	d.
2d.	4	of ſugar at $2\frac{1}{2}$*d.* per ℔.	=	4	13	4
3d.	$8\frac{1}{2}$	5	=	19	16	8
4th.	7	$4\frac{3}{4}$	=	15	10	4
5th.	$4\frac{3}{4}$	$3\frac{1}{2}$	=	7	15	2
6th.	$9\frac{1}{2}$	8	=	35	9	4

CASE VIII.

To find the value of a hundred weight, when the price of 1℔ *is any number of pounds and ſhillings; or ſhillings pence and farthings:* Multiply the price of 1℔ by 7, its product by 8, and *this* product by 2; which laſt product will be the anſwer required.

EXAMPLES.

1. What will 1 *cwt.* of tobacco coſt, at 5*s.* $7\frac{1}{2}$*d.* per ℔?

	£.	s.	d.	
	0	5	$7\frac{1}{2}$	*price of* 1 ℔.
			7	
	1	19	$4\frac{1}{2}$	*price of* 7℔.
			8	
	15	15	0	*price of* 56 ℔ *or* $\frac{1}{2}$ *cwt.*
			2	
Anſ. £.	31	10	0	*price of* 112 ℔ *or* 1 *cwt.*

O *Queſtions.*

Questions.					*Answers.*		
		s.	*d.*		£.	*s.*	*d.*
2d.	1 Cwt. at	3	$10\frac{1}{2}$ per ℔ =		21	14	0
3d.	1 ditto —	9	6 —— =		53	4	0
4th.	1 ditto —	16	$11\frac{1}{2}$ —— =		94	19	4
5th.	1 ditto —	19	$8\frac{1}{4}$ —— =		110	5	0
6th.	1 ditto — £.1	7	10 —— =		155	17	4

PRACTICAL QUESTIONS *in* WEIGHTS *and* MEASURES.

1. What is the weight of 4 hogſheads of ſugar, each weighing 7*cwt.* 3*qrs.* 19℔? *Anſ.* 31*cwt.* 2*qrs.* 20℔.

2. What is the weight of 6 cheſts of tea, each weighing 3*cwt.* 2*qrs.* 9℔? *Anſ.* 21*cwt.* 1*qr.* 26℔.

3. If I am poſſeſſed of $1\frac{1}{2}$ dozen of ſilver ſpoons, each weighing 3*oz.* 5*pwt.*—2 dozen of tea ſpoons, each weighing 15*pwt.* 14*gr.*—3 ſilver cans, each 9*oz.* 7*pwt.*—2 ſilver tankards, each 21*oz.* 15*pwt.* and 6 ſilver porringers, each 11*oz.* 18*pwt.* Pray, what is the weight of the whole? *Anſ.* 18℔ 4*oz.* 3*pwt.*

4. In 35 pieces of cloth, each meaſuring $27\frac{3}{4}$ yards, how many yards? *Anſ.* $971\frac{1}{4}$ *yards.*

5. How much brandy in 9 caſks, each containing 45*gal.* 3*qts.* 1*pt.*? *Anſ.* 412*gal.* 3*qts.* 1*pt.*

6. If I have 9 fields, each of which contains 12 acres, 2 roods and 25 poles; how many acres are there in the whole? *Anſ.* 113*A.* 3*r.* 25*p.*

COMPOUND DIVISION*

Is the dividing of numbers of different denominations: In doing which, always begin at the higheſt, and when you have divided that, if any thing remain, reduce it to the next lower denomination, and ſo on, till you have divided the whole, taking care to ſet down your quotient figures under their reſpective denominations.

INTRODUCTORY EXAMPLES.

	1.			2.			3.		
	£.	*s.*	*d.*	£.	*s.*	*d.*	£.	*s.*	*d.*
Divide	549	17	9 *by* 5	3)197	13	$7\frac{1}{2}$	4)731	5	$10\frac{1}{2}$
Quot.	£.109	19	$6\frac{1}{2}$						

4.

* To divide a number conſiſting of ſeveral denominations by any ſimple number whatever, is the ſame as dividing all the parts or members of which that number is compoſed by the ſame number. And this will be true when any of the parts are not an exact multiple of the diviſor; for, by conceiving the number, by which it exceeds that multiple, to have its proper value by being placed in the next lower denomination, the dividend will ſtill be divided into parts, and the true quotient found as before: Thus, £.41 17*s.* 6*d.* divided by 6, will be the ſame as £.36 117*s.* 42*d.* divided by 6, which is equal to £.6 19*s.* 7*d.* as by the rule.

4. £. s. d. 2)97 19 10½

5. £. s. d. 6)37 11 4¾

6. £. s. d. 7)193 15 9½

7. £. s. d. 8)739 12 1½

8. £. s. d. 9)471 18 10½

9. £. s. d. 10)79 13 9½

10. £. s. d. 11)58 19 11½

11. £. s. d. 12)13 17 9½

In the first example, having divided the pounds, the 4, which remains, is 4 pounds, which are equal to 80 ſhillings, and 17 in the ſhillings make 97; I then find 5 is contained 19 times in 97, and 2 over: I ſet down 19 under the ſhillings, and reduce the 2 ſhillings, which remain, into pence, and they make 24, and the 9 pence, in the queſtion, added, make 33: Then, how often 5 in 33; I find it 6 times, and 3 over: I ſet down 6 under the pence, and reduce the 3 pence, which remain, to farthings, and they make 12; then, how often 5 in 12; I find it to be twice: I therefore ſet down $\frac{1}{2}$*d.* and the 2 which remains, is $\frac{2}{5}$ of a farthing, which I make no account of.

12. *T. cwt. qr.* ℔ *oz. dr.* 3)29 13 2 25 12 13

13. *T. cwt. qr.* ℔ 4)6 11 3 19

14. *Cwt. qr.* ℔ 5)14 1 12

15. ℔ *oz. dr.* 6)10 13 9

16. ℔ *oz.* 7)20 13

17. ℔ *oz. pwt. gr.* 8)7 10 15 2

18. ℔ *oz. pwt. gr.* 9)56 9 13 19

19. ℔ *oz. pwt. gr.* 10)849 11 12 14

20. ℔ *oz. pwt. gr.* 12)529 10 7 14

21. ℔ ℥ ʒ ℈ *gr.* 2)37 10 5 1 12

22. ℔ ℥ ʒ ℈ *gr.* 5)37 11 6 2 17

23. ℔ ℥ ʒ ℈ *gr.* 9)348 9 4 0 19

24. *Yds. qr. n.* 4)76 3 2

25. *E. E. qr. n.* 5)97 4 1

26. *E. Fl. qr. n.* 7)58 2 2

27. *E. Fr. qr. n.* 12)17 5 1

28. *Deg. m. f. p. ft. in. bar.* 6)97 55 7 35 4 2 1

29. *M. f. p.* 7)5 5 19

30. *P. ft. in.* 8)9 13 10

31.

31.	32.	33.	34.
Yds. ft. in.	Ac. r. p.	Yr. m. w. d. h. m. s.	Y. m. d.
9)8 2 11	3)76 3 37	4)25 10 3 5 21 39 45	5)5 9 25

35.	36.	37.
M. w. d. h. m.	M. d. h. m.	
6)6 3 5 10 29	7)9 21 12 45	8)3s 25° 55′ 25″

38.	39.
9)19° 45′ 38″	12)189° 37′ 29″

40. Suppose that two places lie east and west of each other, and it is found by observation that it is noon at the former 2 hours, 6′ 30″ sooner than at the latter; how many degrees are they asunder?

4′)2*h* 6′ 30″.

31° 37′ 30″ *Ans.*

Reduce the hours and minutes to minutes, then, minutes divided by 4′ give degrees in the quotient.

41. The longitude of Cambridge is 4*h* 44′ 17″, and that of Philadelphia, 5*h* 0′ 43″; how many degrees difference?

5*h*	0′	43″
4	44	17
4′)0	16	26
4°	6′	30″ *Ans.*

Practical Questions.

CASE I.

Having the price of any number of yards, &c. within the pence table, to find the price of unity, or 1 yard. If the quantity do not exceed 12, proceed by setting down the price and dividing it by the quantity; which quotient will be the price of one yard, required; but if the quantity exceed 12, then divide by 2 such numbers, as, when multiplied together, will produce the quantity, and the last quotient will be the value of 1 yard, required.

Note. This case proves the first and second cases in Compound Multiplication.

1. If 9 yards of cloth cost £.4 3*s.* 7½*d.* what is it per yard?

£.	s.	d.
9)4	3	7½
0	9	3½ *Ans.*

2. If

2. If 7 ells cost £.5 17s. 5d. what cost 1 ell ?
3. If 11 sheep cost £.6 5s. 9d. what did each cost ?
4. If 12 gallons of rum cost £.8 11s. 9½d. what is it per gallon ?
5. If 84 cows cost £.253 13s. what is the price of each ?
Ans. £.3. 0s. 4⅔d.

6. If 132 bushels of corn cost £.20 12s. 6d. what is that per bushel ? *Ans.* 3s. 1½d.

Note. When the given quantity (or divisor) is large, and not a composite number, the operation may be performed by Long Division.

CASE II.

Having the price of a hundred weight, to find the price of 1 lb. Divide the given price by 8, that quotient by 7, and this quotient by 2, and the last quotient will be the price of 1 lb required.

1. If 1 *cwt.* of flax cost £.2 7s. 8d. what is that per lb ?

```
    £.  s.  d.
 8)2   7   8
 -----------
 7)0   5  11½
 -----------
 2)0   0  10d.0 6/7 qr.
 -----------
   0   0  5 3/28 d. price of one pound.
```

2. At £.3 10s. per *cwt.* what cost 1 lb ? *Ans.* 7½d.
3. At £.6 6s. per *cwt.* what cost 1 lb ? *Ans.* 1s. 1½.
4. At £.42 11s. 8d. per *cwt.* what cost 1 lb ? *Ans.* 7s. 7¼d.
5. At £.19 5s. per *cwt.* what cost 1 lb ? *Ans.* 3s. 5¼d.

CASE III.

Having the price of several hundred weight, to find the price per lb. Divide the whole price by the number of hundreds, which will give the price per *cwt.* and then proceed as in the last Case.

1. If 5 *cwt.* of sugar cost £.13 8s. 4d. what is that per lb ?

```
    £.  s.  d.
 5)13   8   4
 -----------
 8) 2  13   8 price of 1 cwt.
 -----------
 7) 0   6   8½d. price of 14 lb or 1/8 cwt.
 -----------
 2) 0   0  11½d. price of 2 lb or 1/56 cwt.
 -----------
    0   0   5¾ price of 1 lb.
```

2. If 8 *cwt.* of cocoa cost £15. 17s. 4d. what is that per lb ?
Ans. 4¼d.

3. If 3¼ *cwt.* sugar cost £.9 17s. 2d. what is that per lb ?
Ans. 6½d.

4. If

4. If $1\frac{3}{4}$ *cwt.* of cotton wool cost £.6 10*s.* 8*d.* what is that per ℔? *Anſ.* 8*d.*

Note. This Caſe proves the 7th, in Compound Multiplication.

CASE IV.

Having the price of any number of yards, &c. to find the price of 1 *yard.* Divide the price by the quantity, beginning at the higheſt denomination, and, if any thing remain, reduce it into the next, and every inferior denomination, and, at each reduction, divide as before, remembering, each time, to add the odd ſhillings, pence, &c. if there be any, and you will have the value of unity required.

Note. If there be $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$ of a yard, pound, &c. multiply both the price and quantity by 4, and then proceed as above directed.

1. If $95\frac{1}{2}$ ℔ of figs coſt £.16 13*s.* $6\frac{3}{4}$*d.* what are they per ℔?

	℔		£.	s.	d.	
Quantity =	$95\frac{1}{2}$	*Price* =	16	13	$6\frac{3}{4}$	
Mult. by	4				4	
Produces	382 *for a diviſor.*	*Product.*	£.66	14	3	*for a dividend.*

$$
\begin{array}{l}
\quad\ \text{£.}\quad s.\quad d.\quad\ \text{£.}\quad s.\quad d. \\
382)66\ \ 14\ \ 3\ (0\ \ 3\ \ 5\tfrac{3}{4}\tfrac{250}{382} \text{ per ℔.} \\
\qquad\ 20 \\
\hline
382)1334(3 \\
\qquad 1146 \\
\hline
\qquad\ 188 \\
\qquad\quad 12 \\
\hline
382)2259(5 \\
\qquad 1910 \\
\hline
\qquad\ 349 \\
\qquad\quad\ 4 \\
\hline
382)1396(3 \\
\qquad 1146 \\
\hline
\qquad\ 250
\end{array}
$$

2. If 147 buſhels of rye coſt £.47 12*s.* 6*d.* what is it per buſhel? *Anſ.* 6*s.* $5\frac{3}{4}$*d.*

3. If $33\frac{1}{4}$ yards of baize coſt £.25 13*s.* $9\frac{1}{2}$*d.* what is it per yard? *Anſ.* 15*s.* $5\frac{1}{4}$*d.* $\frac{95}{133}$.

4. If $172\frac{3}{4}$ gallons of wine coſt £.43 5*s.* 2*d.* what is it per gallon? *Anſ.* $5\frac{5}{691}$*s.*

5. If

5. If a dozen ſheep coſt £.7 2*s.* 9*d.* what are they apiece?
Anſ. 11*s.* 10$\frac{3}{4}$*d.*

Note. This proves the 3d and 4th Caſes in Multiplication.

PRACTICAL QUESTIONS *in* MONEY.

1. Divide £.273 9*s.* 4*d.* among 5 men and 4 women, and give the men twice as much as the women.

Men. Women.
5 *and* 4
Mult. by 2
10 *ſhares.*
Add 4 *women's ſhares.*
14 *the number of equal ſhares in the whole*=Diviſor.

£. *s.* *d.* £. *s.* *d.*
Divide by 14)273 9 4(19 10 8 = 1 *woman's ſhare.*
14 — 4 *women.*
133 — 78 2 8 = *women's ſhare.*
126
7 — £.19 10 8
20 — 2
14)149(10 — £.39 1 4 = 1 *man's ſhare.*
14 — 5 *men.*
9 — £.195 6 8 = *men's ſhare.*
12 — 78 2 8 = *women's ſhare.*
14)112(8 — £.273 9 4 *Proof.*
112

2. Divide £.120 17*s.* 4*d.* among 7 men and 7 women, and give the women 3 times ſo much as the men.

	£.	*s.*	*d.*	
Anſwer,	4	6	4	= *a man's ſhare.*
	12	19	0	= *a woman's ſhare.*

3. Divide £.39 12*s.* 5*d.* among 4 men, 6 women, and 9 boys: Give each man double to a woman, and each woman double to a boy.

	£.	*s.*	*d.*	
Anſwer,	1	1	5	= *a boy's ſhare.*
	2	2	10	= *a woman's ditto.*
	4	5	8	= *a man's ditto.*

4. Divide 5 guineas among 8 men:—Give A 8*d.* more than B, and B 8*d.* more than C, &c. *Anſ. H's ſhare* = 15*s.* 2*d.*

RULES

For reducing the Federal Coin, and the Currencies of the ſeveral United States; alſo Engliſh, Iriſh, Canada, Novaſcotia, Livres Tournois and Spaniſh milled Dollars, each to the *par* of all the others.

I. *To reduce Newhampſhire, Maſſachuſetts, Rhodeiſland, Connecticut, and Virginia currency:*

1. *To Federal Money.*

Rule.—Reduce the ſhillings, pence and farthings, to decimals, by Inſpection (Caſe 3d, Dec. Frac.) divide the whole by 3, putting the comma one figure further to the right hand in the quotient, than in the pounds of the dividend, and the quotient will be the anſwer in dollars, cents and mills.

1. Reduce

1. Reduce £.349 19*s.* 1*d.* to dollars.

,9 = ½ *the shillings.*
,05 = *odd shilling.*
4 = *qrs. in* 1*d.*

,954 = *decimal.*

3)349,954 D. c. m.
1166,513=1166, 51, 3 *Ans.*

2. Reduce 19*s.* 1¼*d.* to dollars.

,9
,05
,7

,957 = *decimal.*
3),957 D. c.
3,19 = 3 19

3. Reduce 1*s.* to cents.
1*s.* = ,05 then
3)0 5 c. m.
0,166⅔ = 16 6⅔

4. Reduce 1*d.*
1*d.* = 4 *qrs.*
3),004 c. m.
001,3⅓ = 1 3⅓

5. Reduce 1*qr.*
1*qr.* = ,001041 and
3),0 01 041
0,00,347 = 3$\frac{47}{100}$*mills.*

2. *To Newyork and Northcarolina currency.*

Rule.—Add one third to the Newhampshire, &c. sum, and the sum total will be the Newyork, &c. currency.

Reduce £.100 Newhampshire, &c. to Newyork, &c.

£.
3)100
+ 33 6 8

£.133 6 8 *Answer.*

3. *To Pennsylvania, Newjersey, Delaware and Maryland currency.*

Rule.—Add one fourth to the Newhampshire, &c. sum.

Reduce £.100 Newhampshire, &c. to Pennsylvania, &c.

4)100
+ 25

£.125 *Answer.*

4. *To Southcarolina and Georgia currency.*

Rule.—Multiply the Newhampshire, &c. sum by 7, and divide the product by 9, and the quotient is the answer.

Reduce £.100 Newhampshire, &c. to Southcarolina, &c.

100
7

9)700
£.77 15 6⅔ *Answer.*

5. *To English Money.*

Rule.—Deduct one 4th from the Newhampshire, &c. sum.

Reduce £.100 Newhampshire, &c. to English Money.

4)100
— 25

£.75 *Answer.*

6. *To Irish Money.*

Rule.—Multiply the Newhampshire, &c. sum by 13, and divide the product by 16.

Reduce £.100 Newhampshire, &c. to Irish Money.

100

4×3+ the given Sum.

400
3

1200
+ 100

16 = 4×4)1300

4)325

£.81 5 *Ans.*

7. To

7. *To Canada and Novascotia currency.*

Rule.—Multiply the Newhampshire, &c. sum by 5, and divide the product by 6.

Reduce £.100 Newhampshire, &c. to Canada, &c.

```
    100
      5
  6)500
£.83  6  8 Answer.
```

8. *To Livres Tournois.*

Rule.—Multiply the Newhampshire, &c. pounds by $17\frac{1}{2}$, and the product will be livres: Or, multiply the sum in shillings by 7: Divide the product by 8, and the quotient will be livres, sous, &c.

Reduce £.100 Newhampshire, &c. to Livres Tournois.

```
      100          Or, 100
      17½               20
      ---             ----
      700             2000
      100                7
       50             ----
      ---          8)14000
Ans. 1750 Liv.     -----
                Ans. 1750 livres.
```

1*d.* = 1 *sou*, $5\frac{1}{2}$ *den.* 1*s.* = $17\frac{1}{2}$ *sous.*
1£. = $17\frac{1}{2}$ *livres.*

9. *To Spanish milled Dollars.*

Rule 1.—When the sum consists of pounds only: Annex a cypher to the pounds, and divide the whole by 3: The quotient is dollars.

Reduce £.100 Newhampshire, &c. to dollars.

```
  3)1000
Dol.  333⅓ Ans.
```

Rule 2.—When the sum consists of pounds and shillings: Divide the pounds by 3, and the shillings by 6, not separating them in the quotient, and the quotient will be dollars.

Reduce £.152 15*s.* 6*d.* to dollars.

```
£'s. by 3 &  }  £.  s. d.
s. by 6.     }  152 15 6
                509dol. & 1/6 Ans.
```

Note. This article may be applied to the Federal dollar, it being of the same value with a Spanish dollar.

11. *To reduce Federal Money to Newengland and Virginia currency.*

Rule.—Multiply the Federal money by 3, and, if it consist of dollars only, cut off 1 figure, if of cents also, cut off 3, and if of mills, 4 figures at the right hand; then reduce the figures so cut off to farthings, each time cutting off as at first, and the left hand figures are pounds, shillings, &c.

1. Reduce 1166*dolls.* 51*c.* 3*m.* to Newengland currency, or lawful money.

```
   D.   c. m.
   1166,51 3
           3
£.349,95 39
          20
s.19,0780
       12
,936 = 1d. nearly.
```

2. Reduce 45 dollars.

```
   D.
   45
    3
£.13,5
   20
s.10,0
```

3. Reduce

3. Reduce 12*D.* 7*c.* to lawful money.

D. *c.*
12,07
3
£.3,621
20
s.12,420
12
d.5,040

III. *To reduce Newjersey, Pennsylvania, Delaware and Maryland currency.*

1. *To Newhampshire, Massachusetts, Rhodeisland, Connecticut, and Virginia currency.*

Rule.—Deduct one fifth from the Newjersey, &c. sum, and the remainder will be Newhampshire, &c. currency.

Reduce £.100 Newjersey, &c. to Newhampshire, &c.

5)100
— 20
£.80 *Answer.*

2. *To Newyork and Northcarolina currency.*

Rule.—Add one fifteenth to the Newjersey, &c. sum.

Reduce £.100 Newjersey, &c. to Newyork, &c.

15=3×5)100
3)20
+ 6 13 4 + *giv. sum.*
£.106 13 4 *Answer.*

3. *To Southcarolina and Georgia currency.*

Rule.—Multiply the Newjersey, &c. sum by 28, and divide the product by 45, and the quotient is Southcarolina, &c.

Reduce £.100 Newjersey, &c. to Southcarolina, &c.

100
4×7=28
400
7
45=5×9)2800
5)311 2 $2\frac{2}{3}$
£.62 4 $5\frac{1}{3}$ *Answer.*

4. *To English Money.*

Rule.—Multiply the Newjersey, &c. by 3, and divide the product by 5.

Reduce £.100 Newjersey, &c. to English money.

100
3
5)300
£.60 *Answer.*

5. *To Irish Money.*

Rule.—Multiply the Newjersey, &c. by 13, and divide the product by 20.

Reduce £.100 Newjersey, &c. to Irish.

100
4×3 + *the giv. sum.*
400
3
1200
+ 100
20=4×5)1300
4)260
£.65 *Answer.*

6. To

6. *To Canada and Novascotia currency.*

Rule.—Deduct one third from the Newjersey, &c.

Reduce £.100 Newjersey, &c. to Canada, &c.

3)100
— 33 6 8

£.66 13 4 *Answer.*

7. *To Livres Tournois.*

Rule.—Multiply the Newjersey, &c. pounds by 14, and the product will be Livres Tournois—or multiply the sum in shillings by 7; divide the product by 10, and the quotient will be livres, sous, &c.

Reduce £.100 Newjersey, &c. to Livres Tournois.

100 *Or* 100
14 20
400 2000
100 7
Ans. 1400 *Liv.* 10)14000
1400 *as before.*

1*d.* = $1\frac{1}{6}$ *sous.*
1*s.* = 14 *sous.*
1£. = 14 *liv.*

8. *To Spanish milled dollars.*

Rule.—Multiply the Newjersey, &c. pounds by $2\frac{2}{3}$ and the product will be dollars.—Or multiply them by 8: Divide the product by 3, and the quotient will be dollars.—If there be shillings in the given sum, for every 7*s.* 6*d.* add 1 dollar to the quotient.

Reduce £.100 10*s.* Newjersey, &c. to dollars.

100
8
3)800
$266\frac{2}{3}$
10*s.* = $1\frac{1}{3}$
Ans. 268 *dol.*

Or 100
2
200
100 × $\frac{2}{3}$ = $66\frac{2}{3}$
10*s.* = $1\frac{1}{3}$
268 *as before.*

IV. *To reduce Newyork and Northcarolina currency.*

1. *To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.*

Rule.—Deduct one fourth from the Newyork, &c.

Reduce £.100 Newyork, &c. to Newhampshire, &c.

4)100
— 25

£.75 *Answer.*

2. *To Newjersey, Pennsylvania, Delaware and Maryland currency.*

Rule.—Deduct one sixteenth from the Newyork, &c. sum.

Reduce £.100 Newyork, &c. to Newjersey, &c.

16=4×4)100
4)25
—£.6 5

£.93 15 *Answer.*

3. *To Southcarolina and Georgia currency.*

Rule.—Multiply the Newyork, &c. sum by 7, and divide the product by 12: The quotient is Southcarolina, &c.

Reduce £.100 Newyork, &c. to Southcarolina, &c.

100
7
12)700

£.58 6 8 *Answer.*

4. *To English Money.*

Rule.—Multiply the Newyork, &c. sum by 9: Divide the product by 16, and the quotient is English.

Reduce £.100 Newyork, &c. to English money.

100

100
9
16=4×4)900
4)225
£.56 5 *Answer.*

5. *To Irish Money.*

Rule.—Multiply the Newyork, &c. sum by 39: Divide the product by 64, and the quotient is Irish.

Reduce £.100 Newyork, &c. to Irish money.

100
6×6+*thrice the given sum.*
600
6
3600
+300=100×3
64=8×8)3900
8)487 10
£.60 18 9 *Ans.*

6. *To Canada and Novascotia currency.*

Rule.—Multiply the Newyork, &c. sum by 5, and divide the product by 8.

Reduce £.100 Newyork, &c. to Canada, &c.

100
5
8)500
£.62 10 *Ans.*

7. *To Livres Tournois.*

Rule.—Multiply the Newyork, &c. sum in shillings by 21: Divide the product by 32, and the quotient will be livres, sous, &c.

Reduce £.100 Newyork, &c. to Livres Tournois.

100 *Note.* 1*d.*=$1\frac{3}{32}$ *sous.*
20 1*s.*=$13\frac{1}{8}$ *sous.*
2000 1£.=$13\frac{1}{8}$ *liv.*
21
2000
4000
32=4×8)42000
4)5250
Ans. $1312\frac{1}{2}$ *livres.*

8. *To Spanish milled Dollars.*

Rule.—If the Newyork sum be pounds only, annex a cypher to them, then divide by 4, and the quotient is dollars: But if it be pounds and shillings, annex half the shillings to the pounds, and divide as before, and the quotient is dollars.

Reduce £.100 Newyork, &c. to Dollars.

4)1000
250 *Doll. Ans.*

Reduce £.100 8*s.* to Dollars.

4)1004
251 *Doll. Ans.*

V. *To reduce Southcarolina and Georgia currency.*

1. *To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.*

Rule.—Multiply the Southcarolina, &c. sum by 9, and divide the product by 7.

Reduce £100 Southcarolina, &c. to Newhampshire, &c.

100
9
7)900
£.128 11 $5\frac{1}{7}$ *Ans.*

2. To

2. *To Newjersey, Pennsylvania, Delaware and Maryland currency.*

Rule.—Multiply the Southcarolina, &c. sum by 45, and divide the product by 28.

Reduce £.100 Southcarolina, &c. to Newjersey, &c.

$$\begin{array}{r} 100 \\ \hline 9\times5=45 \\ \hline 900 \\ 5 \\ \hline 28=4\times7)4500 \\ \hline 4)642\ \ 17\ \ 1\frac{5}{7} \\ \hline \text{£}.160\ \ 14\ \ 3\frac{3}{7}\ \textit{Ans.} \end{array}$$

3. *To Newyork and Northcarolina currency.*

Rule.—Multiply the Southcarolina, &c. sum by 12, and divide the product by 7.

Reduce £.100 Southcarolina, &c. to Newyork, &c.

$$\begin{array}{r} 100 \\ 12 \\ \hline 7)1200 \\ \hline \text{£}.171\ \ 8\ \ 6\frac{6}{7}\ \textit{Ans.} \end{array}$$

4. *To English Money.*

Rule.—From the Southcarolina, &c. sum, deduct one twenty eighth.

Reduce £.100 Southcarolina, &c. to English Money.

$$\begin{array}{l} 28=4\times7)100 \\ \quad 4)14\ \ 5\ \ 8\frac{4}{7} \\ \hline \quad -\ 3\ \ 11\ \ 5\frac{1}{7}\ \textit{from the given sum.} \\ \quad \text{£}.96\ \ 8\ \ 6\frac{6}{7}\ \textit{Ans.} \end{array}$$

5. *To Irish Money.*

Rule.—Multiply the Southcarolina, &c. sum by 117, and divide the product by 112.

Reduce £.100 Southcarolina, &c. to Irish.

$$\begin{array}{r} 100 \\ \hline 12\times9+9\ \textit{times the giv. sum.} \\ \hline 1200 \\ 9 \\ \hline 10800 \\ +100\times9=900 \\ \hline 112=4\times4\times7)11700 \\ \hline 4)1671\ \ 8\ \ 6\frac{6}{7} \\ \hline 4)417\ \ 17\ \ 1\frac{5}{7} \\ \hline \text{£}.104\ \ 9\ \ 3\frac{3}{7}\ \textit{Answer.} \end{array}$$

6. *To Canada and Novascotia currency.*

Rule.—Multiply the Southcarolina, &c. sum by 15, and divide the product by 14.

Reduce £.100 Southcarolina, &c. to Canada, &c.

$$\begin{array}{r} 100 \\ \hline 5\times3 \\ \hline 500 \\ 3 \\ \hline 14=2\times7)1500 \\ \hline 2)214\ \ 5\ \ 8\frac{4}{7} \\ \hline \text{£}.107\ \ 2\ \ 10\frac{2}{7}\ \textit{Answer.} \end{array}$$

7. *To Livres Tournois.*

Rule.—Multiply the Southcarolina, &c. pounds by $22\frac{1}{2}$, and the product will be livres.

Reduce £.100 Southcarolina, &c. to Livres.

Note 1*d.*$=1\frac{7}{8}$ *sous.*
1*s.*$=1\frac{1}{8}$ *livre.*
1£.$=22\frac{1}{2}$ *livres.*

$$\begin{array}{r} 100 \\ 22\frac{1}{2} \\ \hline 200 \\ 200 \\ 50 \\ \hline \end{array}$$

Ans. 2250 *livres.*

8. *To*

8. *To Spanish milled Dollars.*

Rule.—Multiply the Southcarolina, &c. pounds by 30, and divide the product by 7, and if there be shillings, turn them into dollars and add them.

Reduce £.100 Southcarolina, &c. to Dollars.

```
    100
  10×3=30
  -------
   1000
      3
  -------
 7)3000
  -------
```

Dollars 428$\frac{4}{7}$. *Note,* $\frac{1}{7}$ = 8*d.*

VI. *To reduce English Money.*

1. *To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.*

Rule.—To the English sum add one third.

Reduce £.100 English to Newhampshire, &c.

```
  3)100
  + 33  6  8
  ----------
£.133  6  8 Answer.
```

2. *To Newjersey, Pennsylvania, Delaware and Maryland currency.*

Rule.—Multiply the English money by 5, and divide the product by 3.

Reduce £.100 English to Newjersey, &c.

```
     100
       5
     ---
  3)500
  --------
£.166 13 4 Ans.
```

3. *To Newyork and Northcarolina currency.*

Rule.—Multiply the English money by 16, and divide the product by 9.

Reduce £.100 English to Newyork, &c.

```
    100
    4×4
   ----
    400
      4
   ----
 9)1600
   ----
```

£.177 15 6$\frac{2}{3}$ *Answer.*

4. *To Southcarolina and Georgia currency.*

Rule.—To the English money add one twenty seventh.

Reduce £.100 English to Southcarolina, &c.

```
27=3×9)100
      ----------
      3)11  2  2⅔
      ----------
      + 3 14  0⁸⁄₉
      ----------
   £.103 14  0⁸⁄₉ Ans.
```

5. *To Irish Money.*

Rule.—To the English sum add one twelfth.

Reduce £.100 English money to Irish money.

```
  12)100
    + 8  6  8
  -----------
  £.108  6  8 Answer.
```

6. *To Canada and Novascotia currency.*

Rule.—To the English sum add one ninth.

Reduce £.100 English to Canada, &c.

```
  9)100
  + 11  2  2⅔
  -----------
£.111  2  2⅔ Answer.
```

7. *To Livres Tournois.*

Rule.—Multiply the English pounds by 23$\frac{1}{3}$, and the product will be livres.

Reduce

Reduce £.100 English to Livres Tournois.

$$\begin{array}{l} 100 \\ 23\frac{1}{3} \\ \hline 300 \\ 200 \\ 33\frac{1}{3} \\ \hline \end{array}$$

Note. 1*d.*=$1\frac{17}{18}$ *sous.*
1*s.*=$1\frac{1}{6}$ *livre.*
1£.=$23\frac{1}{3}$ *livres.*

Liv. sou. den.

Ans. $2333\frac{1}{3}$ *Liv.* = 2333 6 8

VII. *To reduce Irish Money.*

1. *To Newhampshire, Massachusetts, Rhodeisland, Connecticut, and Virginia currency.*

Rule.—Multiply the Irish sum by 16, and divide the product by 13.

Reduce £ 100 Irish to Newhampshire, &c.

$$\begin{array}{l} 100 \\ \quad 4\times4 \\ \hline 400 \\ \quad 4 \\ \hline 13)1600 \end{array}$$

£.123 1 $6\frac{6}{13}$ *Answer.*

2. *To Newjersey, Pennsylvania, Delaware and Maryland currency.*

Rule.—Multiply the Irish sum by 20, and divide the product by 13.

Reduce £.100 Irish to Newjersey, &c.

$$\begin{array}{l} 100 \\ \quad 4\times5=20 \\ \hline 400 \\ \quad 5 \\ \hline \end{array}$$

13)2000(153 16 $11\frac{1}{13}$ *Answer.*

$$\begin{array}{lll} 13 & 11 & 12 \\ \hline 70 & 20 & 12 \\ 65 & \hline 13)220(16 & \hline 13)144(11 \\ \hline 50 & 13 & 13 \\ 39 & \hline 90 & \hline 14 \\ \hline 11 & 78 & 13 \\ & \hline & \hline \end{array}$$

3. *To Newyork and Northcarolina currency.*

Rule.—Multiply the Irish sum by 64, and divide the product by 39.

Reduce £.100 Irish to Newyork, &c.

$$\begin{array}{l} 100 \\ \quad 8\times8=64 \\ \hline 800 \\ \quad 8 \\ \hline \end{array}$$

£. s.

39)6400(164 $2\frac{2}{39}$ *Answer.*

$$\begin{array}{l} 39 \\ \hline 250 \\ 234 \\ \hline 160 \\ 156 \\ \hline 4 \end{array}$$

4. *To Southcarolina and Georgia currency.*

Rule.—Multiply the Irish sum by 112, and divide the product by 117.

Reduce £.100 Irish to Southcarolina, &c.

$$\begin{array}{l} 100 \\ \quad 7\times4\times4=112 \\ \hline 700 \\ \quad 4 \\ \hline 2800 \\ \quad 4 \\ \hline \end{array}$$

£. s. d.

117)11200(95 14 $6\frac{42}{117}$ *Ans.*

$$\begin{array}{l} 1053 \\ \hline 670 \\ 585 \\ \hline 85 \end{array}$$

5. To

5. *To English Money.*

Rule.—From the Irish sum deduct one thirteenth.

Reduce £.100 Irish to English money.

```
13)100(7
   91
   --
    9
   20
   --
13)180(13
   13
   --
    50
    39
    --
    11
    12
    --
13)132(10
   13
   --
    2
```

£.	s.	d.
100	0	0
—7	13	$10\frac{2}{13}$
£.92	6	$1\frac{11}{13}$ *Ans.*

6. *To Canada and Novascotia currency.*

Rule.—To the Irish sum add one thirty ninth.

Reduce £100 Irish to Canada, &c.

```
39)100(2
   78
   --
   22
   20
   --
39)440(11
   39
   --
   50
   39
   --
   11
   12
   --
39)132(3
   117
   --
   15 = 5
   39   13
```

100		
+2	11	$3\frac{5}{13}$
£.102	11	$3\frac{5}{13}$ *Ans.*

7. *To Livres Tournois.*

Rule.—Multiply the Irish sum, in pence, by 70; divide that product by 39, and the quotient will be sous, which, divided by 20, will be livres.

Reduce £.100 Irish to Livres Tournois.

100×20×12=24000*d.*

```
        70
     ------2,0
39)1680000)4307,6
           ------sou.
```

Ans. Livres. 2153,,$16\frac{12}{13}$

1*d.*=$1\frac{31}{39}$ *sous.* 1*s*=$21\frac{7}{13}$ *sous.*

1£.=21 *liv.* $10\frac{10}{13}$ *sous.*

VIII. *To reduce Canada and Novascotia currency.*

1. *To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.*

Rule.—To the Canada, &c. sum add one fifth.

Reduce £.100 Canada, &c. to Newhampshire, &c.

```
  5)100
  + 20
  £.120 Answer.
```

2. *To Newyork and Northcarolina currency.*

Rule.—Multiply the Canada &c. sum by 8, and divide the product by 5.

Reduce £.100 Canada, &c. to Newyork, &c.

```
   100
     8
  5)800
 £.160 Answer.
```

3. *To Newjersey, Pennsylvania, Delaware and Maryland currency.*

Rule.—To the Canada, &c. sum add one half.

Reduce £.100 Canada, &c. to Newjersey, &c.

```
  2)100
  + 50
 £.150 Answer.
```

4. *To Southcarolina and Georgia currency.*

Rule.

Rule.—From the Canada, &c. sum deduct one fifteenth.

Reduce £.100 Canada, &c. to Southcarolina, &c.

15=3×5)100

3)20

— 6 13 4

£.93 6 8 *Answer.*

5. *To English Money.*

Rule.—From the Canada, &c. deduct one tenth.

Reduce £.100 Canada, &c. to English money.

10)100

— 10

£.90 *Answer.*

6. *To Irish Money.*

Rule.—From the Canada, &c. deduct one fortieth.

Reduce £.100 Canada, &c. to Irish money.

40)100

— 2 10

£.97 10 *Answer.*

7. *To Livres Tournois.*

Rule.—Multiply the Canada, &c. pounds by 21, and the product will be livres.

Reduce £.100 Canada, &c. to livres Tournois.

100

7×3=21

700 1*d.*=1¾ *sous.*

3 1*s.*=21 *sous.*

1£.= 21 *livres.*

Ans. 2100

8. *To Spanish milled Dollars.*

Rule.—Reduce the Canada, &c. sum to shillings: Divide them by 5, and the quotient is dollars. *Or*, Multiply the pounds by 4, and the product is dollars: And if there be shillings, turn them into dollars, and add them to the product.

Reduce £.100 Canada, &c. to dollars.

100 155 15

20 4

5)2000 620

+ 3 = 15*s.*

Doll. 400 *Ans.*

Doll. 623 *Ans.*

IX. *To reduce Livres Tournois.*

1. *To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.*

Rule.—Multiply the livres by 2: Divide the product by 35, and the quotient will be pounds. Or, Multiply the livres by 8: Divide the product by 7, and the quotient will be shillings.

Reduce 1750 livres to Newhampshire, &c. currency.

1750 *Or,* 1750

2 8

£.

35)3500(100*Ans.* 7)14000

35

00 2,0)200,0

£.100 *as bef.*

2. *To Newyork and Northcarolina currency.*

Rule.—Multiply the livres by 32: Divide the product by 21, and the quotient will be shillings.

Reduce 1312½ livres to Newyork, &c. currency.

1312,5

32

26250

39375

2,0

21)42000(200,0

£.100 *Answer.*

3. To

3. *To Newjersey, Pennsylvania, Delaware and Maryland currency.*

Rule.—Divide the livres by 14, and the quotient will be pounds. Or, Multiply the livres by 10: Divide the product by 7, and the quotient will be shillings.

Reduce 1400 livres to Newjersey, &c. currency.

```
     1400
       10
    ------         Or,
  7)14000      14)1400(£.100.
    ------        14
 2,0)200,0        ---
    -----          00
```

£.100 *Answer.*

4. *To Southcarolina and Georgia currency.*

Rule.—Multiply the livres by 2, divide the product by 45, and the quotient will be pounds. Or, deduct one ninth, and the remainder will be shillings.

Reduce 2250 livres to Southcarolina, &c. currency.

```
   2250                 Or,
      2               9)2250
   ---- £.             — 250
45)4500(100 Ans.      ------
   45               2,0)200,0
   ----               ------
     00             £.100 as bef.
```

5. *To English Money.*

Rule.—Multiply the livres by 6: Divide the product by 7, and the quotient is shillings: Or, deduct one seventh from the livres, and the remainder will be shillings.

Reduce $2333\frac{1}{3}$ livres to English money.

```
   2333⅓              Or,
      6            7)2333⅓
   -----            — 333⅓
  7)14000           ------
   -----          2|0)200|0
2|0)200|0           -----
   -----         £.100 as bef.
Ans. £.100          -----
```

6. *To Irish Money.*

Rule. Reduce the livres to sous, then multiply them by 39: Divide this product by 70, and the quotient will be pence.

Reduce 2153*liv.* $16\frac{12}{13}$*so.* to Irish Money.

```
      20
   -------
   43076 12/13
      39
   -------
   387720
  129228
   -------
7,0)168000,0
   -------
 12)24000
   -------
2,0)200,0
   -----
 £.100 Answer.
```

7. *To Spanish milled Dollars, or to Federal Dollars.*

Rule.—Multiply the livres by 4: Divide the product by 21, and the quotient will be Spanish or Federal Dollars.

Reduce 1000 livres to dollars.

```
   1000                       Or, 1000
      4                              4
   ----                          ----
21)4000(190 { Span. Dol.     21)4000(190 { Fed. Dol.
   21                            21
   ---                           ---
   190 = 190 10/21               190
   189 Dollars.                  189
   ---                           ---
    10                            10
     6                            10
   — s. d. q.                    — d. c. m.
21)60(2 10 1                 21)100(4 7 6 4/21
```

X. To

X. *To reduce Spanish milled Dollars.*

1. *To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.*

Rule.—Multiply the Dollars by 3, and double the right hand figure of the product, for shillings; the left hand figures are pounds.

Reduce 529 dollars to Newhampshire, &c.

$$\begin{array}{r} 529 \\ 3 \\ \hline \end{array}$$

£.158 14 *Answer.*

2. *To Newyork and Northcarolina currency.*

Rule.—Multiply the number of dollars by 4: Double the right hand figure of the product for shillings, and the left hand figures are pounds.

Reduce 529 dollars to Newyork, &c.

$$\begin{array}{r} 529 \\ 4 \\ \hline \end{array}$$

£.211 12 *Answer.*

3. *To Newjersey, Pennsylvania, Delaware and Maryland currency.*

Rule.—Multiply the number of dollars by 3, and divide by 8.

Reduce 529 dollars to Newjersey, &c.

$$\begin{array}{l} \;\;529 \\ \;\;\;\;3 \\ \hline \;\;\;\;\;£. \\ 8)1587(198\ \ 7\ \ 6\ \ \textit{Answer.} \\ \;\;\;8 \\ \hline \;\;\;78 \\ \;\;\;72 \\ \hline \;\;\;\;67 \\ \;\;\;\;64 \\ \hline \;\;\;\;\;8 \end{array}$$

Or,

$$\begin{array}{r} 8)1587 \\ \hline £.198\tfrac{3}{8}\ \textit{Ans.} \end{array}$$

4. *To Southcarolina and Georgia currency.*

Rule.—Multiply the number of dollars by 7, and divide by 30.

Reduce 529 dollars to Southcarolina, &c.

$$\begin{array}{r} 529 \\ 7 \\ \hline 3,0)370,3 \\ \hline \end{array}$$

£.$123\frac{13}{30}$ *Answer.*

*5. *To English Money, at 4s. 6d. per Dollar.*

Rule.—Multiply the dollars by 9, and divide by 40.

Reduce 529 dollars to English money.

$$\begin{array}{r} 529 \\ 9 \\ \hline 4,0)476,1 \\ \hline \end{array}$$

£.$119\frac{1}{40}$ *Answer.*

6. *To Canada and Novascotia currency.*

Rule.—Divide the dollars by 4.

Reduce 529 dollars to Canada, &c. 4)529

£.$132\frac{1}{4}$ *Answer.*

7. *To Livres Tournois.*

Rule.—Multiply the dollars by $5\frac{1}{4}$, and the product will be livres. Or, Multiply them by 21: Divide by 4, and the quotient will be livres.

Reduce 100 Spanish dollars to livres.

$$\begin{array}{r} 100 \\ 5\tfrac{1}{4} \\ \hline 500 \\ 100 \times \tfrac{1}{4} = 25 \\ \hline \end{array}$$

Ans. 525 *livres.*

Or,

$$\begin{array}{r} 100 \\ 21 \\ \hline 4)2100 \\ \hline 525 \end{array}$$

525 *as bef.*

Note,

* *Note,* That in England dollars are Bullion, that is, they are bought and sold by weight, and their value varies as other articles of merchandize.

Note, 1 Cent = $1\frac{1}{20}$ Sous. 1 Dime = $10\frac{1}{2}$ Sous. 1 Dollar = $5\frac{1}{4}$ Livres.

The Pound		*contains*	*Grains of fine Silver.*	*Grains of Stand. silv.*
	Sterling,	$1718\frac{2}{3}$		1858
	Newhamp. &c.	1289		$1393\frac{1}{2}$
	Newyork, &c.	$966\frac{3}{4}$		1045
	Newjersey, &c.	$1031\frac{1}{4}$		$1114\frac{3}{4}$
	Southcarol. &c.	1657,366		1791,819

The proportion of alloy being $\frac{3}{37}$ of the fine silver.

The Federal	*contains*	*Grains of fine*		
Dollar	375,64	Silver	409,78	*grs. of Stand. Silver.*
Eagle	246,268	Gold	268,659	*grs. of Stand. Gold.*

The alloy being $\frac{1}{11}$ of the fine *Silver.* *Gold.* The Subdivisions are in the same proportion.

DUODECIMALS, *or* CROSS MULTIPLICATION,

Is a Rule made use of by workmen and artificers in casting up the contents of their works.

Dimensions are generally taken in feet, inches and parts.

Inches and parts are sometimes called primes, seconds, thirds, &c. and are marked thus; inches or primes (′), seconds (″), thirds (‴), fourths (⁗), &c.

This method of multiplying is not confined to *twelves*; but may be greatly extended: For any number, whether its inferior denominations decrease from the integer in the same ratio, or not, may be multiplied crosswise; and, for the better understanding of it, the learner must observe, that if he multiplies any denomination by an integer, the value of an unit in the product will be equal to the value of an unit in the multiplicand; but if he multiplies by any number of an inferior denomination, the value of an unit in the product will be so much inferior to the value of an unit in the multiplicand as an unit of the multiplier is less than an integer.

Thus, pounds multiplied by pounds are pounds; pounds multiplied by shillings are shillings, &c. shillings multiplied by shillings are twentieths of a shilling; shillings multiplied by pence are twentieths of a penny; pence multiplied by pence are 240ths of a penny, &c.

RULE.

1. Under the multiplicand write the corresponding denominations of the multiplier.

2. Multiply each term in the multiplicand, beginning at the lowest, by the highest denomination in the multiplier, and write the result of each under its respective term, observing, in duodecimals, to carry an unit for every 12, from each lower denomination to its next superior, and for other numbers accordingly.

3. In

3. In the ſame manner multiply all the multiplicand by the primes or ſecond denomination in the multiplier, and ſet the reſult of each term one place removed to the right hand of thoſe in the multiplicand.

4. Do the ſame with the ſeconds in the multiplier, ſetting the reſult of each term two places to the right hand of thoſe in the multiplicand.

5. Proceed in like manner with all the reſt of the denominations, and their ſum will be the anſwer required.

EXAMPLES.

1. Multiply $2\frac{1}{2}$ feet by $2\frac{1}{2}$ feet.

F.	′	
2	6	
2	6	
5	0	
1	3	0
Anſ. 6	3	

Or thus.

$$\begin{array}{r} 2\frac{1}{2} \\ 2\frac{1}{2} \\ \hline 5 \\ 1\frac{1}{4} \\ \hline \end{array}$$

Anſ. $6\frac{1}{4}$ *ſquare feet* = 6*ft.* 36 *in.*

So that the 3 is not 3 inches, but 36 inches, or $\frac{1}{4}$ of a ſquare foot.

Or thus.

$$\begin{array}{r} 2{,}5 \\ 2{,}5 \\ \hline 125 \\ 50 \\ \hline \end{array}$$

Anſ. 6,25 *ſquare feet.*

2. Multiply 9*f.* 8′ 6″ by 7*f.* 9′ 3″

F.	′	″			
9	8	6			
7	9	3			
67	11	6			= *Product by the feet in the multiplier.*
7	3	4	6‴		= *ditto by the primes.*
	2	5	1	6⁗	= *ditto by the ſeconds.*
75	5	3	7	6	*Anſwer.*

3. How many ſquare feet in a board 17 feet 7 inches long, and 1 foot 5 inches wide? *Anſ.* 24*f.* 10′ 11″

4. How many cubic feet in a ſtick of timber 12 feet 10 inches long, 1 foot 7 inches wide, and 1 foot 9 inches thick? *Anſ.* 35*f.* 6′ 8″ 6‴

5. How many cubic feet of wood in a load 6 feet 7 inches long, 3 feet 5 inches high, and 3 feet 8 inches wide? *Anſ.* 82*f.* 5′ 8″ 4‴

6. There is a houſe with 4 tiers of windows, and 4 windows in a tier; the height of the firſt tier is 6*f.* 8′; of the ſecond, 5*f.* 9′; of the third, 4*f.* 6′; and of the fourth, 3*f.* 10′; and the breadth of each is 3*f.* 5′; how many ſquare feet do they contain in the whole? *Anſ.* 283*f.* 7′

The two following queſtions are Sexceſſimals.

7. If

7. If two places differ in longitude 2° 12′; what is their difference of time?

Mult. 2° 12′ 00″ 00‴

by 3′ 59″ 20‴ *the time in which the sun passes through* 1 *degree.*

8′ 46″ 32‴ *Ans.*

8. Two places differ in longitude 31° 27′ 30″; what is the difference, in time, of the sun's coming to the meridian of those places, the sun passing through 15° in an hour?

31° 37′ 30″

4′ 00″ *In* 4′ *of a solar day, or day of* 24 *hours, the sun passes* 1 *deg.*

2 6′ 30″ 00‴ *Answer.*

9. *Mult.* £.3 6 8 *by* £.2 5 7.

		£.	s.	d.
		3	6	8
		2	5	7
£3×£2=£6	=	6	0	0
6s.×£2=12s.	=	0	12	0
8d.×£2=16d.	=	0	1	4
£3×5s.=15s.	=	0	15	0
6s.×5s.=$\frac{30}{20}$s.	=	0	1	6
8d.×5s.=$\frac{40}{20}$d.	=	0	0	2
£3×7d.=21d.	=	0	1	9
6s.×7d.=$\frac{42}{20}$d.	=	0	0	$2\frac{1}{10}$
8d.×7d.=$\frac{56}{240}$d.	=	0	0	$0\frac{7}{30}$
	Ans.	£.7	11	$11\frac{1}{3}$

10. A, B and C bought a drove of sheep in company; A paid £14 5s. B, £13 10s. and C, £11 5s. They agreed to dispose of them at the market; that each man should take 18s. as pay for his time, &c. and that the remainder should be divided in proportion to their several stocks; At the close of the sale, they found themselves possessed of £46 5s. What was each man's gain, exclusive of the pay for his time, &c.?

£14 5 + £13 10 + £11 5 = £39, and £46 5 − £39 = £7 5, and £7 5 − 18s. × 3 = £4 11s. whole gain, and £4 11 ÷ 39 = 2s. 4d. gain in the pound.

A	B	C
£14 5 0	£13 10 0	£11 5 0
× 2 4	× 2 4	× 2 4
1 8 6	1 7 0	1 2 6
4 9	4 6	3 9
A. £1 13 3	B. £1 11 6	C. £1 6 3

	£.	s.	d.
Proof.	1	13	3
	1	11	6
	1	6	3
	£4	11	0

SINGLE RULE OF THREE DIRECT.

The Rule of Three Direct teacheth, by having three numbers given, to find a fourth, that shall have the same proportion to the third, as the second hath to the first.

If *more* require *more*, or *less* require *less*, the question belongs to the Rule of Three Direct.

But

But if *more* require *less*, or *less* require *more*, it belongs to the Rule of Three Inverse.*

RULE.†

1. State the question by making that number, which asks‡ the question, the third term, or putting it in the third place; that, which is of the same name or quality as the demand, the first term; and that, which is of the same name or quality with the answer required, the second term.

2. Multiply the second and third numbers together; divide the product by the first, and the quotient will be the answer to the

* *More* requiring *more*, is when the third term is greater than the first, and requires the fourth term to be greater than the second. And *less* requiring *less*, is when the third term is less than the first, and requires the fourth term to be less than the second.

Also, *more* requiring *less*, is when the third term is greater than the first, and requires the fourth term to be less than the second. And *less* requiring *more*, is when the third term is less than the first, and requires the fourth term to be greater than the second.

† This Rule, on account of its great and extensive usefulness, is sometimes called the *golden rule of Proportion*: For, on a proper application of it and the preceding rules, the whole business of Arithmetic, as well as every mathematical inquiry, depends. The rule itself is founded on this obvious principle, that the magnitude or quantity of any effect varies constantly in proportion to the varying part of the cause: Thus, the quantity of goods bought, is in proportion to the money laid out; the space, gone over by an uniform motion, is in proportion to the time, &c.

As the idea, annexed to the term, *proportion*, is easily conceived, the truth of the rule, as applied to ordinary inquiries, may be made evident by attending to principles, already explained.

It has been shewn, in Multiplication of Money, that the price of one, multiplied by the quantity, is the price of the whole; and in Division, that the price of the whole, divided by the quantity, is the price of one: Now, in all cases of valuing goods, &c. where one is the first term of the proportion, it is plain that the answer, found by this rule, will be the same as that, found by Multiplication of Money; and, where one is the last term of the proportion, it will be the same as that, found by Division of Money.

In like manner, if the first term be any number whatever, it is plain, that the product of the second and third terms will be greater than the true answer, required, by as much as the price in the second term exceeds the price of one, or as the first term exceeds an unit; consequently, this product, divided by the first term, will give the true answer required.

Direct and Inverse proportion are properly only parts of the same general rule; but I have preserved the common distinction, and given some loose definitions, which, to young persons in general, are more intelligible.

Note 1. When it can be done, multiply and divide as in Compound Multiplication, and Compound Division.

2. If the first term, and either the second or third can be divided by any number without a remainder, let them be divided and the quotient used instead of them.

The following methods of operation, when they can be used, perform the work in a much shorter manner than the general rule.

1. Divide the second term by the first: Multiply the quotient into the third, and the product will be the answer.

2. Divide the third term by the first; multiply the quotient into the second, and the product will be the answer.

3. Divide the first term by the second, and the third by that quotient, and the last quotient will be the answer.

4. Divide the first term by the third, and the second by that quotient, and the last quotient will be the answer.

‡ *Note*, The term which asks or moves the question, has generally some words like these before it, viz. What will? What cost? How many? How far? How long? or, How much? &c.

the queſtion, which (as alſo the remainder) will be in the ſame denomination you left the ſecond term in, and which may be brought into any other denomination required.

Two, or more, ſtatings, are ſometimes neceſſary, which may always be known from the nature of the queſtion.

The method of proof is by inverting the queſtion.

But, that I may make the method of working this excellent Rule as intelligible as poſſible to the learner, I ſhall divide it into the ſeveral caſes following:

1. The fourth number is always found in the ſame name in which the ſecond is given, or reduced to; which, if it be not the higheſt denomination of its kind, reduce to the higheſt when it can be done.

2. When the ſecond number is of divers denominations, bring it to the loweſt mentioned, and the fourth will be found in the ſame name to which the ſecond is reduced, which reduce back to the higheſt poſſible.

3. If the firſt and third be of different names, or one or both of divers denominations, reduce them both to the loweſt denomination mentioned in either.

4. When the product of the ſecond and third is divided by the firſt; if there be a remainder after the diviſion, and the quotient be not the leaſt denomination of its kind; then multiply the remainder by that number, which one of the ſame denomination with the quotient contains of the next leſs, and divide this product again by the firſt number; and thus proceed till the leaſt denomination be found, or till nothing remain.

5. If the firſt number be greater than the product of the ſecond and third; then bring the ſecond to a lower denomination.

6. When any number of barrels, bales, or other packages or pieces are given, each containing an equal quantity, let the content of one be reduced to the loweſt name, and then multiplied by the given number of packages or pieces.

7. If the given barrels, bales, pieces, &c. be of unequal contents, (as it moſt generally happens) put the ſeparate content of each properly under one another, then add them together, and you will have the whole quantity.

1. If 6lb of ſugar coſt 9*s.* what will 30lb coſt at the ſame rate?

Here the firſt clauſe (if 6lb of ſugar coſt 9*s.*) ſuppoſes the rate; then follows the queſtion: What will 30lb coſt?

30lb, which moves the queſtion, is the 3d term. 6lb, the ſame kind, is the 1ſt, and 9 ſhillings the 2d.

As 6 : 9 :: 30 : *the Anſwer.*

$$\begin{array}{r} 9 \\ \hline 6)270 \\ \hline 45s. = £.2\ 5s.\ Anſ. \end{array}$$

Again, By inverting the order of the queſtion, it will be,

2. If

2. If 9*s.* buy 6lb of sugar, how much will £.2 5*s.* buy at that rate?

s.	lb	*s.*	
As 9 :	6 ::	45 :	*the Answer.*

6

9)270

30 lb *Answer.*

Again, 3. If 30lb of sugar be worth £.2 5*s.* how much may I buy for 9*s.*?

s.	lb	*s.*	
As 45 :	30 ::	9 :	*the Answer.*

9

45)270(6lb *the Answer.*

270

Again, 4. Suppose £.2 5*s.* will buy 30lb sugar: What will 6lb of the same sugar cost?

lb	*s.*	lb	
As 30 :	45 ::	6 :	*the Answer.*

6

3|0)27|0

9*s.* *Answer.*

N. B. The three last questions are only the first *varied*, being put purely to shew how any question, in this Rule, may be inverted.

5. If 5*yds.* cloth cost £.1 10*s.* what will 20*yds.* ditto come to?

yds.	*s.*	*yds.*
As 5 :	30 ::	20

$30 \div 5 = 6$

20

5)30*s.*

6 *Quot.*

2,0)12,0*s.* = £.6 *Answer.*

Here I divide the 2d term by the 1st, and multiply the quotient into the 3d for the answer.

	yds.	*s.*	*yds.*
Again, 6.	As 5 :	30 ::	20

$20 \div 5 = 4$

120*s.* = £.6.

Here I divide the 3d term by the 1st, and multiply the quotient into the 2d, for an answer.

7. If 20*yds.* cost £.120, how many yards may I have for £30?

£.	*yds.*	£.
As 120 :	20 ::	30

$120 \div 20 = 6$ *quot.* & $30 \div 6 = 5$ *yds. Ans.*

Here I divide the 1st term by the 2d, and then, the 3d term by that quotient for the answer.

	£.	*yds.*	£.
Again, 8.	As 120 :	20 ::	30

$120 \div 30 = 4$ *quot.* $20 \div 4 = 5$ *yds. Ans.*

Here I divide the 1st term by the 3d, and then, the 2d term by that quotient for the answer.

9. If 1*cwt.* of tobacco cost £.5 12 $9\frac{1}{2}$; what will 8*cwt.* ditto cost?

cwt.	£.	*s.*	*d.*	*cwt.*
As 1 :	5	12	$9\frac{1}{2}$ ::	8

8

Ans. £.45 2 4

Here is no need of reducing the middle term, because it can be performed by compound multiplication, the 1st term being an unit.

10. If 8*cwt.* of tobacco cost £.45 2 4; what is that per *cwt.*?

	£.	*s.*	*d.*
8)	45	2	4
Ans.	5	12	$9\frac{1}{2}$

Here there is no need of reducing the middle term, because it may be performed by compound division only, the 3d term being an unit.

11. If

11. If 9*cwt.* 3*qrs.* sugar cost £.27 17*s.* 6*d.* what will 2*cwt.* 1*qr.* 11lb cost?

```
 4              20            4
---            ---           ---
 39            557             9
 28             12            28
---            ----          ---
312            6690           73
 78                           19
----                         ---
1092                         263
```

lb d. lb
As 1092 : 6690 :: 263 : *the answer.*

```
        263
        ----
        2007
       4014
      1338
      --------12
1092)1759470(1611
     1092    ----
     ----2|0)131|4 3d.
      6674  £.6 14/3 Anf.
      6552
      ----
       1227
       1092
       ----
        1350
        1092
        ----
         258
           4
         ---
1092)1032(0qr.
```

Note 1. If you look at the ftating, you will fee that the firft and third terms are of the fame kind, but of different denominations, and therefore are reduced to the fame name or denomination, and, that the demand of the queftion lies on the third term.

2. That the middle term, being given in pounds, fhillings and pence, is reduced to pence.

12. If 57*yds.* coft £69, what will 9*yds.* coft at that rate?

yds. £. *yds.*
As 57 : 69 :: 9

```
        9
       ---
57)621(10£. Anf.
   57
   --
    51
    20
   ----
57)1020(17s.
   57
   ---
   450
   399
   ---
    51
    12
   ---
57)612(10d.
   57
   --
    42
     4
   ---
57)168(2 54/57 qrs.
   114
   ---
```

Here all the terms being whole numbers, there is no need of reducing the middle one until after ftating.

13. If

13. If my income be 109 guineas per annum, I desire to know what I may spend per day, so that I may lay up £.45 at the year's end?

Ans. £.0 5 $10\frac{1}{4}$ $\frac{1}{365}$ *per day.*

Note 1. You must subtract £.45 from the value of 109 guineas.

2. There being 365 days in a year, your question must next be stated thus:

D. *Guin.* £. *D.* *s.* *d.* *qr.*
As 365 : 109 — 45 :: 1 : 5 10 $3\frac{1}{365}$ *the answer.*

14. If my salary be £.43 12*s.* 5*d.* per annum, what does it amount to per week?

Ans. £.0 16 $8\frac{283}{365}$

The Stating.
W. £. *s.* *d.* *W.*
As 52 : 43 12 5 :: 1 : *the Ans.*

Note. As there are 52 weeks and 1 day in a year, you will get the true answer to the above question by the following ratio.

D. £. *s.* *d.* *D.*
As 365 : 43 12 5 :: 7 : *the answer.*

15. Suppose my income to be 16*s.* $8\frac{283}{365}$*d.* per week, what is it per annum?

Ans. £.43 12 5.

The Stating.
D. *s.* *d.* *D.*
As 7 : 16 $8\frac{283}{365}$: : 365 : *the Ans. above.*

Note 1. You must first reduce the middle term to pence.

2. You must multiply by 365 (the denominator of the fraction) and add to the product the 283 which remains; and remember always to do so in similar cases.

3. You must divide by 7, the first term, and the quotient will be the answer in 365ths of a penny, which (in all similar cases) must be first divided by the denominator, and then brought into pounds.

16. If I am to pay 1*s.* 7*d.* per week for pasturing a cow; what must I give per week for 37 cows?

C. *s.* *d.* *C.* £. *s.* *d.*
As 1 : 1 7 : : 37 : 2 18 7 *Ans.*

17. How many yards of cloth may be bought for 195*dol.* 75*c.* of which $9\frac{1}{2}$ yards cost 11*dol.* 2*c.*?

Dol. *c.* *yds.* *Dol.* *c.* *yds.* *qrs.*
As 11 02 : $9\frac{1}{2}$: : 195 75 : 168 3 *Answer.*

18. If I buy 57 yards of cloth for 49 guineas; what did it cost per ell English?

yds. *guin.* *yds.*
As 57 : 49 : : $1\frac{1}{4}$: £.1 10*s.* $1\frac{12}{228}$*d.* *Ans.*

19. A merchant, failing in trade, owes in all £.3475, and has in money and effects but £.2316 13 4: Now, supposing his effects are delivered up, pray what will each creditor receive on the pound?

£. £. *s.* *d.* £.
As 3475 : 2316 13 4 : : 1 : £.0 13*s.* 4*d.* *Ans.*

20. A

20. A owes B £.3475, but B compounds with him for 13*s.* 4*d.* on the pound; pray, what must he receive for his debt?

£. s. d. £. £. s. d.
As 1 : 13 4 :: 3475 : 2316 13 4.

21. If the distance from Newburyport to York be 31 miles; I demand how many times a wheel, whose circumference is $15\frac{1}{2}$ feet, will turn round in performing the journey?

Feet. Cir. M. Cir.
As $15\frac{1}{2}$: 1 :: 31 : 10560 *times, answer.*

22. Bought 9 chests of tea, each weighing 3*cwt.* 2*qrs.* 21lb, at £.4 9*s.* per *cwt.* what came they to?

Cwt. £. s. C. qr. lb *£. s. d.*
As 1 : 4 9 :: 3 2 21 × 9 : 147 13 $8\frac{1}{4}$.

23. What will $37\frac{1}{2}$ gross of buttons come to at 13 *cents* per dozen?

Doz. c. Gross. D. c.
As 1 : 13 :: $37\frac{1}{2}$: 58,50 *Answer.*

24. A farm, containing 125*A.* 3*r.* 27*p.* is rented at £.3 9*s.* per acre; what is the yearly rent of that farm?

A. £. s. A. R. P. £.
As 1 : 3 9 :: 125 3 27 : 434 8*s.* $4\frac{1}{2}$*d.*$\frac{144}{160}$ *Answer.*

25. If a ship cost £.537, what are $\frac{3}{8}$ of her worth?

Eigh. £. Eigh. £. s. d.
As 8 : 537 :: 3 : 201 7 6 *Answer.*

26. If $\frac{7}{16}$ of a ship cost 1163*D.* what is the whole worth?

Sixt. D. Sixt. D. c. m.
As 7 : 1163 :: 16 : 2658,28,5 *Answer.*

27. Bought a cask of wine at 4*s.* 7*d.* per gallon, for 125 dollars: How much did it contain?

s. d. Gal. D. Gal. qt. pt.
As 4 7 : 1 :: 125 : 163 2 $1\frac{5}{55}$ *Answer.*

28. What comes the insurance of £.537 15*s.* to, at £.$4\frac{1}{2}$ per centum?

£. £. £. s. £. s. d.
As 100 : $4\frac{1}{2}$ :: 537 15 : 24 3 $11\frac{1}{2}$ $\frac{8}{10}$ *Ans.*

29. What come the commissions of £.785 to at $3\frac{1}{2}$ guineas per cent.?

£. guin. £. £. s. d.
As 100 : $3\frac{1}{2}$ :: 785 : 38 9 $3\frac{1}{2}$ $\frac{4}{10}$ *Ans.*

30. A merchant bought 9 packages of cloth, at 3 guineas for 7 yards; each package contained 8 parcels, each parcel, 12 pieces, and each piece, 20 yards; what came the whole to, and what per yard?

Yds. guin. pack. £.
As 7 : 3 :: 9 : 10368 *Ans. for the whole cost.*

Yds. guin. yd.
As 7 : 3 :: 1 : £.0 12*s.* *Ans. per yard.*

31. A merchant bought 49 tuns of wine for £.273; freight cost £.27; duties £.12; cellar £.9 10*s*, other charges £.15, and he

he would gain £.55 10s. by the bargain; what muſt I give him for 23 tuns?

Tuns. £. £. £. £. *s.* £. £. *s.* *Tuns.* £.
As 49 : 273+27+12+9 10+15+55 10 :: 23 : 184 *Anſ.*

32. If £.100 gain £.6 in a year, what will £.475 gain in that time?

£. £. £. £. *s.*
As 100 : 6 :: 475 : 28 10 *Anſ.*

33. The earth being 360 degrees in circumference, turns round on its axis in 24 hours; how far does it turn in one minute, in the 43d parallel of latitude; the degree of longitude, in this latitude, being about 51 ſtatute miles?

H. *D.* *M.* *M.* *M.*
As 24 : 360 × 51 :: 1 : $12\frac{3}{4}$ *Anſ.*

34. Shipt for the Weſtindies 225 quintals of fiſh, at 15*s.* 6*d.* per quintal; 37000 feet of boards, at $8\frac{1}{3}$*dolls.* per 1000; 12000 ſhingles, at $\frac{1}{2}$*guin.* per 1000; 19000 hoops at $1\frac{1}{2}$*doll.* per 1000, and 53 half joes; and in return, I have had 3000 gallons of rum at 1*s.* 3*d.* per gallon; 2700 gallons of molaſſes, at $5\frac{1}{2}$*d.* per gallon; 1500lb of coffee, at $8\frac{1}{2}$*d.* per lb, and 19*cwt.* of ſugar, at 12*s.* 3*d.* per *cwt.* and my charges on the voyage were £.37 12*s.* pray, did I gain or loſe, and how much, by the voyage?

Anſwer, loſt £.134 9*s.* 9*d.*

35. If a ſtaff, 4 feet long, caſt a ſhade (on level ground) 7 feet; what is the height of that ſteeple, whoſe ſhade, at the ſame time, meaſures 198 feet?

F. ſh. *F. hei.* *F. ſh.* *F. hei.*
As 7 : 4 :: 198 : $113\frac{1}{7}$ *Anſ.*

*36. Suppoſe a tax of £.755 be laid on a town, and the inventory of all the eſtates in the town amounts to £.9345, what muſt A pay, whoſe eſtate is £.149?

£. £. £. £. *s.* *d.*
As 9345 : 755 :: 149 : 12 0 $9\frac{1095}{9345}$ *Anſ.*

37. If

* It may not be amiſs to ſhew the general method of aſſeſſing town or pariſh taxes. Firſt, then, an inventory of the value of all the eſtates, both real and perſonal, and the number of polls, for which each perſon is rateable, muſt be taken in ſeparate columns: The moſt conciſe way is then to make the total value of the inventory the firſt term, the tax to be aſſeſſed, the ſecond, and £.1. the third, and the quotient will ſhew the value on the pound: 2dly, Make a table, by multiplying the value on the pound by 1, 2, 3, 4, &c.—3dly, From the inventory take the real and perſonal eſtates of each man, and find them ſeparately in the table, which will ſhew you each man's proportional ſhare of the tax for real and perſonal eſtates.

Note. If any part of the tax is averaged on the polls, or otherwiſe, before ſtating to find the value on the pound, you muſt deduct the ſum of the average tax from the whole ſum to be aſſeſſed; for which average, you muſt have a ſeparate column, as well as for the real and perſonal eſtates.

EXAMPLE.

Suppoſe the General Court ſhould grant a tax of £.100000, of which the town of Newburyport is to pay £1321 17*s.* 6*d.* and, of which the polls, being 750, are to pay 5*s.* 3*d.* each;—The town's inventory amounts to £.45000; what will it be on the

37. If 50 gallons of water, in one hour, fall into a ciftern, containg 230 gallons, and by a pipe in the ciftern 35 gallons run out in an hour, in what time will it be filled?

Gal. gal. h. gal. h.
As 50—35 : 1 :: 230 : $15\frac{1}{3}$ *Anf.*

38. A butcher went with £.416, to buy cattle: Oxen, at £.22 each, cows at £.4, fteers at £.3 10*s.* and calves at £.2 10*s.* and of each a like number; how many of each could he purchafe with that fum?

£. £. £. s. £. s. each. £. each.
As 22 + 4 + 3 10 + 2 10 : 1 :: 416 : 13 *Anf.*

39. Said Harry to Dick, my purfe and money are worth $3\frac{1}{4}$ guineas; but the money is worth eleven times as much as the purfe; pray how much money is there in it?

Guin. s. d.
As 12 : 1 :: $3\frac{1}{4}$: 7 7. *then* £.4 11*s.*—7*s.* 7*d.*=£.4 3*s.* 5*d.* *Anf.*

40. How many dozen pair of gloves, at 13 groats per pair, may I have for 125 dollars?

Gr. pr. dol. doz. pr.
As 13 : 1 :: 125 : 14 $5\frac{4}{52}$ *Anf.*

41. There is a ciftern, having four cocks; the firft will empty it in 10 minutes; the fecond in 20 minutes; the third in 40, and

the pound; and what is A's tax, whofe eftate (as by the inventory) is as follows, viz. real £.376, perfonal £.149, and he has 3 polls.

Pol. s. d. Pol. £. s. d.
Firft, As 1 : 5 3 :: 750 : 196 17 6 the average part of the tax to be deducted from £.1321 17*s.* 6*d.* and there will remain £.1125.

£. £. £.
Secondly, As 45000 : 1125 :: 1 : 6*d.* on the pound.

T A B L E.

£.	£. s. d.	£.	£. s. d.	£.	£. s. d.
1 is	0 0 6	20 is	0 10 0	200 is	5 0 0
2 —	0 1 0	30 —	0 15 0	300 —	7 10 0
3 —	0 1 6	40 —	1 0 0	400 —	10 0 0
4 —	0 2 0	50 —	1 5 0	500 —	12 10 0
5 —	0 2 6	60 —	1 10 0	600 —	15 0 0
6 —	0 3 0	70 —	1 15 0	700 —	17 10 0
7 —	0 3 6	80 —	2 0 0	800 —	20 0 0
8 —	0 4 0	90 —	2 5 0	900 —	22 10 0
9 —	0 4 6	100 —	2 10 0	1000 —	25 0 0
10 —	0 5 0				

Now, to find what A's rate will be.
His real eftate being £.376, I find, by the table, that £.300 is £.7 10*s.*
that £.70 is - - - 1 15
and that £.6 is - - - 3

for his real eftate - - £.9 8
In like manner I find his tax for perfonal eftate to be 3 14 6
His 3 polls, at 5*s*, 3*d*, each, are 0 15 9
} + £.9 8*s*,=£.13 18*s*, 3*d*, *Anfwer.*

Real.			Perfonal.			Polls.			Total.		
£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
9	8	0	3	14	6	0	15	9	13	18	3

and the fourth in 80 minutes, in what time will all four, running together, empty it?

$$\text{As}\left\{\begin{matrix}10\\20\\40\\80\end{matrix}\right\}\overset{\textit{Cist.}\ \ \textit{Min.}}{:\ 1\ ::\ 60\ :}\left\{\begin{matrix}6\\3\\1\frac{1}{2}\\\frac{3}{4}\end{matrix}\right\}\ \overset{\textit{Cist.}\ \ \textit{Min.}\ \ \textit{Cist.}\ \textit{Min.}}{\text{As } 11\tfrac{1}{4}\ :\ 60\ ::\ 1\ :\ 5\tfrac{1}{3}}\ \textit{Anf.}$$

$11\frac{1}{4}$ *Cist.*

42. A and B depart from the ſame place, and travel the ſame road; but A goes 5 days before B, at the rate of 20 miles per day; B follows at the rate of 25 miles per day: In what time and diſtance will he overtake A?

M. *M.* *D.* *M.* *D.* *D.* *D.* *M.* *D.* *M.*

As 25—20 : 1 :: 20×5 : 20. *And*, As 1 : 25 :: 20 : 500

43. If the earth revolves 366 times in 365 days, in what time does it perform one revolution?

Revol. *Days.* *Revol.*

As 366 : 365 :: 1 : $23h\ 56'\ 3''\ 56'''$ + = 1 Sidereal day.*

44. If the earth makes one complete revolution in $23h\ 56'\ 3''$+, in what time does it paſs through one degree?

As 366° : $23h\ 56'\ 3''$ :: 1° : $3'\ 55''\ 20'''$ *Anſwer.*

45. If the earth performs its diurnal revolution in a ſolar† day, or 24 hours; in what time does it move one degree?

As 360° : $24h$ :: 1° : $4'$ *Anſwer.*

46. Sold a cargo of flax ſeed in Ireland, for £.1795 10s. Iriſh money; what does that amount to, in Maſſachuſetts currency, £.81 5s. Iriſh being equal to £.100 Maſſachuſetts?

Iriſh. *Maſſ.* *Iriſh.* *Maſſ.*

As £.$81\frac{1}{4}$: £.100 :: £.$1795\frac{1}{2}$: £.2209 16s. 11d. *Anſ.*

Or, As £.13 : £.$1795\frac{1}{2}$ :: £.16 : £.2209 16s. 11d. *as before, becauſe* £.13 *Iriſh are equal to* £.16 *Maſſachuſetts.*

47. My correſpondent in Maryland purchaſed a cargo of flour for me, for £.437 that currency; how much Maſſachuſetts money muſt I remit him, £.125 Maryland being equal to £.100 Maſſachuſetts?

£. £. £. £. s.

As 125 : 100 :: 437 : 349 12 *Anſ.*

£. £. s.

Or, As 5 : 437 :: 4 : 349 12 { Becauſe £.5 Maryland are equal to £.4 Maſſachuſetts.

48. A bill of exchange was accepted at Newburyport for the payment of £.345 10, for the like value delivered in Newyork, at £.$133\frac{1}{3}$ Newyork currency for £.100 Maſſachuſetts ditto; how much money was paid in Newyork, £.75 Maſſachuſetts being equal to £.100 of Newyork?

As

* A ſidereal day is the ſpace of time which happens between the departure of a ſtar from, and its return to the ſame meridian again.

† The ſolar day is that ſpace of time which intervenes between the ſun's departing from any one meridian, and its return to the ſame again.

Maſſ. *N. Y.* *Maſſ.* *N. Y.*

As £.75 : £.100 :: £.345 10*s.* : £.460 13*s.* 4*d.* *Anſ.*

49. When the exchange from Maſſachuſetts to Georgia is £.83⅓ Georgia per £.100 Maſſachuſetts, how much Maſſachuſetts money muſt be paid in Boſton to balance £.457 Georgia currency?

Geor. *Maſſ.* *Geor.* *Maſſ.*

As £.83⅓ : £.100 :: £.457 : £.548 8*s.* *Anſ.*

50. A merchant delivered at Boſton £.320 Maſſachuſetts currency, to receive £.400 in Philadelphia; what was the Maſſachuſetts pound valued at?

B. *P.* *B.* *P.*

As £.320 : £.400 :: £.1 : £.1 5*s.* *Anſ.*

£. £. £. £. *s.*
Or, As 80 : 100 :: 1 : 1 5 { Becauſe £.80 Maſſachuſetts are equal to £.100 Pennſylvania.

51. If I draw a bill of exchange for £.537 10*s.* 6*d.* Maſſachuſetts, to be paid in Ireland, at £.123$\frac{1}{13}$ Maſſachuſetts, per £.100 Iriſh; for how much Iriſh money muſt I draw the bill?

Maſſ. *Iriſh.* *Maſſ.* *Iriſh.*

As £.123$\frac{1}{13}$: £.100 :: £.537 10*s.* 6*d.* : £.436 14*s.* 9¼*d.*

£. £. £. £.
Or, As 16 : 537 10*s.* 6*d.* :: 13 : 436 14*s.* 9¼*d.* { Becauſe £.16 Maſſachuſetts are = £.13 Iriſh.

52. Suppoſe a bill is drawn in Ireland, and payable in Boſton, for £.673 12*s.* 6*d.* Iriſh; how much Maſſachuſetts money comes it to, the exchange at £.81¼ Iriſh per £.100 Maſſachuſetts?

£. £. £. *s.* *d.* £. *s.* *d.*

As 81¼ : 100 :: 673 12 6 : 829 1 6$\frac{6}{13}$.

The value of any quantity of ſilver in any of the currencies of the United States may be found by the following proportion.

As the number of grains, contained in £.1, is to £.1; ſo are the grains, in any given quantity, to its value.

53. What is the value of 1 lb of ſilver in Maſſachuſetts currency; the pound (or 20 ſhillings) containing 1393½ grains?

£. *s.* *d.*

As 1393½ : 1 :: 5760 : 4 2 8.

All queſtions in the Rule of Three, whether direct or inverſe, may be ſolved by the following rule.

Let that number, which is of the ſame name or quality as the number ſought, be the third term; then, conſider whether the number ſought ſhould be more or leſs than the third; if *more*, let the *greater* of the two other terms be the *middle* term, and the *leſs*, the *firſt*; but if the fourth number ought to be *leſs* than the third, then give the *leſs* the *ſecond* place, and the *greater*, the firſt. The queſtion being thus ſtated, the proportion will be; as the firſt term is to the ſecond, ſo is the third to the fourth, or number ſought.

Euclid's Elements V. 14.

Note. The firſt and ſecond terms muſt always be brought into one name, and the third into the loweſt mentioned, then proceed as

in the common method, by multiplying the ſecond and third terms together, and dividing the product by the firſt, and the quotient will be the anſwer, in the ſame name as the third term was reduced into.

54. If 15 yards of cloth coſt £.6, how many yards may I have for £.125?

```
   £.  £.    yds.
As 6 : 125 :: 15
            15
           ----
           625
          125
          ----
        6)1875
          ----
          312½ yards Anſ.
```

55. If 12 men can do a jobb in 20 days; in what time will 18 men do it?

```
   M.   M.    D.
As 18 : 12 :: 20
              20
             ---
          18)240(13⅓ days, Anſ.
             18
             --
              60
              54
              --
               6
```

56. If I give 7s. 9d. for 3 yards, how many yards may I have for £.39?

s. d. £. yds. yds. qrs. n.

As 7 9 : 39 :: 3 : 301 3 $2\frac{90}{93}$ *Anſ.*

Or thus,

State the queſtion in the uſual way, and let the ſecond term keep its proper, or natural place; then, multiply it by the greater or leſs extreme, that is, by the firſt or third number, accordingly as the anſwer ought to be greater or leſs; divide the product by the other term, and the quotient will be the anſwer.

RULE OF THREE DIRECT IN VULGAR FRACTIONS.

RULE.*

Having made the neceſſary preparations, as directed in Multiplication and Diviſion of Vulgar Fractions, ſtate your queſtion as in whole numbers, and invert the firſt term of the proportion; then multiply the three terms continually together, and the product will be the anſwer.

1. If $\frac{5}{8}$ of a yard coſt $\frac{5}{7}$ of a £. what will $\frac{9}{15}$ of an Ell Eng. coſt?

$$\frac{5}{8}yd. = \frac{5}{8} \text{ of } \frac{4}{1} \text{ of } \frac{1}{5} = \frac{5\times4\times1}{8\times1\times5} = \frac{1}{2} \text{ Ell Engliſh.}$$

E.Eng. £. E.Eng.

As $\frac{1}{2} : \frac{5}{7} :: \frac{9}{15}$ And $\frac{2}{1}\times\frac{5}{7}\times\frac{9}{15} = \frac{2\times5\times9}{1\times7\times15} = \frac{90}{105}£. = 17s.\ 1\frac{1}{2}d.\ \frac{30}{35}$ *Anſ.*

2. If $\frac{3}{5}$ *yd.* coſt $\frac{7}{8}£.$ what will $40\frac{1}{2}$ *yds.* come to?

Anſ. £.59 1s. 3d.

3. If 70 buſhels of corn coſt £.$12\frac{3}{5}$, what is it per buſhel?

Anſ. 3s. $7\frac{1}{5}d.$

4. If

* This rule and the next, depend upon the ſame principle as the Rule of Three in whole numbers.

4. If $\frac{7}{16}$ of a ſhip coſt £.51, what are $\frac{3}{32}$ of her worth?
Anſ. £.10 18*s.* $6\frac{3}{4}$*d.* $\frac{3}{7}$.

5. At £.$3\frac{5}{8}$ per *cwt.* what will $9\frac{2}{3}$℔ come to? *Anſ.* 6*s.* 3*d.* $\frac{5}{56}$.

6. A perſon having $\frac{4}{5}$ of a veſſel, ſells $\frac{2}{3}$ of his ſhare for £.319; what is the whole veſſel worth? *Anſ.* £.598 2*s.* 6*d.*

7. A merchant ſold $5\frac{1}{2}$ pieces of cloth, each containing $12\frac{2}{3}$*yds.* at 9*s.* $\frac{1}{2}$*d.* per yard; what did the whole amount to?
Anſ. £.31 9*s.* $10\frac{3}{4}$*d.* $\frac{1}{3}$.

8. A buys of B £.$560\frac{3}{4}$ bank ſtock at £.$85\frac{2}{3}$ per cent. what comes it to? *Anſ.* £.480 7*s.* $6\frac{1}{5}$*d.*

9. A merchant makes inſurance upon a veſſel and cargo, valued at £.3750 16*s.* at $15\frac{1}{2}$ guineas per cent. what does the premium amount to? *Anſ.* 813 18*s.* $5\frac{1}{2}$*d.*

10. A merchant in Holland draws a bill upon his correſpondent in Boſton for 3750 ducats at 8*s.* $4\frac{1}{5}$*d.*: How much Maſſachuſetts currency muſt he receive? *Anſ.* £.1565 12*s.* 6*d.*

11. A gentleman from Boſton being in England, where the price of ſilver is to that of gold, as 1 to $15\frac{1}{14}$, exchanged $158\frac{1}{4}$℔ of ſilver for gold; on his return to Maſſachuſetts, where the price of ſilver is to that of gold, as 1 to $15\frac{15}{31}$, a friend, wanting his gold, gave him the value thereof in ſilver; what weight of ſilver did he gain by the exchange?

	℔ S.	G.	℔ S.	℔ G.		G.	S.	G.	℔ S.
As	$15\frac{1}{14}$:	$\frac{1}{1}$ ::	$158\frac{1}{4}$;	$10\frac{1}{2}$	As	$\frac{1}{1}$:	$15\frac{15}{31}$ ::	$10\frac{1}{2}$:	$162\frac{36}{62}$

Anſ. $4\frac{41}{124}$℔.

12. A merchant bought a number of bales of velvet, each containing $129\frac{17}{27}$ yards, at the rate of 7 dollars for 5 yards, and ſold them out at the rate of 11 dollars for 7 yards; and gained 200 dollars by the bargain; how many bales were there?

	Yds.	Dol.	Yds.	Dol.
As	7 :	11 ::	5 :	$7\frac{6}{7}$

Sold 5 *yards for* $7\frac{6}{7}$ *Dollars.*
Bought 5 *yds. for* 7 *Dollars.*
In 5 *yards gained* $\frac{6}{7}$ *Dollar.*

	Dol.	Yds.	Dol.	Yds.		Yds.	B.	Yds.	B.
As	$\frac{6}{7}$:	5 ::	200 :	$1166\frac{2}{3}$, *and,*	As	$129\frac{17}{27}$:	$\frac{1}{1}$ ::	$1166\frac{2}{3}$:	9 *Anſ.*

Although the method before laid down be univerſally applicable, yet there are other methods more ready and expeditious in ſome particular caſes.

RULE I.

If the firſt and third terms be fractions, and the ſecond a whole number, reduce the firſt and third to one common denominator, then, rejecting the denominators, make the numerator of the firſt, the firſt term, and the numerator of the third, the third term, and work as in whole numbers.

If $\frac{5}{8}$ yard coſt 9*s.* what coſt $\frac{7}{12}$ yard at that rate?

$\frac{5}{8}=\frac{15}{24}$ and $\frac{7}{12}=\frac{14}{24}$. Now, As 15 : 9*s.* :: 14 : 8*s.* $4\frac{4}{5}$*d.* *Anſ.*

RULE II.

RULE II.

If, of the firſt and third terms, one be 1, and the other a fraction; put the denominator of the fraction inſtead of 1, and the numerator in the place of the fraction, and work as in whole numbers, as before.

If 1 acre of land coſt £.12, what will $\frac{5}{8}$ of an acre coſt, at that rate?

	Den.		£.		Num.		£.	s.	
As	8	:	12	::	5	:	7	10	*Anſ.*

RULE III.

If the ſecond term be a fraction likewiſe, (that is, if all the terms be fractions); having reduced the firſt and third to one common denominator, multiply the numerator of the firſt term by the denominator of the ſecond, for a diviſor; and the numerator of the third by the numerator of the ſecond, for a dividend; divide the laſt product by the firſt, and the quotient will be the anſwer.

If $\frac{1}{2}$ yard of cloth coſt $\frac{3}{4}$£. what coſt $\frac{7}{8}$ yard?

$\frac{1}{2} = \frac{4}{8}$, which reduces it to a common denominator; then

As 4 : $\frac{3}{4}$ :: 7

4	7
4	3
16	16)21($1\frac{5}{16}$ = 26*s.* 3*d.* *Anſ.*
	16
	5

To find the value of Gold in Maſſachuſetts Currency.

PROB. I. Given the weight of any quantity of gold, to find its value.

THEOREM I. As 1 Oz. : $5\frac{1}{3}$ £. :: 12 Oz. : 64 £. :: 1 pwt. : $5\frac{1}{3}$ s. :: 1 gr. : $2\frac{2}{3}$ d. (Caſe 1.) $= \frac{2\frac{2}{3}}{1}$ (Caſe 2.) $= \frac{5\frac{1}{3}}{2}$ (Caſe 3.) $= \frac{8}{3}$. Therefore,

Rule 1.—If the given quantity be in grains; ſay, As the denominator is to the number of grains; ſo is the numerator to their value, in pence.

1. What is the value of 18 grains of gold?

By Caſe 1.	*By Caſe* 2.	*By Caſe* 3.
Gr.	Gr.	Gr.
As 1 : 18 :: $2\frac{2}{3}$	As 2 : 18 :: $5\frac{1}{3}$	As 3 : 18 :: 8
$2\frac{2}{3}$	$5\frac{1}{3}$	8
36	90	3)144
12	6	48*d.* = 4*s.*
12)48(4*s.* *Anſ.*	2)96(48*d.* = 4*s.*	
48		

Rule

Rule 2. If the given quantity confiſt of ounces, pennyweights and grains, halve the grains, and then proceed as in multiplication of pounds, ſhillings and pence, making the numerator, in Caſe 2d, the multiplier.

	Oz.	*pwt.*	*gr.*
1. What is the value of	7	8	16 of gold ?

Gr. $16 \div 2 = 8$ *gr.*, then, *oz. pwt. gr.* 7 8 8

7	8	8
		$5\frac{1}{3}$
37	3	4
2	9	$6\frac{2}{3}$
£.39	12	$10\frac{2}{3}$ *Anſ.*

Rule 3.—If the given quantity confiſt of pounds only, multiply by 64, and the product will be the anſwer ; but if it confiſt of pounds, ounces, &c. it will be moſt convenient to reduce the pounds to ounces, and proceed by Rule 2.

1. What is the value of 36lb of gold, at £.64 per lb ?

$$\begin{array}{r} 64 \\ \hline 144 \\ 216 \\ \hline \text{£}.2304 \end{array}$$ *Anſ.*

2. What is the value of 15lb 9*oz.* 12*pwt.* 18*gr.* of gold ?

	12	*pwt. gr.*	*gr.*
oz.	189	12 9	$= 18 \div 2$
		$5\frac{1}{3}$	
	948	3 9	
	63	4 3	
£.	1011	8 0 *Anſ.*	

Prob. 2. To aſcertain the value of any given quantity of gold in Spaniſh milled dollars.

Theorem 2. 1*pwt.* of gold $= 5\frac{1}{3}s.$ 1 dollar $= 6s.$ And $\frac{5\frac{1}{3}}{6} = \frac{16}{18} = \frac{8}{9}.$ Therefore,

Rule. Reduce the given quantity of gold to pennyweights ; then, as the denominator is to the given quantity ; ſo is the numerator to the anſwer in dollars. *Or,*

Divide by the denominator, and multiply the quotient by the numerator. *Or,*

Divide

Divide by the denominator and ſubtract the quotient from the dividend. In either caſe, you will have the anſwer.

1. What is the value of 6*oz.* 6*pwt.* of gold, in Spaniſh dollars?

20

126 *pwt.*

pwt.
As 9 : 126 :: 8
8

9)1008

Anſ. 112 *dolls.*

Or,
9)126

14×8=112 *Anſ.*

Or,
9)126
— 14

112 *Anſ.*

oz. pwt. gr.
2. In 7 13 17 how many dollars? 20

$153\frac{17}{24}$
24

619
307

3689

As $\frac{9}{1}$: $\frac{3689}{24}$ :: $\frac{8}{1}$: $\frac{29512}{216}$

216)29512(136 *doll.*
216

791
648

1432
1296

136
6

216)816(3*s.*
648

168
12

216)2016(9*d.*
1944

72
4

216)288(1*gr.*
216

72

oz. pwt. gr.
3. In 9 8 6 how many dollars? 20

188
pwt.
As 9 : $188\frac{1}{4}$ :: 8
8

9)1506

Anſ. $167\frac{1}{3}$ or 3 *eight pences.*

Or, ſuppoſe it were required to reduce the quantity of gold to dollars, 90ths and 8ths of a 90th?

Find the value for ounces and pennyweights, as in the firſt Example; the quotient will be dollars, and the remainder, (if any) 9ths of a dollar: Then, as one grain is very nearly $\frac{3}{90} + \frac{2}{8}$ of $\frac{1}{90}$ of a dollar; divide the grains by 3, and the quotient will be 9ths of a dollar: Then multiply this remainder into $\frac{3}{90} + \frac{2}{8}$, and add all to the other work.

4. In

In 5*oz.* 19*pwt.* 17*gr.* how many dollars, 90ths and 8ths of a 90th?

$$
\begin{array}{l}
\quad\ \ oz.\ pwt. \\
9 : 5\ \ 19 :: 8 \\
\quad\ \ 20 \\
\hline
\quad\ 119 \\
\quad\quad\ 8 \\
\hline
9)952 \\
\hline
\end{array}
\qquad
\begin{array}{l}
\quad gr. \\
3)17(5\ ninths = \frac{50}{90} \\
\quad\ 15 \\
\hline
\quad\ \ 2 \times \overline{\frac{3}{90} + \frac{2}{8}} = \frac{6}{90} + \frac{4}{8}
\end{array}
$$

$105\frac{7}{9} = \frac{70}{90}$, then $105\frac{70}{90} + \frac{50}{90} + \frac{6}{90} + \frac{4}{8} = 106\frac{36}{90}, \frac{4}{8}$ *Anſ.*

Prob. 3. To aſcertain the weight of gold equivalent to any given ſum, currency.

Rule 1. If the given ſum be in pence, reverſe Rule 1. Theorem 1. that is; As the numerator 8 is to the given ſum in pence; ſo is the denominator 3 to the weight required, in grains.

What weight of gold is equal to 4*s.*?

$$
\begin{array}{l}
\quad\quad\ d. \\
As\ 8 : 48 :: 3 \\
\quad\quad\quad\ 3 \\
\hline
\quad 8)144 \\
\hline
\end{array}
\qquad
\begin{array}{l}
12 \\
\hline
48d.
\end{array}
$$

Anſ. 18 *grains.*

Rule 2. If the given ſum be in pounds, ſhillings and pence. As $\frac{5\frac{1}{3}}{1}$ is equal to $\frac{16}{3}$; therefore, divide the given ſum by 8, and that quotient by 2; add the two quotients together, double the laſt denomination, and you will have the anſwer.

What quantity of gold is equivalent to £.45 13*s.* 4*d.*?

Mark the pounds, ſhillings and pence, as oz. pwt. and gr.

$$
\begin{array}{rrrl}
 & oz. & pwt. & gr. \\
8) & 45 & 13 & 4 \\
\hline
2) & 5 & 14 & 2 \\
 & 2 & 17 & 1 \\
\hline
 & 8 & 11 & 3+3 \\
\hline
Oz. & 8 & 11 & 6\ Anſ. \\
\hline
\end{array}
\left.\right\}\ Add.
$$

Prob. 4. To find the weight of gold equivalent to any given number of dollars.

Rule. As the numerator 8 is to the number of dollars; ſo is the denominator 9 to the anſwer in pennyweights: *Or,* divide the dollars by the numerator 8, and add the quotient to the dividend.

Or,

Or, divide as before, and multiply the quotient by the denominator 9. In either caſe you will have the anſwer.

Required the weight of gold equal to 76 dollars.

As 8 : 76 :: 9
$$\begin{array}{r} 9 \\ \hline 8)684 \\ \hline \end{array}$$
Anſ. $85\frac{1}{2}$*pwt.* = 4 *oz.* 5 *pwt.* 12 *gr.*

Or thus, 8)76
$$\begin{array}{r} 9\frac{1}{2} \\ \hline \end{array}$$
Anſ. $85\frac{1}{2}$*pwt.*

Or, $9\frac{1}{2} \times 9 = 85\frac{1}{2}$*pwt.*

RULE OF THREE DIRECT IN DECIMALS.

Rule.

Having reduced your fractions to decimals, and ſtated your queſtion as in whole numbers, multiply the ſecond and third together; divide by the firſt, and the quotient will be the anſwer.

1. If $\frac{5}{8}$ of a yard coſt $\frac{7}{12}$ of a pound; what will $9\frac{2}{3}$ yards come to?

$\frac{5}{8} = ,625$ $\frac{7}{12} = ,583 +$ and $\frac{2}{3} = ,666 +$

As ,625 *Yd.* : ,583 *£.* :: 9,666 *Yd.*

$$\begin{array}{r} 9,666 \\ ,583 \\ \hline 28998 \\ 77328 \\ 48330 \\ \hline \end{array}$$

,625)5,635278(9,0164 + = £.9 0*s.* $3\frac{3}{4}$*d.* *Anſ.*

$$\begin{array}{r} 5\;625 \\ \hline 1027 \\ 625 \\ \hline 4028 \\ 3750 \\ \hline 2780 \\ 2500 \\ \hline 280 \end{array}$$

2. If 1*oz.* of ſilver coſt 6*s.* 8*d.* what is the price of a bowl, which weighs 1℔ 7*oz.* 13*gr.*? *Anſ.* £.6 6*s.* 10*d.*

3. If $9\frac{3}{4}$ yards coſt £.3 7*s.* 6*d.* what will $\frac{1}{2}$ yard come to? *Anſ.* 3*s.* $5\frac{1}{2}$*d.*

4. If 1*hhd.* ſugar, weighing 9*cwt.* 3*qrs.* 14℔ coſt £.27 13*s.* 7*d.* what will 3*cwt.* 1*qr.* 17℔ come to? *Anſ.* £.9 10*s.* $8\frac{1}{4}$*d.*

5. A tobacconiſt bought 5*hhds.* of tobacco, each weighing 8*cwt.* 2*qrs.* 19℔, for £.161 16*s.* 8*d.* what was it per ounce? *Anſ.* $\frac{1}{2}$*d.*

6. There is a ciſtern, which has 3 cocks, the firſt will empty it in $\frac{1}{4}$ of an hour, the ſecond in $\frac{3}{4}$, and the third in $1\frac{1}{2}$ hour; in what time will it be emptied, if all three run together?

As

	h. Cist.	h. Cist.	Cist.	h.	Cist.
As	,25 : 1 : : 1 : 4	As 6 : 1 : : 1 : ,1666+ = 10 min. Ans.			
	,75 : 1 : : 1 : 1,333+				
	1,5 : 1 : : 1 : 0,666+				
	6 Cist.				

7. A conduit has a cock, which, running into a ciftern, will fill it in 12 minutes: This ciftern has 3 cocks; the firft will empty it in $1\frac{1}{4}$ hour, the fecond in $37\frac{1}{2}$ minutes, and the third in $\frac{1}{2}$ an hour: In what time will the ciftern be filled, if all four run together?

	h. Cist. h. Cist.		
As	,2 : 1 : : 1 : 5	the filling Cock.	5 Cist. filled in an hour.
	1,25 : 1 : : 1 : 0,8	emptying Cocks.	4,4 do. emptied in do.
	,625 : 1 : : 1 : 1,6		,6 Cist. difference.
	,5 : 1 : : 1 : 2		
	4,4		

Cist. h. Cist. h. h. m.
Then, as ,6 : 1 : : 1 : 1,66+ = 1 40 Ans.

D. d. c.
8. If 19 yards coft 25, 7 5 what will $435\frac{1}{2}$ yards come to?

yds. D. d. c. yds.
As 19 : 25, 7 5 : : 435,5
25,75

217 75
3048 5
21775
8710

D. d. c. m.
19)11214,125(590,2 1 7 $\frac{2}{19}$ Ans.
95

171
171

41
38

32
19

135
133

2

D. d. c. m.
9. If 345 yards of tape coft 5, 1 7 5, what will one yard coft?

yds. D. d. c. m. yd.
As 345 : 5, 1 7 5 : : 1
1

d. c. m.
345)5, 1 7 5(,0 1 5 Ans.
3 4 5

1 7 2 5
1 7 2 5

D. d. c. m.
10. If I give 12,8 2 5 for 675 tops, how many tops will 19 mills buy?

D. d. c. m. T. d. c. m.
As 12, 8 2 5 : 675 : : ,0 1 9
,019

6075
675

12,825)12,825(1 top, Ans.
12,825

RULE

RULE OF THREE INVERSE, OR RECIPROCAL PROPORTION,

Teaches, by having three numbers given, to find a fourth, which ſhall have the ſame proportion to the ſecond, as the firſt has to the third.

Therefore, the greater the third term is, in reſpect to the firſt, the leſs will the fourth term be, in reſpect to the ſecond; or, the leſs the third term is in proportion to the firſt, the greater the fourth muſt be in proportion to the ſecond; and this is called *reciprocal, inverted* or *indirect* Proportion.

The principal difficulty, that will embarraſs the learner, will be, to diſtinguiſh when the proportion is direct, and when indirect. This is done by an attentive conſideration of the ſenſe and tenor of the queſtion propoſed: For if thereby it appears that, when the third term of the ſtating is leſs than the firſt, the anſwer muſt be leſs than the ſecond; or when the third is greater than the firſt, the anſwer muſt be greater than the ſecond; then the proportion is direct: But, if the third be leſs than the firſt, and yet the ſenſe of the queſtion requires the fourth to be greater than the ſecond; or if the third being greater than the firſt, the anſwer muſt be leſs than the ſecond, the proportion is inverſe.

RULE.*

State and reduce the terms as in the Rule of Three Direct; then, multiply the firſt and ſecond terms together, and divide the product by the third; the quotient will be the anſwer in the ſame denomination as the middle term was reduced into.

If there be fractions in your queſtion, they muſt be ſtated as before directed, and if they be vulgar, invert the third term: Then multiply the three terms continually together, and the product will be the anſwer.

EXAMPLES.

1. How much ſhalloon, that is $\frac{3}{4}$ yard wide, will line $6\frac{3}{4}$ yards of cloth which is $1\frac{1}{4}$ yard wide?

	yd.	yds.	qrs.			qrs.	qrs.	qrs.
As	$1\frac{1}{4}$:	$6\frac{3}{4}$ ::	3		As	5 :	27 ::	3
	4	4					5	
	5	27				3)	135	
						4)	45	
							$11\frac{1}{4}$ *yards, Anſ.*	

The

* The reaſon of this rule may be explained from the principles of Compound Multiplication and Compound Diviſion, in the ſame manner as the direct rule.—*For example*, If 4 men can do a piece of work in 12 days, in what time will 8 men do it?

As

The same by Vulgar Fractions.

First. $1\frac{1}{4}=\frac{5}{4}$, $6\frac{3}{4}=\frac{27}{4}$, and 3*qrs.* $=\frac{3}{4}$. Then,

$$\overset{yd.}{\text{As } \tfrac{5}{4}} : \overset{yds.}{\tfrac{27}{4}} :: \overset{yd.}{\tfrac{3}{4}}. \text{ And } \tfrac{5}{4}\times\tfrac{27}{4}\times\tfrac{4}{3}=\frac{5\times27\times4}{4\times4\times3}=\tfrac{540}{48}=\tfrac{45}{4}=11\tfrac{1}{4} \text{ yds. } \textit{Ans.}$$

The same by Decimal Fractions.

$1\frac{1}{4}=1,25$. $6\frac{3}{4}=6,75$. And 3*qrs.* $=,75$. Then,

```
   yd.   yds.   yd.
As 1,25 : 6,75 :: ,75
              1,25
              ----
              3375
             1350
             675
             -----
,75)8,4375(11,25yds. Ans.
    75
    --
     93
     75
     --
     187
     150
     ---
      375
      375
```

2. What length of board $7\frac{1}{2}$ inches wide, will make a square foot?

In. br. In. len. In. br. In. len.
As 12 : 12 :: $7\frac{1}{2}$: $19\frac{1}{5}$ *Ans.*

3. How many yards of carpet, $2\frac{3}{4}$ feet wide, will cover a floor, which is 18 feet long and 16 feet wide?

ft. ft. ft. ft. yds.
As 16 : 18 :: $\overline{2\frac{3}{4}\times3}$: $34\frac{10}{11}$ *Ans.*

{ Note, *I multiply* $2\frac{3}{4}$ *by* 3, *because* 3 *feet* = 1 *yard.*

4. Suppose I lend a friend £.350 for 5 months, he promising the like kindness; but, when requested, can spare but £.125, how long may I keep it to balance the favour?

£. Mo. £. Mo.
As 350 : 5 :: 125 : 14 *Ans.*

5. Suppose 450 men are in a garrison, and their provisions are calculated to last but 5 months; how many must leave the garrison, that the same provisions may be sufficient for those who remain 9 months?

Mo. M. Mo. M. M.
As 5 : 450 :: 9 : 250, and 450 — 250 = 200 *men, Ans.*

6. If a man perform a journey in 15 days, when the day is 12 hours long, in how many will he do it, when the day is but 10 hours?

h. d. h. d.
As 12 : 15 :: 10 : 18 *Ans.*

7. If

As 4 men : 12 days :: 8 men : $\frac{4\times12}{8}=6$ *days, the Answer.*

And here the product of the first and second terms, that is, 4 times 12, or 48, is evidently the time in which one man would perform the work. Therefore, 8 men will do it in one eighth part of the time, or 6 days.

7. If a piece of land, 40 rods in length, and 4 in breadth, make an acre, how wide muſt it be, when it is but 19 rods long, to make an acre?

Leng. Br. Leng. Br. ft. in.
As 40 : 4 :: 19 : 8 6 $11\frac{7}{19}$ *Anſ.*

8. If, when wheat is 6*s.* per buſhel, the twopenny loaf weigh 9,6*oz.* what ought it to weigh, when wheat is 7*s.* 6*d.* per buſhel?

s. oz. s. d. oz. pwt. gr.
As 6 : 9,6 :: 7 6 : 7 13 14,4 *Anſ.*

9. If a piece of board be 30 inches in length, what breadth will make $1\frac{1}{2}$ ſquare foot?

ſq. ft. in. in. in.
As 1,5 : 1 :: 30 : 7,2 *Anſ.*

10. If 9 men can build a houſe in 5 months, by working 14 hours per day, in what time will the ſame number of men do it, when they work only 10 hours per day?

h. mo. h. mo.
As 14 : 5 :: 10 : 7 *Anſ.*

11. A wall, which was to be built 24 feet high, was raiſed 8 feet by 6 men, in 12 days: How many men muſt be employed, to finiſh the wall in 4 days?

ft. m. ft. m.
As 8 : 6 :: $\overline{24-8}$: 12 to finiſh it in 12 days. And,
d. m. d. m.
As 12 : 12 :: 4 : 36 to finiſh it in 4 days.

12. There is a ciſtern having a pipe, which will empty it in 6 hours: How many pipes, of the ſame capacity, will empty it in 20 minutes?

h. pi. min. pi.
As 6 : 1 :: 20 : 18 *Anſ.*

13. What number of men muſt be employed to finiſh in 9 days, what 15 men would be 30 days about?

da. m. da. m.
As 30 : 15 :: 9 : 50 *Anſ.*

14. If a field will feed 6 cows 91 days, how long will it feed 21 cows?

c. d. c. d.
As 6 : 91 :: 21 : 26 *Anſ.*

15. How much in length, that is $8\frac{5}{7}$ inches broad, will make a foot ſquare?

in. in. in. in.
As $\frac{144}{1}$: $\frac{1}{1}$ :: $8\frac{5}{7}$: $16\frac{32}{61}$ *Anſ.*

16. How much in length, that is $13\frac{7}{8}$ poles in breadth, will make a ſquare acre?

po. po. po. po.
As $\frac{160}{1}$: $\frac{1}{1}$ :: $13\frac{7}{8}$: $11\frac{59}{111}$ *Anſ.*

17. A regiment of ſoldiers, conſiſting of 745 men, is to be clothed, each ſuit to contain $3\frac{1}{2}$ yards of cloth, which is $1\frac{3}{8}$ yard wide,

wide, and lined with ſhalloon $\frac{7}{8}$ yard wide; how many yards of ſhalloon will line them?

As $\overline{745 \times 3\frac{1}{2}}$: $1\frac{3}{8}$ :: $\frac{7}{8}$: $4097\frac{1}{2}$ *yards, Anſ.*

18. If a ſuit of clothes can be made of $4\frac{1}{8}$ yards of cloth, $1\frac{3}{8}$ yard wide; how many yards of coating $\frac{7}{8}$ of a yard wide will it require for the ſame perſon?

	yd.		*yds.*		*yd.*		*yds.*	*qr.*	*n.*	
As	$1\frac{3}{8}$	:	$4\frac{1}{8}$	::	$\frac{7}{8}$	:	6	1	$3\frac{5}{7}$	*Anſ.*

ABBREVIATIONS.

To know whether a fraction, when abbreviated, be equivalent in all reſpects to the original given fraction.

RULE.

As the numerator of the fraction, in its loweſt terms, is to its denominator; ſo will the numerator of the original fraction be to its own denominator.

Or, as one numerator is to the other; ſo will one denominator be to the other, &c.

A owes B £.75 13*s.* 6*d.* now £.100 of A's money is equal to £.140 of B's; what muſt A pay to ſatisfy the ſaid debt?

$\frac{100}{140} = \frac{5}{7}$, *therefore,*

£.	s.	d.
75	13	6
		5
7)378	7	6
£.54	1	$0\frac{6}{7}$ *Anſ.*

Now, to prove whether $\frac{5}{7}$ be equal to $\frac{100}{140}$.

Num. Den. Num. Den. — *Num. Num. Den. Den.*

As 5 : 7 :: 100 : 140——*Or,* as 5 : 100 :: 7 : 140.

COMPOUND PROPORTION, OR DOUBLE RULE OF THREE,

Teaches to reſolve ſuch queſtions as require two, or more, ſtatings by ſimple proportion; and that, whether direct or inverſe: It is compoſed (commonly) of 5 numbers to find a ſixth, which, if the proportion be direct, muſt bear ſuch proportion to the 4th and 5th as the 3d bears to the 1ſt and 2d; but if inverſe, the 6th number muſt bear ſuch proportion to the 4th and 5th, as the firſt bears to the 2d and 3d.

FIRST

FIRST METHOD.*

By two, or more, proportions in the Single Rule of Three.

RULE.

1. Let either of the two numbers, of which the queſtion is raiſed, be put in the third place, and the correſpondent number, of the ſame name or kind, in the firſt; the ſecond will be that, which has no correſpondent number given.

2. Three of the five given numbers being thus ſtated, find a fourth proportional.

3. Put this fourth number for a ſecond number of a ſecond ſtating, the remaining number of which the queſtion is raiſed, the third, and its correſpondent number of the ſame name, the firſt, then will the fourth number reſulting be the anſwer.

If a principal of £.100 gain £.6 intereſt in a year; what will a principal of £.400 gain in 9 months?

Here of the five given numbers, £.100 principal, £.6 intereſt, and a year or 12 months, are conjoined in form of a ſuppoſition, and thereupon a queſtion is raiſed concerning £.400 for 9 months; wherefore, let either of the two numbers, £.400 or 9 months, be put for the third number of the firſt ſtating, and its correſponding term, £.100 or 12 months, for the firſt.

£.	£.	£.
As 100 :	6 ::	400
		6
	1,00)	24,00
		24 *Anſ.*

Mo.	£.	Mo.
As 12 :	24 ::	9
		9
	12)	216
		£.18 *Anſ.*

Or thus,

Mo. £. Mo. £. — As 12 : 6 :: 9 : $4\frac{1}{2}$.

£. £. £. £. — And, As 100 : $4\frac{1}{2}$ :: 400 : 18.

Such queſtions as, when ſtated, are found to have both ſtatings direct, may be ſolved more readily by one compound ſtating, thus: Place the two terms, of which the queſtion is raiſed, under one another in the third place, their correſpondent terms under each other in the firſt, and the remaining term in the middle: Then multiply both theſe firſt terms together, and the third terms together, and ſo the double ſtating is reduced to a ſimple one of the *Rule of Three Direct*; *viz.* the product of the two

* The reaſon of this rule may be ſhewn from the nature of direct and inverſe proportion:—For in this rule, every row is a particular ſtating in one of thoſe rules; and, therefore, if all the ſeparate dividends be collected into one dividend, and all the diviſors into one diviſor, their quotients muſt be the anſwer ſought: Thus, in example 1ſt.

£. £. £. — Mo. £. £. Mo.

As 100 : 6 :: 400 : $\frac{400\times6}{100}$, and as 12 : $\frac{400\times6}{100}$ :: 9 : $\frac{400\times6\times9}{100\times12}$ by the Rule of Three Direct.

two firſt terms is the firſt of a ſimple ſtating ; the ſecond term is the ſecond, and the product of the two third terms is the third, to find a fourth proportional—Thus,

$$\text{As}\begin{Bmatrix}100\\12\end{Bmatrix} : 6 :: \begin{Bmatrix}400\\9\end{Bmatrix}$$

So the firſt example will ſtand thus.

$$\begin{Bmatrix}\pounds.\ 100\\ \textit{Mo.}\ 12\end{Bmatrix} : \pounds.6 :: \begin{Bmatrix}400\ \pounds.\\ 9\ \textit{Mo.}\end{Bmatrix}$$

$$\begin{array}{ccc} 12|00 & & 36|00 \\ & & 6 \\ \hline & 12)216 & \\ \hline & \pounds.18\ \textit{Anſ.} & \end{array}$$

SECOND METHOD.

Always place the three conditional terms in this order : That number which is the principal cauſe of gain, loſs or action, poſſeſſes the firſt place ; that which denotes the ſpace of time or diſtance of place, the ſecond ; and that, which is the gain, loſs or action, the third : This being done, place the other two terms, which move the queſtion, under thoſe of the ſame name, and if the blank place, or term ſought, fall under the third place, then the queſtion is in direct proportion ; therefore,

Rule 1.—Multiply the three laſt terms together, for a dividend, and the two firſt for a diviſor :—But if the blank fall under the firſt or ſecond place ; then, the proportion is inverſe ; therefore,

Rule 2.—Multiply the firſt, ſecond and laſt terms together for a dividend, and the other two for a diviſor, and the quotient will be the anſwer.

1. If £.100 gain £.6 in a year ; what will £.400 gain in 9 months?

£. P. Mo. £. Int.
100 : 12 : : 6 *Terms in the ſuppoſition, or conditional terms.*
400 : 9 *Terms which move the queſtion.*

Here, the blank falling under the third place, the queſtion is in direct proportion, and the anſwer muſt be found by the firſt Rule ; therefore, $400 \times 9 \times 6 = 21600$ *For the dividend, and*
$100 \times 12 = 1200$ *For the diviſor.*

See the work at large.

$$\begin{array}{rrcrcl} & \pounds.\ Pr. & & Mo. & & \pounds.\ Int. \\ & 100 & : & 12 & :: & 6 \\ & 400 & : & 9 & & \\ & 9 & & & & \\ \cline{2-2} 100 & 3600 & & & & \\ 12 & 6 & & & & \\ \hline \multicolumn{2}{l}{12{,}00)216{,}00(} & & & & \end{array} \qquad \begin{array}{l} 12{,}00)216{,}00(18\pounds.\ \textit{Anſ.} \\ \quad\quad 12 \\ \quad\quad \overline{96} \\ \quad\quad 96 \\ \quad\quad \overline{} \end{array}$$

2. If £.100 will gain £.6 in a year; in what time will £.400 gain £.18?

£. Mo. £.
100 : 12 :: 6 *Terms in the ſuppoſition.*
400 : :: 18 *Terms which move the queſtion.*

Here, the blank falling under the 2d place, the queſtion is in reciprocal or inverſe Proportion, and the anſwer muſt be ſought by the ſecond Rule; therefore,

$100 \times 12 \times 18 = 21600$ *For the dividend.*
$400 \times 6 = 2400$ *For the diviſor.*

£. Pr. Mo. £. Int.
100 : 12 :: 6
400 : :: 18
6 12
—— ——
2400 216
100
——
24|00)216|00(9 *months, Anſ.*
216
——

3. If £.400 gain £.18 in 9 months; what is the rate per cent. per annum?

Pr. Mo. Int.
400 : 9 :: 18
100 : 12
18
——
96
12
——
400 216
9 100
—— ——
36|00)216|00(£.6 *Anſ.*
216
——

4. What principal, at 6 per cent. per ann. will gain £.18 in 9 months?

Pr. Mo. Int.
100 : 12 :: 6
9 :: 18
12
——
9 216
6 100
—— £.
54)21600(400 *Anſ.*
216
——
00

5. If 8 men ſpend £.32 in 13 weeks; what will 24 men ſpend in 52 weeks?

M. W. £.
8 : 13 :: 32
24 : 52 £.384 *Anſ.*

6. If the freight of 9 *hhds.* of ſugar, each weighing 12 *cwt.* 20 leagues, coſt £.16; what muſt be paid for the freight of 50 tierces ditto, each weighing $2\frac{1}{2}$ *cwt.* 100 leagues?

hhds. leag. £.
9 : 20 :: 16
50 : 100

£. s. d.
92 11 $10\frac{2}{9}$ *Anſ.*

7. There

7. There was a certain edifice completed in a year by 20 workmen; but the ſame being demoliſhed, it is neceſſary that juſt ſuch an one ſhould be built in 5 months. I demand the number of men to be employed about it?

m.		*mo.*		*ed.*	
20	⁝	12	::	1	
		5	::	1	- - 48 *men, Anſwer.*

8. If 6 men build a wall 20 feet long, 6 feet high and 4 feet thick, in 16 days, in what time will 24 men build one 200 feet long, 8 feet high and 6 feet thick?

m.		*da.*		*ft.*	
6	⁝	16	::	$20 \times 6 \times 4$	
24	⁝		::	$200 \times 8 \times 6$	80 *days, Anſwer.*

THIRD METHOD.

That number, which is of the ſame name as the number ſought, muſt be the laſt term, on the right hand; then, take any two of the other numbers, which are of one kind, and if, when compared with the laſt number, *more* be required; ſet the greater in the ſecond place, (next to the laſt term, with four dots between) and the leſs in the firſt, (at the left hand, with two dots between:) But, if *leſs* be required, let the leſs ſtand in the ſecond place, and the greater in the firſt: When theſe three numbers are properly ſtated, take any two others, of one kind, which remain in the queſtion, and compare them with the laſt number, as before, to find whether they require a greater or a leſs anſwer, and ſet them accordingly, immediately to the left hand of thoſe, already ſtated, with dots, as before directed; thus proceed with every two remaining numbers, until all ſtand in one continued line.

Place A over the firſt, third, fifth, &c. numbers, omitting the laſt, and call them *antecedents*; and C over the ſecond, fourth, ſixth, &c. and call them *conſequents.* This being done, multiply all the antecedents continually together, for your firſt term; and all the conſequents continually together, for the ſecond. Then will the proportion be, As the product of the antecedents is to the product of the conſequents; ſo will the laſt number be to the anſwer.

Euclid's Elements, V. 12.

Take the 8th queſtion in the ſecond method.

If 6 men build a wall 20 feet long, 6 high and 4 thick, in 16 days, in what time will 24 men build one 200 feet long, 8 high, and 6 thick?

A

A	C	A	C	A	C	A	C	
ft. th.	*ft. th.*	*ft. hi.*	*ft. hi.*	*ft. lon.*	*ft. lon.*	*men.*	*men.*	
4 :	6 ::	6 :	8 ::	20 :	200 ::	24 :	6 ::	16

```
                    20                200
                    24                200
                   ----              ----
                    80               1200
                    40                  8
                   ----              ----
                   480               9600
                     6                  6
                   ----              -----
                  2880              57600
                     4
                  -----
                 11520
```

```
As 11520 : 57600 :: 16
              16
          ------
          345600
          57600
          ------
1152|0)92160|0(80 days, Answer.
          9216
          ----
             0
```

COMPARISON OF WEIGHTS AND MEASURES.

EXAMPLES.

1. If 78 pence Massachusetts be worth 1 French crown, How many Massachusetts pence are worth 320 French crowns?

```
    Fr. cr.  d.    Fr. cr.
As    1   : 78 ::  320
                    78
                  ----
                  2560
                  2240
                  ----
                  24960 Answer.
```

2. If 24 yards at Boston make 16 ells at Paris, How many ells at Paris will make 128 yards at Boston?

Bost. *Par.* *Bost.* *Par.*
As 24*yds.* : 16 *ells* :: 128*yds.* : $85\frac{1}{3}$*ells*, *Ans.*

3. If 60lb at Boston make 56lb at Amsterdam, How many lb at Boston will be equal to 350 at Amsterdam?

Ams. *Bost.* *Ams.* *Bost.*
As 56lb : 60lb :: 350lb : 375lb *Ans.*

 4. If

4. If 95lb Flemish make 100lb American, how many American lbs are equal to 550lb Flemish ?

	Flem.		*Amer.*		*Flem.*		*Amer.*	
As	95lb	:	100lb	: :	550lb	:	$578\frac{90}{95}$lb	*Ans.*

CONJOINED PROPORTION

Is when the coins, weights or measures of several countries are compared in the same question ; or, in other words, it is joining many proportions together, and by the relation, which several antecedents have to their consequents, the proportion between the first antecedent and the last consequent is discovered, as well as the proportion between the others in their several respects.

This rule may generally be so abridged by cancelling equal quantities on both sides, and abbreviating commensurables, that the whole operation may be performed with very little trouble, and it may be proved by as many statings in the Single Rule of Three as the nature of the question may require.

CASE I.

When it is required to find how many of the first sort of coin, weight, or measure, mentioned in the question, are equal to a given quantity of the last.

RULE.

Place the numbers alternately, that is, the antecedents at the left hand, and the consequents at the right, and let the last number stand on the left hand ; then multiply the left hand column continually for a dividend, and the right hand for a divisor, and the quotient will be the answer.

EXAMPLES.

1. Suppose 100 yards of America = 100 yards of England, and 100 yards of England = 50 canes of Thoulouse, and 100 canes of Thoulouse = 160 ells of Geneva, and 100 ells of Geneva = 200 ells of Hamburgh : How many yards of America are equal to 379 ells of Hamburgh ?

Antecedents.		*Consequents.*	*Abridged.*	
			Ant.	*Con.*
100 of America	=	100 of England.		
100 of England	=	50 of Thoulouse.	5	8
100 of Thoulouse	=	160 of Geneva.	379	
100 of Geneva	=	200 of Hamburgh.		
379 of Hamburgh ?				

Therefore, $\frac{379\times5}{8}=236\frac{7}{8}$ *yds.* of America = 379 *ells* of Hamburgh.

ILLUSTRATION.

The two 100's on both sides cancel each other. Let the last cyphers of the three next antecedents and consequents be cancelled, which is dividing by 10. Then divide the second antecedent and consequent

consequent by 5, and the quotients will be 2 on the side of the antecedents, and 1 on the side of the consequents; then 2 will measure the third antecedent and consequent, and the quotients will be 5 and 8. 10 will measure the 4th antecedent and consequent, and the quotients will be 1 and 2. Now, there being 2 left on each side, they cancel each other, and as there is no farther room for abridging by reason of the odd number 379, the operation is finished, and the answer found, as before.

2. If 20lb at Boston make 23lb at Antwerp, and 155 at Antwerp make 180 at Leghorn; How many at Boston are equal to 144 at Leghorn?

Antecedents.	*Consequents.*	
lb	lb	
20 of Boston =	23 of Antwerp.	$20\times155\times144=446400$ *divid.*
155 of Antwerp =	180 of Leghorn.	$23\times180=4140$ *divisor.*
144 of Leghorn.		$414|0)44640|0(107\frac{19}{23}$ *Answer.*

Or, abridged $\frac{155\times144}{23\times9}=107\frac{19}{23}$

3. If 12lb at Boston make 10 at Amsterdam, 100lb at Amsterdam 120lb at Paris, How many lb at Boston are equal to 80lb at Paris? *Ans.* 80lb.

4. If 140 braces at Venice be equal to 150 braces at Leghorn, and 7 braces at Leghorn be equal to 4 American yards, How many Venetian braces are equal to 32 American yards? *Ans.* $52\frac{4}{15}$.

5. If 40lb at Newburyport make 36 at Amsterdam, and 90lb at Amsterdam make 116 at Dantzick, How many lb at Newburyport are equal to 260lb at Dantzick? *Ans.* $224\frac{4}{29}$lb.

CASE II.

When it is required to find how many of the last sort of coin, weight or measure, mentioned in the question, are equal to a given quantity of the first.

Rule.

Place the numbers alternately, beginning at the left hand, and let the last number stand on the right hand; then multiply the first row for a divisor, and the second for a dividend.

Examples.

1. If 12lb at Boston make 10lb at Amsterdam, and 100lb at Amsterdam 120lb at Paris, How many at Paris are equal to 80lb at Boston?

	Left.	*Right.*	
Boston	12	10	$10\times120\times80=96000$
Amsterdam	100	120	$\frac{96000}{1200}=80$ *Ans.*
		80	$12\times100=1200$

2. If 40lb at Newburyport make 36 at Amsterdam, and 90lb at Amsterdam make 116 at Dantzick, How many lb at Dantzick are equal to 244 at Newburyport? *Ans.* $283\frac{1}{2}\frac{2}{5}$lb.

ARBITRATION

ARBITRATION OF EXCHANGES.

By this term is underſtood how to chooſe, or determine the beſt way of remitting money from abroad with advantage ; which is performed by conjoined proportion : Thus,

Suppoſe a merchant has effects at Amſterdam to the amount of 3530 dollars, which he can remit by way of Liſbon at 840 rees per dollar, and thence to Boſton, at 8*s.* 1*d.* per milree (or 1000 rees :) *Or*, by way of Nantz, at $5\frac{2}{5}$ livres per dollar, and thence to Boſton at 6*s.* 8*d.* per crown : It is required to arbitrate theſe exchanges, that is, to chooſe that which is moſt advantageous ?

1 dollar at Amſterdam = 840 rees at Liſbon.
1000 rees at Liſbon = 97*d.* at Boſton.
3530 dollars at Amſterdam.

$$\frac{840 \times 97 \times 3530}{1000 \times 1} = £.1198\ 8s.\ 8\tfrac{4}{10}d. \text{ by way of Lisbon.}$$

1 dollar at Amſterdam = $5\frac{2}{5}$ livres at Nantz.
6 livres at Nantz = 80 pence at Boſton.
3530 dollars at Amſterdam.

$$\frac{5\frac{2}{5} \times 80 \times 3530}{1 \times 6} = £.1059 \text{ by way of Nantz.}$$

Here it may be obſerved that the difference is £.139 8*s.* $8\frac{4}{10}$*d.* in favour of remitting by way of Liſbon rather than by Nantz, which depends on the *courſe* of exchange, at that time ; but the *courſe* may vary ſo, that, in a ſhort time, by way of Nantz may be better ; hence appears the neceſſity and advantage of an extenſive correſpondence, to acquire a thorough knowledge in the *courſes* of exchange, to make this kind of remittance.

FELLOWSHIP.

The Rules of Fellowſhip are thoſe by which the accompts of ſeveral merchants, or other perſons, trading in partnerſhip, are ſo adjuſted, that each may have his ſhare of the gain, or ſuſtain his ſhare of the loſs, in proportion to his ſhare of the joint ſtock, together with the time of its continuance in trade.

SINGLE FELLOWSHIP

Is, when the ſtocks are employed for any certain equal time.

RULE.*

As the whole ſtock is to the whole gain or loſs, ſo is each man's particular ſtock, to his particular ſhare of the gain, or loſs.

PROOF.

* That their gain or loſs, in this rule, is in proportion to their ſtocks, is evident : For, as the times, in which the ſtocks are in trade, are equal, if I put in $\frac{1}{2}$ of the whole ſtock, I ought to have $\frac{1}{2}$ of the gain ; if my part of the ſtock be $\frac{1}{4}$, my ſhare of the gain or loſs ought to be $\frac{1}{4}$ alſo. And generally the ſame ratio that the whole ſtock has to the whole gain or loſs, muſt each perſon's particular ſtock have to his reſpective gain or loſs.

PROOF. Add all the particular ſhares of the gain or loſs together, and, if it be right, the ſum will be equal to the whole gain or loſs.

EXAMPLES.

1. Divide the number 360 into 4 ſuch parts, which ſhall be to each other as 3, 4, 5 and 6.

$$\text{As } 3+4+5+6 : 360 :: \left\{\begin{array}{l} 3 : 60 \\ 4 : 80 \\ 5 : 100 \\ 6 : 120 \end{array}\right\} \textit{Anſwer.}$$

360 *Proof.*

2. A, B, C and D companied;—A put in £.145; B, £.219; C, £.378, and D, £.417, with which they gained £.569: What was the ſhare of each?

			£.	s.	d.	
Whole ſtock.	*Gain.*	145 :	71	3	$8\frac{1}{2}\ \frac{1058}{1159}$	A's *ſha.*
As 145+219+378+417 :	569 : :	219 :	107	10	$3\frac{3}{4}\ \frac{375}{1159}$	B's *dit.*
		378 :	185	11	$6\ \frac{552}{1159}$	C's *dit.*
		417 :	204	14	$5\frac{1}{4}\ \frac{333}{1159}$	D's *dit.*
			£.569	—	— —	*Proof.*

3. A, B, C and D are concerned in a joint ſtock of £.168 2s. 6d. of which A's part is £.25 10s.; B's £.37 15s.; C's £.49, and D's £.55 17 6d.—Upon the adjuſtment of their accompts, they have loſt £.73 13s. 4d. What is the loſs of each?

Anſ. A's *loſs* £.11 3s. $5\frac{1}{2}$d. B's £.16 10s. $9\frac{3}{4}$d. C's £.21 9s. $4\frac{3}{4}$d. & D's £.24 9s. $7\frac{3}{4}$d.

4. A and B companied; A put in £.45, and took $\frac{3}{5}$ of the gain; What did B put in? 5—3=2. Then, As 3 : 45 :: 2 : 30 *Anſ.*

5. A, B and C freighted a ſhip with 68900 feet of boards: A put in 16520 feet; B 28750; and C the reſt; but, in a ſtorm, the Captain threw overboard 26450 feet; How much muſt each ſuſtain of the loſs? *Anſ.* A, $6341\frac{3}{4}$ feet. B, $11036\frac{3}{4}$ & C, $9071\frac{1}{2}$ *do.*

6. A gentleman died, leaving three ſons and a daughter, to whom he bequeathed his eſtate in the following manner: To the eldeſt ſon he gave 312 moidores, to the ſecond 312 guineas, to the the third 312 piſtoles, and to the daughter 312 dollars; but when his debts were paid, there were but 312 half joes left; What muſt each have in proportion to the legacies which had been bequeathed them?

Anſ. 1ſt ſon £.293 0s. 2d.—2d ſon £.227 17s. $10\frac{3}{4}$d.—3d ſon £.179 1s. $2\frac{1}{2}$d. and the daughter £.48 16s. $8\frac{1}{4}$d.

7. A ſhip, worth £.780, being loſt at ſea, of which $\frac{1}{6}$ belonged to A, $\frac{1}{2}$ to B, and the reſt to C; What loſs will each ſuſtain, ſuppoſing £.450 to have been inſured upon her?

780—450=330, Then $\frac{1}{6}$)330 $\frac{1}{2}$)330 $\frac{1}{3}$)330

£.55=A's £.165=B's £.110=C's *ſhare.*

8. A

8. A and B venturing equal ſums of money, cleared by joint trade £.140 :—By agreement, as A executed the buſineſs, he was to have 8 per cent. and B was to have 5 per cent. What was A allowed for his trouble?

$$\text{As } \overset{£.}{8}+\overset{£.}{5} : \overset{£.}{140} :: \overset{£.}{8} : \overset{£.}{86\tfrac{2}{13}} \quad \text{And, as } \overset{£.}{8}+\overset{£.}{5} : \overset{£.}{140} :: \overset{£.}{5} : \overset{£.}{53\tfrac{11}{13}}.$$

Anſ. £.32 6*s.* $1\frac{3}{4}$*d.* $\frac{5}{13}$.

9. A bankrupt is indebted to A £.120, to B £.230, to C £.340, and to D £.450, and his whole eſtate amounts only to £.560 : How muſt it be divided among the creditors?

Anſ. A, £.58 18*s.* $11\frac{1}{4}$*d.* B, £.112 19*s.* $7\frac{3}{4}$*d.* C, £.167 0*s.* 4*d.* *and* D, £.221 1*s.* $0\frac{1}{2}$*d.*

10. A, B and C put their money into a joint ſtock; A put in £.40 : B and C together, £.170 : They gained £.126, of which B took £.42; What did A and C gain, and B and C put in reſpectively?

As £.210 the whole ſtock : £.126 the whole gain : : £.40 A's ſtock : £.24 A's gain.

As £.24 A's gain : £.40 A's ſtock : : £.42 B's gain : £.70 B's ſtock. Then £.170−£.70=£.100 C's ſtock; and the whole gain £.126—£.66 A's and B's gain=£.60 C's gain.

11. A, B and C companied;—A put in £.40; B £.60, and C, a ſum unknown : They gained £.72; of which C took £.32 for his ſhare : What did A and B gain, and C put in?

The whole gain £.72—C's gain £.32=£.40, A's and B's gain: Then, As £.100, A's and B's ſtock : £.40 their gain : : £.40 A's ſtock : £.16, his gain. Again, As £.16 A's gain : £.40, his ſtock : : £.32, C's gain : £.80, his ſtock.

12. A, B and C put in £.720, and gained £.540, of which, ſo often as A took up £.3, B took 5, and C 7; What did each put in and gain?

$$\text{As } \overset{£.}{3}+\overset{£.}{5}+\overset{£.}{7} : \overset{£.}{540} :: \left\{\begin{array}{l} \overset{£.}{3} : \overset{£.}{108} \text{ A's gain.} \\ 5 : 180 \text{ B's ditto.} \\ 7 : 252 \text{ C's ditto.} \end{array}\right\}$$

$$\text{And, as } 3+5+7 : \overset{£.}{720} :: \left\{\begin{array}{l} 3 : 144 \text{ A's Stock.} \\ 5 : 240 \text{ B's ditto.} \\ 7 : 336 \text{ C's ditto.} \end{array}\right\}$$

Or, you may find a common multiplier to multiply the proportions by, or multiplicand to be multiplied by the given proportions, thus, 15)720(48 multiplicand to find the ſtocks.—And 15)540(36 multiplicand to find the grains.

$$\left.\begin{array}{l} 48\times 3=\overset{£.}{144} \text{ A's ſtock} \\ 48\times 5=240 \text{ B's ditto.} \\ 48\times 7=336 \text{ C's ditto.} \end{array}\right\} \text{And} \left\{\begin{array}{l} 36\times 3=\overset{£.}{108} \text{ A's gain,} \\ 36\times 5=180 \text{ B's ditto.} \\ 36\times 7=252 \text{ C's ditto, as before.} \end{array}\right.$$

13. A

13. A, B, C and D companied; and gained a fum of money, of which A, B and C took £.120, B, C and D, £.180, C, D, and A, £.160, and D, A and B, £.140; What diftinct gain had each?

The fum of thefe 4 numbers is £.600, and as each man's money is named 3 times, therefore $\frac{1}{3}$, *viz.* £.200 is the whole gain—Therefore £.200—£.120 A's, B's and C's gain=£.80 D's gain;—And £.200—£.180 B's, C's and D's gain=£.20 A's gain.—£.200—£.160 C's, D's and A's gain=£.40 B's gain.—And £.200—£.140 D's, A's and B's gain=£.60 C's gain.

14. Two merchants companied; A put in £.40, and B 288 ducats. They gained £.135, of which A took £.60. What was the value of a ducat?

As £.60, A's gain : £.40, his ftock :: £.135, the whole gain—£.60, A's gain : £.50, B's gain.

$$\begin{array}{llllll} & \textit{Duc.} & \textit{£.} & & \textit{Duc.} & \textit{s. d.} \\ \text{And, as} & 288 & : 50 & :: & 1 & : 3\ 5\tfrac{2}{3}\ \textit{Anf.} \end{array}$$

15. Four men fpent, at a reckoning, 20 fhillings, of which they agreed that A fhould pay $\frac{3}{4}$, B, $\frac{1}{2}$, C, $\frac{1}{4}$, and D, $\frac{1}{8}$. What muft each pay in that proportion?

$$\text{As } \overset{\text{£.}}{\tfrac{3}{4}} + \overset{\text{£.}}{\tfrac{1}{2}} + \overset{\text{£.}}{\tfrac{1}{4}} + \overset{\text{£.}}{\tfrac{1}{8}} : \overset{s.}{20} :: \left\{ \begin{array}{rrcrr} \textit{s.} & \textit{d.} & & \textit{s.} & \textit{d.} \\ 15 & 0 & : & 9 & 2\tfrac{10}{13} \\ 10 & 0 & : & 6 & 1\tfrac{11}{13} \\ 5 & 0 & : & 3 & 0\tfrac{12}{13} \\ 2 & 6 & : & 1 & 6\tfrac{6}{13} \end{array} \right\} \textit{Anfwer.}$$

DOUBLE FELLOWSHIP,*

Or *Fellowfhip with Time*, is occafioned by the fhares of partners being continued unequal times.

RULE.

Multiply each man's ftock, or fhare, by the time it was continued in trade. *Then,*

As the whole fum of the products, is to the whole gain or lofs, fo is each man's particular product, to his particular fhare of the gain or lofs.

EXAMPLES.

1. A, B and C hold a pafture in common, for which they pay £.40 per annum. A put in 9 oxen for 5 weeks; B, 12 oxen for 7 weeks, and C 8 oxen for 16 weeks. What muft each pay of the rent?

$9\times5=45$. $12\times7=84$, and $8\times16=128$, then $128+84+45=257$

As

* When the times are equal, the fhares of the gain or lofs are evidently as the ftocks, as in Single Fellowfhip; and when the ftocks are equal, the fhares are as the times; wherefore, when neither are equal, the fhares muft be as their products.

```
As 257 : 40 :: 45
            45
           ---
           200
           160
           ---
    257)1800(7
        1799
        ----
           1
          20
          --
    257)20(0
         12
         --
    257)240(0
          4
         --
    257)960(3
        771
        ---
        189
```

```
As 257 : 40 :: 84
            84
            --
           160
           320
           ---
    257)3360(13
        257
        ---
         790
         771
         ---
          19
          20
          --
    257)380(1
        257
        ---
        123
         12
        ---
    257)1476(5
        1285
        ----
         191
           4
         ---
    257)764(2
        514
        ---
        250
```

```
As 257 : 40 :: 128
                40
               ---
      257)5120(19
          257
          ----
          2550
          2313
          ----
           237
            20
           ---
      257)4740(18
          257
          ----
          2170
          2056
          ----
           114
            12
          ----
      257)1368(5
          1285
          ----
            83
             4
           ---
      257)332(1
          257
          ---
           75
```

2. Four merchants traded in company, A put in £.100 for five months, B, £.150 for 7 months, C, 220 for 8 months, and D, £.310 for 9 months; but by misfortunes at sea, they lost £.145. What must each man sustain of the loss?

Answer, { A, £.11 17*s.* 8¼ $\frac{51}{61}$. C, £.41 16*s.* 8½ $\frac{38}{61}$. B, £.24 19*s.* 2 $\frac{40}{61}$. D, £.66 6*s.* 4½ $\frac{54}{61}$. }

3. A, with a capital of £.100 began trade January 1st 1787, and meeting with success in his business, he took in B as a partner, on the 1st day of March following, with a capital of £.150. Three months after that, they admit C as a third partner, who brought into stock £.180, and after trading together until the 1st of January 1788, they found there had been gained since A's commencing business, £.177 13*s.* How must this be divided among the partners?

Ans. A, £.53 16*s.* 8*d.* B, £.67 5*s.* 10*d.* C, £.56 10*s.* 6*d.*

4. Two merchants entered into partnership for 18 months; A, at first, put into stock £.100, and at the end of 8 months he put in £.50 more; B, at first, put in £.275, and at 4 months' end took out £.70. Now, at the expiration of the time, they found they had gained £.263. What is each man's just share?

Ans. A, £.96 9*s.* 6*d.* B, £.166 10*s.* 6*d.*

5. A and B companied; A put in, the 1st of January, £.150; but B could not put in any until the first of May; What did he then put in, to have an equal share with A at the year's end?

As

$$\begin{array}{ccccccc} & M. & £. & & M. & & \\ \text{As } & 12 & : 150 & :: & 8 & : & \frac{150\times12}{8} = £225 \textit{ Anf.} \end{array}$$

6. A, B and C companied; A put in the firft of March, 30*l.* B, the 1ft of May, put in 80 yards of broadcloth; and on the 1ft of June C put in 120 dollars. On the firft of January following, they reckoned their gains, of which A and B took 228*l.* B and C 215*l.* 10*s.* and C and A 187*l.* 10*s.* What was the whole gain, and the gain of each? What did they value a yard of cloth at? and, What was C's dollar worth?

228*l.*+215*l.* 10*s.*+187*l.* 10*s.*=631*l.* and 631÷2=315*l.* 10*s.* the whole gain; then, 315*l.* 10*s.*—228=87*l.* 10*s.* C's gain. 315*l.* 10*s.*—215*l.* 10*s.*=100*l.* A's gain, and 315*l.* 10*s.*—187*l.* 10*s.* =128*l.* B's gain. To find the value of one yard of cloth, fay, As 100*l.* A's gain : 30*l.* his ftock :: 128*l.* B's gain : 38*l.* 8*s.* then, inverfely, As 10 months : 38*l.* 8*s.* :: 8 months : 48*l.* the value of the whole cloth.

As 80 *yds.* : 48*l.* :: 1 *yd.* : 12*s.* anfwer. Now, to find the value of a dollar. As 100*l.* A's gain : 30*l.* his ftock :: 87*l.* 10*s.* C's gain : 26*l.* 5*s.* then, inverfely, As 10 months : 26*l.* 5*s.* :: 7 months : 37*l.* 10*s.*=120 dollars. Laftly: As 120 dollars : 37*l.* 10*s.* :: 1 dollar : 6*s.* 3*d.* anfwer.

7. A, B and C companied and put in together 1911*l.* A's money was in 3 months, B's 5 months, and C's 7 months; they gained 117*l.* which was fo divided, as that the $\frac{1}{2}$ of A's gain was equal to $\frac{1}{3}$ of B's, and $\frac{1}{4}$ of C's gain; What did each gain and put in?

Suppofe A's gain was 2*l.* then muft B have 3*l.* and C 4*l.* by the queftion:

$$\text{Then, as } 2+3+4 : \overset{£.}{117} \text{ the whole gain} :: \left\{\begin{array}{l} 2 : \overset{£.}{26} \text{ A's gain.} \\ 3 : 39 \text{ B's ditto.} \\ 4 : 52 \text{ C's ditto.} \end{array}\right.$$

Then, divide each man's gain by his time; and as the fum of the quotients is to each particular quotient, fo is the whole ftock, to each man's particular ftock.

A's ftock, 693*l.* 2*s.* 2*d.* B's ftock 623*l.* 15*s.* 1$\frac{1}{2}$*d.* C's ftock 594*l.* 1*s.* 10$\frac{1}{4}$*d.* *Anf.*

FELLOWSHIP BY DECIMALS.

RULE.*

Divide the whole gain, or lofs, by the whole ftock, and the quotient multiplied feverally by each man's ftock, will give the gain, or lofs, of each.

EXAMPLES.

1. A, B and C companied, A put in 40*l.* 5*s.*; B, 80*l.* 10*s.* and C, 161*l.* with which they gained 120*l.*; What is each man's fhare of the gain?

A's

* This is no more than Divifion of Decimals.

A's Stock = 40,25
B's ditto = 80,5
C's ditto = 161

Sum total = 281,75)120,000000(,4259+

,4259	,4259	,4259
40,25	80,5	161
21295	21295	4259
8518	340720	25554
170366	£34,28495	4259
£17,142475	20	£68,5699
20	5,69900	20
2,849500	12	11,3980
12	8,388	12
10,194000	4	4,776
4	1,552	4
0,776000		3,104

Proof. A's gain 17*l.* 2*s.* 10*d.* + B's gain 34*l.* 5*s.* 8¼*d.* + C's gain 68*l.* 11*s.* 4¾*d.* = 119*l.* 19*s.* 11*d.*

2. A, B and C companied; A put in 200*l.* B, 150*l.* and C 50*l.* with which they gained 800*l.*; What is the ſhare of each?

800÷400=2; then 200×2=400*l.*=A's gain, 150×2=300*l.*= B's gain, and 50×2=100=C's gain.

3. A, B, C and D trade, and gain 200*l.* which is to be divided in the following manner, viz. ſo often as A has 6*l.* B muſt have 10*l.* C, 14*l.* and D, 20*l.* What is the ſhare of each?

6+10+14+20=50, and $\frac{200}{50}$ = 4, quotient; then 6×4=24*l.* A's gain; 10×4=40*l.* B's gain; 14×4=56 C's; and 20×4= 80*l.* D's gain.

PRACTICE,*

Is a contraction of the Rule of Three direct, when the firſt term happens to be an unit, or one; and has its name from its daily uſe among merchants and tradeſmen, being an eaſy and conciſe method of working moſt queſtions which occur in trade and buſineſs.

The method of proof is by the Rule of Three, Compound Multiplication, or by varying the order of them.

Before

*GENERAL RULE.

1. Suppoſe the price of the given quantity to be 1*l.* or 1*s.* &c. then will the quantity itſelf be the anſwer at the ſuppoſed price.

2. Divide the given price into aliquot parts, either of the ſuppoſed price, or of one another, and the ſum of the quotients belonging to each will be the true anſwer required.

EXAMPLE.

[*For the remainder of this note, ſee page* 164.]

Before the queſtions, hereafter given, can be wrought, the following Tables muſt be perfectly gotten by heart.

TABLES.

Aliquot, or even Parts of Money.

Pts. of a ſhil. of a £.

d.		s.		£.
6	=	$\frac{1}{2}$	=	$\frac{1}{40}$
4	=	$\frac{1}{3}$	=	$\frac{1}{60}$
3	=	$\frac{1}{4}$	=	$\frac{1}{80}$
2	=	$\frac{1}{6}$	=	$\frac{1}{120}$
$1\frac{1}{2}$	=	$\frac{1}{8}$	=	$\frac{1}{160}$
1	=	$\frac{1}{12}$	=	$\frac{1}{240}$
$\frac{3}{4}$	=	$\frac{1}{16}$	=	$\frac{1}{320}$
$\frac{1}{2}$	=	$\frac{1}{24}$	=	$\frac{1}{480}$
$\frac{1}{4}$	=	$\frac{1}{48}$	=	$\frac{1}{960}$

Parts of 2 Shill.

d.		2s.
1	is	$\frac{1}{24}$
$1\frac{1}{2}$	—	$\frac{1}{16}$
2	—	$\frac{1}{12}$
3	—	$\frac{1}{8}$
4	—	$\frac{1}{6}$
6	—	$\frac{1}{4}$
8	—	$\frac{1}{3}$

Parts of a Pound.

s.	d.		£.
10	0	is	$\frac{1}{2}$
6	8	—	$\frac{1}{3}$
5	0	—	$\frac{1}{4}$
4	0	—	$\frac{1}{5}$
3	4	—	$\frac{1}{6}$
2	6	—	$\frac{1}{8}$
1	8	—	$\frac{1}{12}$
1	4	—	$\frac{1}{15}$
1	3	—	$\frac{1}{16}$
1	0	—	$\frac{1}{20}$
0	10	—	$\frac{1}{24}$
0	8	—	$\frac{1}{30}$
0	5	—	$\frac{1}{48}$
0	$2\frac{1}{2}$	—	$\frac{1}{96}$

Aliquot, or even Parts of Weight.

Parts of a Cwt.

Qrs.	℔		Cwt.
2	0	is	$\frac{1}{2}$
1	0	—	$\frac{1}{4}$
0	16	—	$\frac{1}{7}$
0	14	—	$\frac{1}{8}$
0	8	—	$\frac{1}{14}$
0	7	—	$\frac{1}{16}$
0	4	—	$\frac{1}{28}$

Pts. of $\frac{1}{2}$ Cwt.

℔		$\frac{1}{2}$ Cwt.
28	is	$\frac{1}{2}$
14	—	$\frac{1}{4}$
8	—	$\frac{1}{7}$
7	—	$\frac{1}{8}$
4	—	$\frac{1}{14}$

Pts. of $\frac{1}{4}$ Cwt.

℔		$\frac{1}{4}$ Cwt.
14	is	$\frac{1}{2}$
7	—	$\frac{1}{4}$
4	—	$\frac{1}{7}$
2	—	$\frac{1}{14}$

Parts of a Ton.

Cwt.	qr.		T.
10	0	is	$\frac{1}{2}$
5	0	—	$\frac{1}{4}$
4	0	—	$\frac{1}{5}$
2	2	—	$\frac{1}{8}$
2	0	—	$\frac{1}{10}$
1	1	—	$\frac{1}{16}$
1	0	—	$\frac{1}{20}$

Parts of 240.

180	is	$\frac{3}{4}$
120	—	$\frac{1}{2}$
80	—	$\frac{1}{3}$
60	—	$\frac{1}{4}$
40	—	$\frac{1}{6}$

Parts of 480.

360	is	$\frac{3}{4}$
160	—	$\frac{1}{3}$

Another TABLE of Aliquot *parts of* Money.

Parts of a Shill.

d.		s.
10	is	$\frac{5}{6}$
9	—	$\frac{3}{4}$
8	—	$\frac{2}{3}$
$7\frac{1}{2}$	—	$\frac{5}{8}$
$4\frac{1}{2}$	—	$\frac{3}{8}$

Parts of a Pound.

s.	d.		£.
18	0	is	$\frac{9}{10}$
17	6	—	$\frac{7}{8}$
16	8	—	$\frac{5}{6}$
16	0	—	$\frac{8}{10}$
15	0	—	$\frac{3}{4}$
14	0	—	$\frac{7}{10}$

Parts of a Pound.

s.	d.		£.
13	4	is	$\frac{2}{3}$
12	6	—	$\frac{5}{8}$
12	0	—	$\frac{6}{10}$
8	0	—	$\frac{4}{10}$
7	6	—	$\frac{3}{8}$
6	0	—	$\frac{3}{10}$

A TABLE *of* DISCOUNT *per cent.*

£.	s.	d.	
$1\frac{1}{4}$ *per cent.* is	0	3	*on the Pound.*
$2\frac{1}{2}$	0	6	
$3\frac{3}{4}$	0	9	
5	1	0	
$6\frac{1}{4}$	1	3	
$7\frac{1}{2}$	1	6	

£.	s.	d.	
$8\frac{3}{4}$ *per cent.* is	1	9	*on the Pound.*
10	2	0	
$12\frac{1}{2}$	2	6	
15	3	0	
$17\frac{1}{2}$	3	6	
20	4	0	

£.	s.	d.	
$22\frac{1}{2}$ *per cent.* is	4	6	*on the Pound.*
25	5	0	
30	6	0	
35	7	0	
40	8	0	
45	9	0	
50	10	0	

CASE I.

When the price of 1 *yd.* ℔, *&c. is an even part of one shilling*:— Find the value of the given quantity at 1*s.* per yard, ℔, &c. then draw a line underneath, and divide it by *that even part*, and the quotient will be the answer in shillings, which must always be brought into pounds.

EXAMPLES.

1st. What will $354\frac{1}{2}$ yards cost, at $\frac{1}{4}d.$ per yard?

		s.	*d.*	
$\frac{1}{4}d.$	$\frac{1}{48}$	354	6	*value of* $354\frac{1}{2}$ *yards, at* 1*s. per yard.*
Ans. £0		7	$4\frac{1}{2}$	*value of* $354\frac{1}{2}$ *yards, at* $\frac{1}{4}d.$ *per yard.*

Or thus.

£.	*s.*	*d.*		*s.*	*d.*
8)17	14	6	=	354	6
6)2	4	$3\frac{3}{4}$			
	7	$4\frac{1}{2}$	*Ans. as before.*		

Or divide by 8 *and* 6, *thus,*

8)354 6

6)44 $3\frac{3}{4}$

7 $4\frac{1}{2}$ *Ans. as before.*

2d. What will $759\frac{3}{4}$ yards come to, at 3*d.* per yard?

		s.	*d.*	
3*d.*	$\frac{1}{4}$	759	9	*value at* 1*s. per yard.*
	2|0)18|9		$11\frac{1}{4}$	
Ans. £9		9	$11\frac{1}{4}$	*value at* 3*d. per yard.*

Or thus,

		£.	*s.*	*d.*	
3*d.*	$\frac{1}{4}$	37	19	9	*value at* 1*s. per yard.*
Ans.		£9	9	$11\frac{1}{4}$	*value of* $759\frac{3}{4}$ *yards at* 3*d. per yard.*

Questions. Yds.		*Answers.* £.	*s.*	*d.*
3d. 642	at $\frac{1}{4}d.$ *per yd.*	0	13	$4\frac{1}{2}$
4th. $918\frac{1}{4}$	— $\frac{1}{2}d.$ ——	1	18	$3\frac{1}{8}$
5th. $739\frac{1}{2}$	— 1*d.* ——	3	1	$7\frac{1}{2}$
6th. $567\frac{1}{2}$	— $1\frac{1}{2}d.$ ——	3	10	$11\frac{1}{4}$
7th. $685\frac{3}{4}$	at 2*d.* ——	5	14	$3\frac{1}{2}$
8th. $475\frac{1}{4}$	— 4*d.* ——	7	18	5
9th. $913\frac{1}{2}$	— 6*d.* ——	22	16	9

CASE II.

[*Conclusion of the note begun page* 162.]

EXAMPLE. What is the value of 468 yards, at 2*s.* $9\frac{1}{4}d.$ per yard?

	£468	*s.*	*d.*	*Answer at*	£1	*s.*	*d.*
2*s.* 6*d.* is $\frac{1}{8}$ =	58	10	0	*ditto at*	0	2	6
3*d.* is $\frac{1}{10}$ =	5	17	0	*ditto at*	0	0	3
$\frac{1}{4}d.$ is $\frac{1}{12}$ =	0	9	9	*ditto at*	0	0	$0\frac{1}{4}$
The full price =	£64	16	9		0	2	$9\frac{1}{4}$

In this example it is plain that the quantity 468 is the answer at 1*l.*; consequently as 2*s.* 6*d.* is $\frac{1}{8}$ of a pound, $\frac{1}{8}$ part of that quantity, or 58*l.* 10*s.* is the price at 2*s.* 6*d.*; in like manner, as 3*d.* is the $\frac{1}{10}$ part of 2*s.* 6*d.* so $\frac{1}{10}$ part of 58*l.* 10*s.* or 5*l.* 17*s.* is the answer at 3*d.* and as $\frac{1}{4}d.$ is $\frac{1}{12}$ of 3*d.* so $\frac{1}{12}$ of 5*l.* 17*s.* or 9*s.* 9*d.* is the answer as $\frac{1}{4}d.$—Now, the sum of all these parts is equal to the whole price (2*s.* $9\frac{1}{4}d.$) so the sum of the answers belonging to each price will be the answer at the full price required, and the same will be true in any example whatever.

CASE II.

When the price is pence, and no even part of a ſhilling ;—Find the value of the given quantity at 1*s.* per yard ; divide the pence into aliquot parts, for diviſors, and the ſum of the quotients ariſing from them, will be the anſwer.

EXAMPLES.

1ſt. What will $487\frac{1}{2}$ yards come to at 5*d.* per yard ?

		£.	s.	d.	
3*d.*	$\frac{1}{4}$	24	7	6	*value of* $487\frac{1}{2}$ *yards, at* 1*s. per yard.*
2*d.*	$\frac{1}{6}$				
		6	1	$10\frac{1}{2}$	*value of ditto, at* 3*d. per yard.*
		4	1	3	*value of ditto, at* 2*d. per yard.*
	Anſ.	£.10	3	$1\frac{1}{2}$	*value of ditto, at* 5*d. per yard.*

Queſtions. Yds.	*Anſwers.* £. s. d.
2d. $568\frac{1}{4}$ at 7*d.*	16 11 $5\frac{3}{4}$
3d. $683\frac{3}{4}$ — 8*d.*	22 15 10
4th. $912\frac{1}{2}$ — 9*d.*	34 4 $4\frac{1}{2}$
5th. $649\frac{1}{4}$ at 10*d.*	27 1 $0\frac{1}{2}$
6th. $745\frac{3}{4}$ — 11*d.*	34 3 $7\frac{1}{4}$

CASE III.

When the price is pence or farthings, and an even part of a pound, Cut off the right hand figure of the given quantity, and the cypher, in the aliquot part, (if it has one) and divide by the remaining figure or figures. When you come to the remainder, double it, and divide as before. The anſwer will be pounds, ſhillings, &c. If there be no cypher in the diviſor, then none ſhould be cut off from the dividend.

d.
| $\frac{1}{4}$ | $\frac{1}{96.0}$ | 3795 lb at $\frac{1}{4}$*d.*
96=8 × 12)379|5
8)31 12 6
Anſ. £3 19 $0\frac{3}{4}$

d.
| $\frac{1}{2}$ | $\frac{1}{48.0}$ | 3795 *yds. at* $\frac{1}{2}$*d.*
48=6 × 8)379|5
6)47 8 9
Anſ. £7 18 $1\frac{1}{2}$

d.
| $\frac{3}{4}$ | $\frac{1}{32.0}$ | 3795 *yds. at* $\frac{3}{4}$*d.*
32=4 × 8)379|5
4)47 8 9
Anſ. £11 17 $2\frac{1}{4}$

d.
| 1 | $\frac{1}{24.0}$ | 3795 *yds. at* 1*d.*
24=4 × 6)379|5
4)63 5
Anſ. £15 16 3

d.
| $1\frac{1}{2}$ |

d.
| $1\frac{1}{2}$ | $\frac{1}{16.0}$ | 3795 *yds. at* $1\frac{1}{2}$*d.*
16=4×4)379|5
4)94 17 6
£23 14 $4\frac{1}{2}$ *Anf.*

d.
| 2 | $\frac{1}{12.0}$ | 379|5 *yds. at* 2*d.*
£31 12 6 *Anf.*

d.
| 3 | $\frac{1}{8.0}$ | 379|5 *yds. at* 3*d.*
£47 8 9 *Anf.*

d.
| 4 | $\frac{1}{6.0}$ | 379|5 *yds. at* 4*d.*
£63 5 *Anf.*

d.
| 6 | $\frac{1}{4.0}$ | 379|5 *yds. at* 6*d.*
£94 17 6 *Anf.*

d.
| 8 | $\frac{1}{3.0}$ | 379|5 *yds. at* 8*d.*
£126 10 *Anf.*

d.
| 10 | $\frac{1}{24}$ | 3795 *yds.*
24=4×6)3795
4)632 10
£158 2 6 *Anf.*

CASE IV.

When the price is between one and two ſhillings ;—Find the value of the quantity at 1*s.* per yard, &c. which value being divided by *thoſe even parts* which the pence are of 1*s.* and the quotient or quotients, ariſing therefrom, added thereto ; the ſum will be the anſwer.

EXAMPLES.

1ſt. What will $758\frac{1}{2}$ yards, at 1*s.* 9*d.* per yard, come to ?

		£.	s.	d.	
6*d.*	$\frac{1}{2}$	37	18	6	*value at* 1*s. per yard.*
3*d.*	$\frac{1}{4}$	18	19	3	*value at* 6*d. per yard.*
		9	9	$7\frac{1}{2}$	*value at* 3*d. per yard.*

Anſ. £66 7 $4\frac{1}{2}$ *value of* $758\frac{1}{2}$ *yds. at* 1*s.* 9*d. per yard.*

	Queſtions. Yds.		Anſwers. £.	s.	d.
2d.	$987\frac{1}{2}$ at	$12\frac{1}{2}$*d.*	51	8	$7\frac{3}{4}$
3d.	793 —	$12\frac{3}{4}$*d.*	42	2	$6\frac{3}{4}$
4th.	$847\frac{1}{2}$—	1*s.* 1*d.*	45	18	$1\frac{1}{2}$
5th.	$686\frac{3}{4}$—	1*s.* $1\frac{1}{2}$*d.*	38	12	7
6th.	$591\frac{1}{4}$—	1*s.* 2*d.*	34	9	$9\frac{1}{2}$
7th.	$573\frac{1}{2}$—	1*s.* 3*d.*	35	16	$10\frac{1}{2}$
8th.	$846\frac{1}{2}$—	1*s.* 4*d.*	56	8	8
9th.	$647\frac{3}{4}$ at	1*s.* 5*d.*	45	17	$7\frac{3}{4}$
10th.	$896\frac{1}{4}$ —	1*s.* 6*d.*	67	4	$4\frac{1}{2}$
11th.	458 —	1*s.* 7*d.*	36	5	2
12th.	964 —	1*s.* 8*d.*	80	6	8
13th.	$752\frac{1}{2}$—	1*s.* 10*d.*	68	19	7
14th.	$649\frac{3}{4}$—	1*s.* 11*d.*	62	5	$4\frac{1}{4}$

CASE V.

CASE V.

When the price is any even number of shillings under 40;—Multiply the given quantity by half the price, and double the first figure of the product for shillings. The rest of the product will be pounds.

N. B. If the price be 2*s.* you need only double the unit figure for shillings. The other figures will be pounds.

EXAMPLES.

1st. What will 746 yards cost at 2*s.* per yard?

746

Ans. £74 12 *value at* 2*s. per yard.*

Note. The above is done, by saying, twice 6 (the unit figure) is 12. The other figures, viz. 74, are pounds.

2d. What will 567¾ *yds.* at 2*s.* per yard come to? *Ans.* £56 15/6.

N. B. Before I double the unit figure, viz. 7, I consider that ¾ of a yard, at 2*s.* per yard, will amount to 1*s.* 6*d.* Then I double 7, which makes 14*s.* and 1*s.* 6*d.* added, makes 15*s.* 6*d.* The other figures are pounds.

	Questions. Yds.		*Answers.* £.	s.	d.
3d.	129½	at 4*s. per yard.*	25	18	0
4th.	697	— 6*s.* ———	209	2	0
5th.	845	— 8*s.* ———	338	0	0
6th.	917¼	— 10*s.* ———	458	12	6
7th.	528½	— 12*s.* ———	317	2	0
8th.	646¾	— 14*s.* ———	452	14	6
9th.	591	— 16*s.* ———	472	16	0
10th.	845½	— 18*s.* ———	760	19	0
11th.	645¼	— 24*s.* ———	774	6	0

CASE VI.

When the price wants an even part of 2*s.*—First find the value of the whole quantity at 2*s.* per lb, yard, &c. then divide it by *that even part* which is wanting, and subtract this quotient from the value at 2*s.* The remainder will be the answer

EXAMPLES.

1. What will 95½ yards cost at 22*d.* per yard?

		£.	s.	d.	
2*d.*	$\frac{1}{12}$	9	11	0	*value at* 2*s. per yard.*
		0	15	11	*value at* 2*d. per yard.*
	Ans.	£8	15	1	*value at* 1*s.* 10*d. per yard.*

	Questions. Yds.		*Answers.* £.	s.	d.
2d.	64	at 23*d. per yd.*	6	2	8
3d.	128	— 22½*d.* ———	12	0	0
4th.	246½	— 21*d.* ———	21	11	4½
5th.	375¼	at 20*d. per. yd.*	31	5	5
6th.	486	— 18*d.* ———	36	9	0
7th.	754	— 16*d.* ———	50	5	4

CASE VII.

CASE VII.

When the price is between 2s. and 3s.—First find the value of the quantity at 2s. per yard, &c. which value being divided by *those even parts*, which the pence are of 2s. and those quotients added thereto, the sum will be the answer.

EXAMPLES.

1st. What will $148\frac{1}{2}$ yards come to at 2s. 7d. per yard?

		£.	s.	d.	
4d.	$\frac{1}{6}$	14	17	0	*value at 2s. per yard.*
3d.	$\frac{1}{8}$	2	9	6	*ditto at 4d. per yard.*
		1	17	$1\frac{1}{2}$	*ditto at 3d. per yard.*
	Ans.	£19	3	$7\frac{1}{2}$	*value at 2s. 7d. per yard.*

	Questions. Yds.		*Answers.* £.	s.	d.
2d.	$266\frac{1}{4}$	at 2s. 1d. *per yard.*	27	14	$8\frac{1}{4}$
3d.	344	— 2s. $1\frac{1}{2}d.$ ———	36	11	0
4th.	$543\frac{1}{2}$	— 2s. 2d. ———	58	17	7
5th.	$655\frac{3}{4}$	— 2s. 3d. ———	73	15	$5\frac{1}{4}$
6th.	716	— 2s. 4d. ———	83	10	8
7th.	813	— 2s. 5d. ———	98	4	9

CASE VIII.

When there are pence in the price which are an even part of a shilling, besides an even number of shillings under 20; First find the value of the quantity at the shillings per yard, &c. according to Case 5th; then suppose the quantity to stand as shillings per yard; divide it by *that even part*, which the pence are of 1s. and this quotient being added to the value before found, the sum will be the answer.

EXAMPLES.

1st. What will $156\frac{1}{2}$ yards come to, at 6s. 4d. per yard?

			Yds.			
			$156\frac{1}{2}$			
		s. d.	3			
4d.	$\frac{1}{3}$	156 6				
			£46	19	0	*value of $156\frac{1}{2}$ yards at 6s. per yard.*
		52s. 2d. =	2	12	2	*value of ditto at 4d. per yard.*
		Ans.	£49	11	2	*value of ditto at 6s. 4d. per yard.*

Questions.

Questions.					Answers.		
	Yds.	s.	d.		£.	s.	d.
2d.	$17\frac{1}{2}$ at	4	$\frac{1}{2}$	*per yard.*	3	10	$8\frac{3}{4}$
3d.	$59\frac{3}{4}$ —	6	$\frac{3}{4}$	———	18	2	$2\frac{3}{4}$
4th.	$68\frac{1}{4}$ —	8	1	———	27	11	$8\frac{1}{4}$
5th.	96 —	10	$1\frac{1}{2}$	———	48	12	0
6th.	$67\frac{1}{2}$ —	12	2	———	41	1	3

CASE IX.

When the price is any odd number of ſhillings under 40; Find the value of the greateſt even number contained in the price, according to Caſe 5th, and add thereto the value of the quantity at 1*s.* per yard, &c. which ſum will be the anſwer: *Or*, Multiply the quantity by the price, according to the 1ſt or 2d Caſe in Simple Multiplication, and divide the product by 20, the quotient will be the anſwer: *Or*, laſtly, if the price be under 13*s.* find the value of the quantity at 1*s.* per yard, &c. and multiply it by the number of ſhillings in the price of 1 yard; the product will be the anſwer.

EXAMPLES.

1ſt. What will 186 yards coſt, at 3*s.* per yard?

£.	s.	
18	12	*value at 2s. per yd.*
9	6	*ditto at 1s. per yd.*
£ 27	18	*Anſ.*

Or thus.

	£.	s.	
	9	6	*value at 1s. per yard.*
		3	
Product	£ 27	18	*Anſ.*

2d. What will 647 yards coſt, at 17*s.* per yard?

	647		
	8		
	£ 517	12	*value at 16s. per yard.*
	32	7	*ditto at 1s. per yard.*
Anſ.	£ 549	19	*ditto at 17s. per yard.*

Questions.					Answers.		
	Yds.	s.			£.	s.	d.
3d.	$169\frac{1}{4}$ at	5	*per yd.*		42	6	3
4th.	$248\frac{3}{4}$ —	7	———		87	1	3
5th.	139 at	9	———		62	11	0
6th.	782 —	25	———		977	10	0

CASE X.

When the price is an even part of a pound; Find the value of the given quantity, at one pound per yard, &c. then draw a line underneath, and divide it by *that part;* the quotient will be the anſwer.

EXAMPLES.

1st. What will $156\frac{3}{4}$ yards of cloth come to, at 3*s.* 4*d.* per yard?

s.	d.		£.	s.	d.	
3	4	$\frac{1}{6}$	156	15	0	*price at 1l. per yard.*
		Anf.	£26	2	6	*price at 3s. 4d. per yard.*

Questions. Yds.	s. d.	*Answers.* £. s. d.	*Questions.* Yds.	s. d.	*Answers.* £. s. d.
2d. $516\frac{3}{4}$	at 1 0 *per yd.*	25 16 9	7th. $429\frac{1}{4}$	— 4 0 —	85 17 0
3d. 624	— 1 3 —	39 0 0	8th. $687\frac{1}{2}$	— 5 0 —	171 17 6
4th. $710\frac{1}{2}$	— 1 4 —	47 19 4	9th. 843	— 6 8 —	281 0 0
5th. 648	— 1 8 —	54 0 0	10th. $486\frac{3}{4}$	— 10 0 —	243 7 6
6th. $419\frac{3}{4}$	— 2 6 —	52 9 $4\frac{1}{2}$			

CASE XI.

When the price wants an even part of a pound; First find the value of the given quantity at 1*l.* per yard, &c. then divide it by *that even part* which is wanting, and ſubtract this quotient therefrom; the remainder will be the anſwer.

EXAMPLES.

1st. What will $167\frac{1}{2}$ yards coſt, at 17*s.* 6*d.* per yard?

s.	d.		£.	s.	d.	
2	6	$\frac{1}{8}$	167	10	0	*value at 1l. per yard.*
			20	18	9	*ditto at 2s. 6d. per yard.*
		Anf.	£146	11	3	*value at 17s. 6d. per yard.*

Questions. Yds.	s. d.	*Answers.* £. s. d.	*Questions.* Yds.	s. d.	*Answers.* £. s. d.
2d. $347\frac{1}{2}$	at 13 4 *per yd.*	231 13 4	4th. 614	— 16 0 —	491 4 0
3d. $485\frac{3}{4}$	— 15 0 —	364 6 3	5th. $912\frac{1}{4}$	— 17 6 —	798 4 $4\frac{1}{2}$

CASE XII.

When the price is ſhillings, pence and farthings, and not an even part of a pound; Multiply the given quantity by the ſhillings in the price of 1 yard, &c. and take parts of parts from the quantity for the pence, &c. then add them together, and their ſum will be the anſwer, in ſhillings, &c. *Or,* you may let the given quantity ſtand as pounds per yard, &c. then draw a line underneath, and take parts of parts therefrom; which add together, and their ſum will be the anſwer.

N. B. I adviſe the learner to work the following examples both ways, by which means he will be able to diſcover the moſt conciſe method of performing ſuch queſtions, in buſineſs, as may fall under this caſe.

1ft. What will $248\frac{1}{2}$ yards, at 7*s.* 6*d.* per yard, come to?

d.		s.	d.	
6	$\frac{1}{2}$	248	6	*value of $248\frac{1}{2}$ yards, at 1s. per yard.*
			7	
		1739	6	*value of ditto at 7s. per yard.*
		124	3	*value of ditto at 6d. per yard.*
		2|0)186|3	9	
	Anſ.	£93 3	9	*value of ditto at 7s. 6d. per yard.*

Or thus,

d.		£.	s.	d.	
6	$\frac{1}{2}$	12	8	6	*value of $248\frac{1}{2}$ yards, at 1s. per yard.*
Multiply by				7	
		86	19	6	*value of ditto at 7s. per yard.*
		6	4	3	*value of ditto, at 6d. per yard.*
Anſ.		£93	3	9	

By the latter part of this caſe,

s. d.		£.	s.	d.	
5 0	$\frac{1}{4}$	248	10	0	*value of $248\frac{1}{2}$ yards, at 1l. per yard.*
2 6	$\frac{1}{2}$	62	2	6	*value of ditto, at 5s. per yard.*
		31	1	3	*value of ditto, at 2s. 6d. per yard.*
	Anſ.	£93	3	9	*value of ditto, at 7s. 6d. per yard.*

Queſtions.	Yds.		s.	d.		*Anſwers.* £.	s.	d.
2d.	$68\frac{1}{2}$	at	4	6	*per yd.*	15	8	3
3d.	124	—	5	8	—	35	2	8
4th.	146	—	14	9	—	107	13	6
5th.	$218\frac{1}{2}$	at	12	6	—	136	11	3
6th.	645	—	4	$1\frac{1}{2}$	—	133	0	$7\frac{1}{2}$

CASE XIII.

When the price of the yard, ℔, &c. is pounds, ſhillings and pence:— Firſt, multiply the quantity by the pounds, and if the ſhillings and pence be an even part of a pound, divide the given quantity by *that part*, and add the quotient to the product for the anſwer. But if they be not an even part of a pound, you muſt take parts of parts, and add them together as before. *Or*, reduce the pounds and ſhillings into ſhillings, and multiply the quantity thereby, after which, take parts for the pence, and add the whole together, and their ſum will be the anſwer in ſhillings, &c.

N. B. The learner ſhould work the following queſtions both ways.

EXAMPLES.

EXAMPLES.

1st. What will 156 yards of broadcloth come to, at 3*l.* 6*s.* 8*d.* per yard?

	£.	s.	d.	
6/8 | $\frac{1}{3}$	156	0	0	*value at 1l.*
	3			*per yard.*
	468	0	0	
	52	0	0	
Anf.	£520	0	0	

Or thus.

d.		s.	
4	$\frac{1}{3}$	156	*value at 1s. per yard.*
4	$\frac{1}{3}$	66	*fhillings in the price of 1 yd.*
		936	
		936	
		10296	*value at 3l. 6s. per yard.*
		52	
		52	
		2|0)1040|0	
		£520 0 0	*Anf.*

Questions. Yds.	£. s. d.		Anfwers. £. s. d.
2d. $345\frac{1}{2}$	at 6 5 0	*per yd.*	2159 7 6
3d. $59\frac{3}{4}$	— 3 6 8	——	199 3 4
4th. 75	— 5 3 4	——	387 10 0
5th. 68	— 4 6 0	——	292 8 0
6th. 59	— 6 7 6	——	376 2 6
7th. $112\frac{1}{2}$	— 3 8 8	——	386 5 0
8th. 125	— 4 9 7	——	559 17 11

CASE XIV.

When the quantity is any number lefs than 1000, *and the price not more than* 12*d. per yard, &c.*—Find the value of the whole quantity at 1*d.* per yard, which may be done by dividing it by 12, *mentally*, fetting down the quotient only in pounds, or fhillings, or both. Then multiply this fum by the pence in the price of 1 yard, and the product will be the anfwer.

1. What will $759\frac{1}{2}$ yards coft, at 7*d.* per yard?

	£.	s.	d.	
	0	63	$3\frac{1}{2}$	*value at 1d. per yard.*
Or,	3	3	$3\frac{1}{2}$	*value at 1d. per yard.*
Mult. by			7	
Anf.	£22	3	$0\frac{1}{2}$	*value of $759\frac{1}{2}$ yards, at 7d. per yard.*

Questions. Yds.	d.		Anfwers. £. s. d.
2d. $975\frac{1}{2}$	at 2	*per yd.*	8 2 7
3d. 846	— $3\frac{1}{2}$	——	12 6 9
4th. $793\frac{3}{4}$	— $4\frac{3}{4}$	——	15 14 $2\frac{1}{4}$
5th. 684	— $5\frac{1}{2}$	——	15 13 6
6th. $984\frac{1}{2}$	— $6\frac{3}{4}$	——	27 13 $9\frac{1}{4}$
7th. $440\frac{1}{2}$	— 9	——	16 10 $4\frac{1}{2}$

CASE XV.

When the price is fuch a number of fhillings and pence, as, when reduced into pence, may be produced by any two numbers in the multiplication

tion table, and when the quantity does not exceed 1000 ;—First find the value of the whole at 1*d.* per yard, &c. according to the last case ; then multiply this sum by the component parts of the pence in the price, and the last product will be the answer.

EXAMPLES.

1st. What will $439\frac{1}{2}$ yards cost, at 6*s.* 9*d.* per yard ?

£.	s.	d.	
1	16	$7\frac{1}{2}$	*value at* 1*d. per yard.*
		9	
16	9	$7\frac{1}{2}$	
		9	
Ans. £ 148	6	$7\frac{1}{2}$	*value at* 81*d. or* 6*s.* 9*d. per yard.*

N. B. In 6*s.* 9*d.* there being 81 pence, I multiply by 9 twice; because 9 times 9 is 81.

Questions. Yds.	s. d.		*Answers.* £.	s.	d.
2d. $984\frac{1}{2}$	at 1 2	*per yd.*	57	8	7
3d. $849\frac{3}{4}$	— 2 8	——	113	6	0
4th. 657	— 3 6	——	114	19	6
5th. 593	— 4 8	——	138	7	4

CASE XVI.

When the quantity is 240 ; As many pence as there are in the price of 1 yard, &c. so many pounds will the quantity amount to.

N. B. One farthing per yard will come to 5*s.* at a halfpenny per yard to 10*s.* and at three farthings, to 15*s.*

EXAMPLES.

1st. What will 240 yards come to, at 2*s.* $7\frac{1}{2}$*d.* per yard ?

Ans. £ 31 10*s.*

N. B. The price is 2*s.* $7\frac{1}{2}$*d.* per yard. Now, as in 2*s.* 7*d.* there are 31 pence, so the quantity being 240 comes to 31 pounds. Then according to the same rule, the halfpenny per yard comes to 10*s.* Therefore the answer to the question is 31*l.* 10*s.*

Questions. Yds.	s. d.		*Answers.* £.	s.	d.
2d. 240	at 1 $7\frac{3}{4}$	*per yd.*	19	15	0
3d. 240	— 2 9	——	33	00	0
4th. 240	— 3 4	——	40	00	0
5th. 240	at 7 $8\frac{1}{2}$	——	92	10	0
6th. 240	— 8 $3\frac{1}{4}$	——	99	15	0

CASE XVII.

When the quantity is not less than 228, *nor more than* 252 ;—First find the value of 240 yards, &c. by the last Case ; then multiply the price of 1 yard by the number *above* or *under* 240, and add or subtract

ſubtract this product to or from the value of 240 yards, as the queſtion may require; and the ſum or remainder will be the anſwer.

EXAMPLES.

1ſt. What will 248 yards come to, at 16*s.* $5\frac{1}{2}$*d.* per yard?

	£.	s.	d.	
	197	10	0	*value of* 240 *yards.*
16*s.* $5\frac{1}{2}$*d.* *multiplied by* 8 =	6	11	8	*value of* 8 *yards.*
Anſ.	£204	1	8	*value of* 248 *yards.*

2d. What will 229 yards coſt, at 5*s.* $9\frac{3}{4}$*d.* per yard?

	£.	s.	d.	
	69	15	0	*value of* 240 *yards.*
5*s.* $9\frac{3}{4}$*d.* *mult. by* 11 =	3	3	$11\frac{1}{4}$	*value of* 11 *yards.*
Anſ.	£66	11	$0\frac{3}{4}$	*value of* 229 *yards.*

Queſtions.	Yds.		s.	d.		£.	s.	d.	*Anſwers.*
3d.	228	at	6	$10\frac{1}{2}$	*per yd.*	78	7	6	
4th.	252	—	4	6	—	56	14	0	
5th.	230	—	9	$8\frac{3}{4}$	—	111	17	$8\frac{1}{2}$	
6th.	$251\frac{1}{2}$	—	3	$10\frac{1}{2}$	—	48	14	$6\frac{3}{4}$	
7th.	$231\frac{1}{4}$	—	8	2	—	94	8	$6\frac{1}{2}$	
8th.	250	—	5	$9\frac{1}{4}$	—	72	2	$8\frac{1}{2}$	

CASE XVIII.

When the quantity is 480;—Find the value of 240 yards, &c. by Caſe the 16th, and multiply this ſum by 2. The product will be the anſwer.

N. B. If the quantity be 12 over, or under 480, proceed according to the directions given in the laſt caſe.

EXAMPLES.

1ſt. What will 480 yards coſt at 2*s.* $9\frac{1}{2}$*d.* per yard?

	£.	s.	d.	
	33	10	0	*value of* 240 *yards.*
Multiplied by			2	
Anſ.	£67	0	0	*value of* 480 *yards.*

2d. What will 468 yards come to at 5*s.* $8\frac{1}{2}$*d.* per yard?

	£.	s.	d.	
	68	10	0	*value of* 240 *yards.*
			2	
	137	00	0	*value of* 480 *yards.*
Subtract	3	8	6	*value of* 12 *yards*
Anſ.	£133	11	6	*value of* 468 *yards.*

3d. What

3d. What will 492 yards come to, at 3*s.* $8\frac{3}{4}$*d.* per yard?

£.	s.	d.	
44	15	0	*value of* 240 *yards.*
		2	
89	10	0	*value of* 480 *yards.*
Add 2	4	9	*value of* 12 *yards.*
Anſ. £91	14	9	*value of* 492 *yards.*

N. B. Any perſon, who is expert in figures, may find the value, *mentally*, of 480 yards, almoſt as eaſily as 240, it being nothing more than doubling the amount of 240.

	Yds.		s.	d.		£.	s.	d.	
Queſtions.	4th. 469	at	6	$9\frac{1}{2}$	*per yd.*	159	5	$3\frac{1}{2}$	*Anſwers.*
	5th. 470	—	3	$4\frac{3}{4}$	—	79	16	$0\frac{1}{2}$	
	6th. 471	—	5	$3\frac{1}{2}$	—	124	12	$4\frac{1}{2}$	
	7th. 472	—	4	9	—	112	2	0	
	8th. 483	—	8	$10\frac{3}{4}$	—	214	16	$8\frac{1}{4}$	

CASE XIX.

When the quantity is 160;—Find the value of 480 yards, and divide it by 3. The quotient will be the anſwer.

Note. If there be 12 yards over, or under, 160, proceed as before directed.

EXAMPLES.

1ſt. What will 160 yards come to, at 3*s.* $4\frac{1}{2}$*d.* per yard?

	£.	s.	d.	
Divide by 3)	81	0	0	*value of* 480 *yards.*
Anſ.	£27	0	0	*value of* 160 *yards.*

2d. What coſt 148 vards, at 4*s.* 2*d.* per yard?

	£.	s.	d.	
Divide by 3)	100	0	0	*value of* 480 *yards.*
	33	6	8	*value of* 160 *yards.*
Subtract	2	10	0	*value of* 12 *yards.*
Anſ.	£30	16	8	*value of* 148 *yards.*

3d. What will 172 yards amount to, at 5*s.* $7\frac{3}{4}$*d.* per yard?

	£.	s.	d.	
Divide by 3)	135	10	0	*value of* 480 *yards.*
	45	3	4	*value of* 160 *yards.*
Add	3	7	9	*value of* 12 *yards.*
Anſ.	£48	11	1	*value of* 172 *yards.*

Queſtions.

		Yds.		s. d.		£. s. d.	
Questions.	4th.	149	at	12 $6\frac{1}{2}$	per yard.	93 8 $8\frac{1}{2}$	Answers.
	5th.	150	—	3 8	——	27 10 0	
	6th.	166	—	12 8	——	105 2 8	
	7th.	152	—	5 $2\frac{3}{4}$	——	39 14 10	
	8th.	153	—	6 3	——	47 16 3	
	9th.	171	—	7 9	——	66 5 3	

CASE XX.

When the quantity is 120; First find the value of 240 yards, &c. then divide it by 2, and the quotient will be the answer.

Note. If there be 12 over, or under, 120, proceed as before directed.

EXAMPLES.

1st. What will 120 yards cost, at 3*s.* $7\frac{1}{2}$*d.* per yard?

	£.	s.	d.	
Divide by 2)	43	10	0	*value of* 240 *yards.*
Ans.	£21	15	0	*ditto of* 120 *yards.*

2d. What will 108 yards come to, at 4*s.* 7*d.* per yard?

	£.	s.	d.	
Divide by 2)	55	0	0	*value of* 240 *yards.*
	27	10	0	*value of* 120 *yards.*
Subtract	2	15	0	*ditto of* 12 *yards.*
Ans.	£24	15	0	*value of* 108 *yards.*

3d. What is the value of 132 yards, at 4*s.* $3\frac{1}{2}$ per yard?

	£.	s.	d.	
Divide by 2)	51	10	0	*value of* 240 *yards.*
	25	15	0	*value of* 120 *yards.*
Add	2	11	6	*value of* 12 *yards.*
Ans.	£28	6	6	*value of* 132 *yards.*

		Yds.		s. d.		£. s. d.	
Questions.	4th.	109	at	3 $9\frac{1}{2}$	per yard.	20 13 $3\frac{1}{2}$	Answers.
	5th.	110	—	4 3	——	23 7 6	
	6th.	125	—	15 7	——	97 7 11	
	7th.	112	—	4 9	——	26 12 0	
	8th.	113	—	5 6	——	31 1 6	
	9th.	130	—	6 4	——	41 3 4	

CASE XXI.

When the quantity is 80 *yards, &c.* One third part of the value of 240 will be the answer.

N. B. If

N. B. If there be 12 over, or under, proceed as before directed; except when the quantity is found in the Multiplication Table; for, then, Case the 2d. of Compound Multiplication will be more concise; or, when the price is an even part of a pound, Case 10th of Practice, is to be preferred.

EXAMPLES.

1st. What cost 80 yards of cloth, at 7*s.* 9¾*d.* per yard?

£. *s.* *d.*
Divide by 3)93 15 0 *value of* 240 *yards.*

Ans. £31 5 0 *ditto of* 80 *yards.*

2d. What will 68 yards cost, at 4*s.* 9¼*d.* per yard?

£. *s.* *d.*
Divide by 3)57 15 0 *value of* 240 *yards.*

19 5 0 *value of* 80 *yards.*
Subtract 2 17 9 *value of* 12 *yards.*

Ans. £16 7 3 *value of* 68 *yards.*

3d. What will 92 yards cost, at 6*s.* 4*d.* per yard?

£. *s.* *d.*
Divide by 3)76 0 0 *value of* 240 *yards.*

25 6 8 *value of* 80 *yards.*
Add 3 16 0 *value of* 12 *yards.*

Ans. £29 2 8 *value of* 92 *yards.*

		Yds.		*s.*	*d.*		£.	*s.*	*d.*	
Questions.	4th.	69	at	10	7½	*per yard.*	36	13	1½	*Answers.*
	5th.	70	—	3	7¾	——	12	15	2½	
	6th.	71	—	4	5	——	15	13	7	
	7th.	84	—	16	10	——	70	14	0	
	8th.	86	—	14	9	——	63	8	6	
	9th.	90	—	2	9	——	12	7	6	

CASE XXII.

When the quantity is 60 *yards, &c.* One fourth of the value of 240 will be the answer.

Note. If there be 12 over, or under, proceed as before directed, and observe the exception made in the last case.

EXAMPLES.

1st. What will 60 yards of cloth cost, at 3*s.* 9¾*d.* per yard?

£. *s.* *d.*
Divide by 4)45 15 0 *value of* 240 *yards.*

Ans. £11 8 9 *value of* 60 *yards.*

2d. What will 48 yards, at 4*s.* $5\frac{1}{2}$*d.* per yard, come to?

	£.	s.	d.		
Divide by 4)	53	10	0	*value of*	240 *yards.*
	13	7	6	*value of*	60 *yards.*
Subtract	2	13	6	*value of*	12 *yards.*
Anſ.	£10	14	0	*value of*	48 *yards.*

3d. What will 72 yards come to, at 5*s.* $4\frac{1}{2}$*d.* per yard?

	£.	s.	d.		
Divide by 4)	64	10	0	*value of*	240 *yards.*
	16	2	6	*value of*	60 *yards.*
Add	3	4	6	*value of*	12 *yards.*
Anſ.	£19	7	0	*value of*	72 *yards.*

		Yds.		*s.*	*d.*		£.	s.	d.	
Queſtions.	4th.	49	at	6	$10\frac{1}{2}$	*per yard.*	16	16	$10\frac{1}{2}$	*Anſwers.*
	5th.	50	—	9	4	——	23	6	8	
	6th.	51	—	10	3	——	26	2	9	
	7th.	66	—	6	7	——	21	14	6	
	8th.	69	—	17	5	——	60	1	9	
	9th.	70	—	9	3	——	32	7	6	

CASE XXIII.

When the quantity is 180; Three fourths of the value of 240 will be the anſwer.

EXAMPLES.

1ſt. What will 180 yards coſt, at 5*s.* $5\frac{1}{2}$*d.* per yard?

	£.	s.	d.		
Divide by 2)	65	10	0	*value of*	240 *yards.*
Divide by 2)	32	15	0	*value of*	120 *yards.*
	16	7	6	*value of*	60 *yards.*
Anſ.	£49	2	6	*value of*	180 *yards.*

		Yds.		*s.*	*d.*		£.	s.	d.	
Queſtions.	2d.	180	at	3	$9\frac{1}{4}$	*per yard.*	33	18	9	*Anſwers.*
	3d.	180	—	4	$9\frac{1}{2}$	——	43	2	6	
	4th.	180	—	6	$4\frac{3}{4}$	——	57	11	3	
	5th.	180	—	7	3	——	65	5	0	
	6th.	180	—	8	$9\frac{1}{2}$	——	79	2	6	
	7th.	180	—	13	$9\frac{1}{2}$	——	124	2	6	

CASE XXIV.

When the price of one hundred weight is of ſeveral denominations, and the quantity likewiſe; Multiply the price by the integers, (that is the

the whole numbers) and take parts for the reſt from the price of an integer; which, added together, will be the anſwer.

EXAMPLES.

1ſt. What will 9 *Cwt.* 3*qrs.* 14℔ of ſugar come to, at 4*l.* 17*s.* 4*d.* per Cwt.?

qrs.	℔		£.	s.	d.		Cwt.	qr.	℔
2	0	$\frac{1}{2}$	4	17	4	*price of* 1 *Cwt.*			
1	0	$\frac{1}{2}$			9				
0	14	$\frac{1}{2}$							
			43	16	0	*price of*	9	0	0
			2	8	8	*price of*	0	2	0
			1	4	4	*price of*	0	1	0
			0	12	2	*price of*	0	0	14
		Anſ.	£48	1	2	*price of*	9	3	14

Questions	Cwt.	qrs.	℔		£.	s.	d.		£.	s.	d.
2d.	8	1	16	Tobacco, at	5	17	9	*per cwt.*	49	8	$2\frac{1}{4}$
3d.	7	3	19	———	7	12	8	———	60	9	$0\frac{1}{2}$
4th.	12	1	24	———	3	18	10	———	49	2	$7\frac{1}{4}$
5th.	16	2	17	———	2	15	11	———	46	11	1
6th.	72	3	27	———	8	11	5	———	625	11	$10\frac{3}{4}$
7th.	59	1	14	Sugar, at	1	8	7	———	84	17	$1\frac{1}{2}$
	℔		oz.								
8th.	27		10	Coffee, at	0	1	4	*per* ℔	1	16	10
	℔	oz.	pwt. gr.								
9th.	13	10	12 8	Silv. at	4	7	6	*per* ℔	60	14	$11\frac{1}{2}$
	oz.	pwt.	gr.								
10th.	17	6	16	Gold, at	3	16	8	*per oz.*	66	8	$10\frac{2}{3}$

Queſtions. — *Anſwers.*

CASE XXV.

When the price is at any of the rates in the ſecond Practice Table of aliquot parts; Multiply the given quantity by the numerator, and divide that product by the denominator; if the price be pence, the quotient will be the anſwer in ſhillings; if ſhillings, the anſwer will be pounds.

EXAMPLES.

1ſt. What will 379 yards, at $4\frac{1}{2}d.$ per yard come to?

```
    379
      3
   ----
 8)1137
   ----
2|0)14|2  1½
   ----
Anſ. £7  2  1½
```

2d. What will 149 yards, at 6*s.* per yard come to?

```
    149
      3
   ----
1|0)44|7
   ----
Anſ. £44 14
```

Queſtions.

		Yds.		s.	d.		£.	s.	d.	
Questions.	3d.	127	at	0	$7\frac{1}{2}$	per yd.	3	19	$4\frac{1}{2}$	Answers.
	4th.	159	—	0	8	———	5	6	0	
	5th.	173	—	0	9	———	6	9	9	
	6th.	241	—	0	10	———	10	0	10	
	7th.	249	—	7	6	———	93	7	6	
	8th.	357	—	12	6	———	223	2	6	
	9th.	345	—	13	4	———	230	0	0	
	10th.	323	—	14	0	———	226	2	0	
	11th.	287	—	15	0	———	215	5	0	
	12th.	253	—	16	8	———	210	16	8	

CASE XXVI.

When the price is any even number of ſhillings, if it be required to know what quantity of any thing may be bought for ſo much money; Annex a cypher to the money, and divide it by half of the price, and the quotient will be the quantity to be purchaſed.

EXAMPLES.

1ſt. How many yards of cloth, at 18*s.* per yard, may I have for £345?

Half the price = 9)3450 = *money with a cypher annexed.*

$383\frac{1}{3}$ *yards, Anſ.*

			s.		£.		Yds.	
Questions.	2d.	How many *yds*, at	2	*per yd.* for	427 ?	——	4270	Anſwers.
	3d.	————	4	————	312	——	1560	
	4th.	————	6	————	917	——	$3056\frac{2}{3}$	
	5th.	————	8	————	195	——	$487\frac{1}{2}$	
	6th.	————	10	————	247	——	494	
	7th.	————	12	————	439	——	$731\frac{2}{3}$	

CASE XXVII.

To find the diſcount of any invoice, or bill of parcels, at any rate per cent. Multiply the pounds in the invoice by the amount of the diſcount of 1 pound, at the rate per cent. and take parts for the ſhillings and pence; then add them together, and the ſum will be the diſcount required.

N. B. The diſcount for 1 pound at any rate per cent. is in the 3d. Practice Table.

EXAMPLES.

1ſt. What is the diſcount of an invoice, amounting to £65 18*s.* 4*d.* at £$7\frac{1}{2}$ per cent?

OPERATION

OPERATION.

	£.	s.	d.
The difcount of 65*l.* at 5*l.* per cent. is 65*s.* or	3	5	0
The difcount of 65*l.* at $2\frac{1}{2}$*l.* per cent. is -	1	12	6
The difcount of 10*s.* being half a pound, at $7\frac{1}{2}$*l.* per cent. is - - - - - -	0	0	9
The difcount of 3*s.* 4*d.* being $\frac{1}{6}$ of a pound at $7\frac{1}{2}$*l.* per cent. is - - - -	0	0	3
The fum is,	£4	18	6 *Anf.*

2d. What is the difcount of a bill of parcels, amounting to 8*l.* 17*s.* 8*d.* at $2\frac{1}{2}$*l.* per cent ?

	£.	s.	d.
The difcount of 8*l.* is - - -	0	4	0
The difcount of 10*s.* is - - -	0	0	3
The difcount of 6*s.* 8*d.* is - -	0	0	2
The difcount of 1*s.* is - - -	0	0	$0\frac{1}{4}$
Anf.	£0	4	$5\frac{1}{4}$

N. B. When the rate per cent. is any even part of £100, it may be performed by dividing the amount by *that even* part.

3d. What is the difcount of an invoice amounting to 57*l.* 13*s.* 9*d.* at $12\frac{1}{2}$*l.* per cent. ?

		£.	s.	d.
$12\frac{1}{2}$	$\frac{1}{8}$	57	13	9
		£7	4	$2\frac{1}{2}$ *Anf.*

CASE XXVIII.

To find the value of goods fold by particular quantities, viz. I. By the fcore. II. Round timber. III. By 5 fcore to the hundred. IV. By 112 to the hundred. V. By 6 fcore to the hundred. VI. By the great grofs. VII. By the 1000.

I. *To find the value of goods fold by the fcore.*

The price of one is given, to find the price of one fcore.

If the given price be fhillings and pence, or only pence, divide the given price, in pence, by 12. The quotient will be the anfwer in pounds, and the remainder will be fo many times 1*s.* 8*d.*

EXAMPLES.

1ft. At 9*d.* each ; What is that per fcore ?

12)9*d.*(,75 = £0 15 0 *Anf.*

Or by inverting the queftion.

1 fcore = 20 = 1*s.* 8*d.*

9

15*s.* 0

2d. At 4*s.* 9*d.* each ; What is that per fcore ?

4*s.* 9*d.*

12

12)57*d.*

£4 15 *Anf.*

It

It may be remarked, that when the price is ſhillings and pence, the anſwer will be juſt ſo many pounds as there are ſhillings, and ſo many times 1*s.* 8*d.* as there are pence. If farthings are given, for $\frac{1}{4}$*d.* reckon 5*d.* for $\frac{1}{2}$*d.* 10*d.* and for $\frac{3}{4}$*d.* 1*s.* 3*d.*

TABLE *of* Aliquot Parts. 20 *the* Integer.

2 is $\frac{1}{10}$	6 is $\frac{3}{10}$	12 is $\frac{6}{10}$	16 is $\frac{8}{10}$
4 — $\frac{1}{5}$	8 — $\frac{4}{10}$	14 — $\frac{7}{10}$	18 — $\frac{9}{10}$
5 — $\frac{1}{4}$	10 — $\frac{1}{2}$	15 — $\frac{3}{4}$	

3d. What coſt 7 ; at 2*s.* 9*d.* per ſcore ?

		s.	*d.*
5	$\frac{1}{4}$	2	9
2	$\frac{1}{10}$		
		0	$8\frac{1}{4}$
		0	$3\frac{1}{4}$
7	=	0	$11\frac{1}{2}$

4th. What coſt 17 ; at 19*s.* 10*d.* per ſcore ?

		s.	*d.*
10		19	10
5	$\frac{1}{2}$	9	11
2	$\frac{1}{4}$	4	$11\frac{1}{2}$
	$\frac{1}{10}$	1	$11\frac{3}{4}$
17	=	16	$10\frac{1}{4}$

II. *Round Timber.*

Forty feet make a load or ton of round timber.

If the given price of a foot be ſhillings,

RULE.

Multiply the given price by 2, and the product will be the anſwer in pounds.

5th. What coſt a ton, at 3*s.* per foot ? 3*s.* × 2 = 6*l.* *Anſ.*

6th. What coſt a ton at 9*s.* per foot ? 9*s.* × 2 = 18*l.* *Anſ.*

If the given price of 1 foot be pence only, or ſhillings and pence, divide the given price, in pence, by 6. The quotient will be the anſwer in pounds, and the remainder will be ſo many times 3*s.* 4*d.*

7th. What coſt 40 feet, at 17*d.* per foot ?

6)17

£2 16 8 *Anſ.*

8th. At 1*s.* 9*d.* per foot, What coſt a ton ?

6)21

£3 10 *Anſ.*

If the given price of a foot be farthings only, or pence and farthings, divide the given price, in farthings, by 6 ; then divide *that quotient* by 4, and *this laſt quotient* will be the anſwer.

9th. At $\frac{3}{4}$*d.* per foot, What coſt a ton ?

6)3

4)0 10

£0 2 6 *Anſ.*

10th. At $13\frac{1}{4}$ per foot, What coſt a ton ?

$13\frac{1}{4}$
4

6)53

4)8 1 6 8

£2 4 2 *Anſ.*

Or,

Or, ſuppoſe every ſhilling in the price to be 2*l.* every penny to be 3*s.* 4*d.* and every farthing to be 10*d.*

11th. What coſt 40 feet at $\frac{3}{4}d.$ per foot?

$\frac{3}{4}d. \times 10 = £0\ 2\ 6$ *Anſ.*

12th. What coſt 40 at $15\frac{1}{2}d.$ per foot?

s.	d.			£	s.	d.
1	0	× 2	=	£2	0	0
3	4	× 3	=	0	10	0
0	$\frac{1}{2}$	× 10	=	0	1	8
				£2	11	8 *Anſ.*

III. *To find the value of goods ſold by 5 ſcore to the hundred.*

1ſt. If the given price be pounds and ſhillings, or ſhillings only.

RULE.

Multiply the given price, in ſhillings, by 5, and the quotient will be the anſwer in pounds.

13th. At 19*s.* per yard, What coſt 100 yards?

19*s.*
5
£95 *Anſ.*

14th. At 4*l.* 13*s.* per cwt. What coſt 100 cwt. or 5 tons?

4 13
20
93
5
£465 *Anſ.*

2d. If the given price of 1 be pence only, or ſhillings and pence.

RULE.

Multiply the given price, in pence, by 5; then divide that product by 12. The quotient will be pounds, and the remainder ſo many times 1*s.* 8*d.*

15th. If 1 yard coſt 9*d.* What coſt 100 yards?

9
5
12)45
£3 15 *Anſ.*

16th. What coſt 100 buſhels, at 35*s.* 4*d.* per buſhel?

s. *d.*
35 4
12
424
5
12)2120
£176 13 4 *Anſ.*

Or,

35*s.* 4*d.*
| 4*d.* | $\frac{1}{3}$ | 5
175
1 13 4
£176 13 4

Here 5 is divided by $\frac{1}{3}$.

3d. If

3. If the given price of 1 be ſhillings and pence; Multiply the price by 5, and the product, under the place of ſhillings, will be the anſwer in pounds, and the product under the place of pence, will be ſo many times 1*s.* 8*d.*

17th. At 2*s.* 5*d.* per buſhel; What coſt 100 buſhels?

s.	*d.*
2	5
	5
12	1

Anſ. £12 1 8

18th. At 25*s.* 3*d.* per ton; What coſt 100 tons?

s.	*d.*
25	3
	5
126	3

1*s.* 8*d.* × 3 = 5*s.*

Anſ. £126 5

4. To find the price of *one*, at ſo much per hundred of 5 ſcore.

GENERAL RULE.

Multiply the given price by 12; divide the product by 5, and the quotient will be the anſwer in pence.

But if the price be pounds only;

RULE.

Divide the given price by 5, and the quotient will be the anſwer in ſhillings.

19th. If 100 *yds.* coſt 65*l.* What coſt 1 *yd.*?

5)65

13*s.* *Anſ.*

20th. If 100 *yds.* coſt 2*l.* 18*s.* 4*d.* What is that per yard?

£.	*s.*	*d.*
2	18	4
		12
5)35	0	0

7*d.* *Anſwer.*

21ſt. If 100 yards coſt 11*l.* 7*s.* 9*d.* What coſt 1 yard?

£.	*s.*	*d.*
11	7	9
		12
5)136	13	0
12)27	6	7

2*s.* $3\frac{1}{4}$*d.* *Anſ.*

In dividing 27 by 12 (in the 21ſt queſtion) the quotient is 2*s.* and the remainder 3*d.* the 6 is $\frac{6}{20}$ of a penny = one farthing, and the 7 is of no account.

TABLE *of Aliquot Parts.* 100 *the* Integer.

5 is	$\frac{1}{20}$	25 is	$\frac{1}{4}$	50 is	$\frac{1}{2}$	75 is	$\frac{3}{4}$
10 —	$\frac{1}{10}$	30 —	$\frac{3}{10}$	60 —	$\frac{6}{10}$	80 —	$\frac{8}{10}$
20 —	$\frac{1}{5}$	40 —	$\frac{4}{10}$	70 —	$\frac{7}{10}$	90 —	$\frac{9}{10}$

22d. At

22d. At 3*l*. 7*s*. 6*d*. per 100; What will 23 coſt?

		£.	s.	d.	
20	$\frac{1}{5}$	3	7	6	
		0	13	6	Add.
2	$\frac{1}{10}$	0	1	4	
1	$\frac{1}{2}$	0	0	8	
23		£0	15	6	*Anſ.*

23d. At 2*l*. 1*s*. 10*d*. per 100; What coſt 18?

		£.	s.	d.	
20	$\frac{1}{5}$	2	1	10	
		0	8	$4\frac{2}{5}$	Sub.
2	$\frac{1}{10}$	0	0	10	
18	=	£0	7	$6\frac{2}{5}$	*Anſ.*

24th. At 5*l*. 9*s*. 6*d*. per 100; What coſt 35?

		£.	s.	d.	
		5	9	6	
				3	
	10)	16	8	6	
5	$\frac{1}{6}$	1	12	10	Add.
			5	$5\frac{2}{3}$	
		£1	18	$3\frac{2}{3}$	*Anſ.*

IV. *To find the value of goods, ſold by* 112lb *the Cwt.*

The price of 1lb is given to find the value of 1 cwt.

RULE.

For a farthing, account 2*s*. 4*d*. per cwt. For a half penny, 4*s*. 8*d*. For three farthings, 7*s*. And for every penny, 9*s*. 4*d*. per cwt.

25th. What coſt 1*cwt*. at $3\frac{1}{2}$*d*. per lb?

	£	s.	d.	
At 1*d*. per lb 1*cwt*. coſts -		9	4	
			3	
At 3*d*. - -	£1	8	0	Add.
At $\frac{1}{2}$*d*. - -	0	4	8	
	£1	12	8	*Anſ.*

26th. At $8\frac{3}{4}$*d*. per lb, What coſt 1*cwt*.?

	£	s.	d.	
At 1*d*. per lb 1*cwt*. coſts -		9	4	
			8	
At 8*d*. - -	£3	14	8	Add.
At $\frac{3}{4}$ - -	0	7	0	
	£4	1	8	*Anſ.*

V. *To find the value of goods ſold by* 6 *ſcore to the hundred.*

The price of 1 is given, to find the price of 1 hundred.

RULE.

Suppoſe every penny in the price to be ſo many pounds, and for the farthings, ſuch a part of a pound, as they are of a penny; then, half of that ſum will be the anſwer.

27th. At $4\frac{1}{2}d.$ per yard, What cost 120 yards?

£. s.
2)4 10

£2 5 *Ans.*

28th. At 16s. $9\frac{1}{4}d.$ per yard; What cost 120 yards?

s. d.
16 $9\frac{1}{4}$
12

201

2)201*l.* 5*s.*

£100 12 6 *Ans.*

To find the price of *one*, at so much per hundred of 6 score.

RULE.

Multiply the price by 2, then call the pounds so many pence, and the shillings, such a part of a penny, as they are of a pound, and you will have the answer.

29th. If 120 *yds.* cost 3*l.* 12*s.* What cost 1 yard?

£. s.
3 12
2

7 4

Ans. $7\frac{1}{5}d.$

30th. If 120 *yds.* cost 5*l.* 18*s.* 6*d.* What cost 1 yard?

£. s. d.
5 18 6
2

11 17 0

Ans. $11\frac{3}{4}d. + \frac{2}{5}$ *of a farthing.*

TABLE *of Aliquot Parts.* 120 *the* Integer.

Also,

6	is	$\frac{1}{20}$	24	is	$\frac{1}{5}$	36	is	$\frac{3}{10}$	72	is	$\frac{3}{5}$	96	is	$\frac{4}{5}$
10	——	$\frac{1}{12}$	30	——	$\frac{1}{4}$	45	——	$\frac{3}{8}$	75	——	$\frac{5}{8}$	100	——	$\frac{5}{6}$
12	——	$\frac{1}{10}$	40	——	$\frac{1}{3}$	48	——	$\frac{2}{5}$	80	——	$\frac{2}{3}$	105	——	$\frac{7}{8}$
15	——	$\frac{1}{8}$	60	——	$\frac{1}{2}$	50	——	$\frac{5}{12}$	84	——	$\frac{7}{10}$	108	——	$\frac{9}{10}$
20	——	$\frac{1}{6}$				70	——	$\frac{7}{12}$	90	——	$\frac{3}{4}$	110	——	$\frac{11}{12}$

31st. At 3*l.* 17*s.* 6*d.* per hundred; What cost 14?

		£. s. d.
12	$\frac{1}{10}$	3 17 6
		0 7 9 } *Add.*
2	$\frac{1}{6}$	0 1 $3\frac{1}{2}$
14		= £0 9 $0\frac{1}{2}$ *Ans.*

32d. At 2*l.* 13*s.* $6\frac{1}{2}$*d.* per hundred; What cost 49?

		£.	s.	d.	
40	$\frac{1}{3}$	2	13	$6\frac{1}{2}$	
8	$\frac{1}{5}$	0	17	10	$0\frac{2}{3}$
1	$\frac{1}{8}$	0	3	6	$3\frac{1}{3}$
		0	0	5	1
49	=	£1	1	10	1 *Ans.*

33d. At 1*l.* 19*s.* 3*d.* per hundred; What cost 75?

£.	s.	d.
1	19	3
		5
8)9	16	3
£1	4	$6\frac{1}{4}$ *Ans.*

VI. *To find the value of goods sold by the great gross.*

Note. 12 make 1 dozen, 12 dozen 1 small gross, 12 small gross 1 great gross.

The price of 1 dozen being given, in pence, to find the price of a great gross.

RULE.

Multiply the price of 1 dozen, in pence, by 3, then divide that product by 5, and the quotient will be the answer in pounds, &c. For proof, do the contrary.

N. B. *If the price of 1 be given, the price of 1 small gross is found after the same manner.*

34th. What cost 1 great gross, at 18*d.* per dozen?

$$\begin{array}{r} 18 \\ 3 \\ \hline 5)54 \\ \hline £10\ \ 16\ \textit{Ans.} \end{array}$$

35th. At 4*s.* 3*d.* per dozen; What cost 1 great gross.

$$\begin{array}{r} 4s.\ 3d. \\ 12 \\ \hline 51d. \\ 3 \\ \hline 5)153 \\ \hline \textit{Ans.}\ £30\ \ 12 \end{array}$$

Or,

$$\begin{array}{r} s.\ \ d. \\ 4\ \ 3 \\ 12 \\ \hline 2\ \ 11\ \ 0 \\ 12 \\ \hline £30\ \ 12\ \ 0 \end{array}$$

Or,

$$\begin{array}{rrrr} & & £. & s. \\ & 144 = & 7 & 4 \\ & & & 4 \\ \hline 3d. & \frac{1}{4} & 28 & 16 \\ & & 1 & 16 \\ \hline & & £30 & 12 \end{array} \Big\} \textit{Add.}$$

TABLE *of Aliquot Parts.* 144 *the* Integer.

						Also,								
12	is	$\frac{1}{12}$	36	is	$\frac{1}{4}$	32	is	$\frac{2}{9}$	84	is	$\frac{7}{12}$	128	is	$\frac{8}{9}$
16	—	$\frac{1}{9}$	48	—	$\frac{1}{3}$	60	—	$\frac{5}{12}$	96	—	$\frac{2}{3}$	132	—	$\frac{11}{12}$
18	—	$\frac{1}{8}$	72	—	$\frac{1}{2}$	64	—	$\frac{4}{9}$	108	—	$\frac{3}{4}$			
24	—	$\frac{1}{6}$				80	—	$\frac{5}{9}$	120	—	$\frac{5}{6}$			

36th. At

36th. At 2*l.* 12*s.* 9*d.* per great groſs ; What coſt 45 dozen ?

Doz.		£. s. d.
36	$\frac{1}{4}$	2 12 9
9	$\frac{1}{4}$	0 13 $2\frac{1}{4}$ } *Add.*
		0 3 $3\frac{1}{2}$
45	=	£0 16 $5\frac{3}{4}$ *Anſ.*

37th. What coſt 117 dozen, at 9*l.* 13*s.* 7*d.* per great groſs ?

Doz.		£. s. d
		9 13 7
		3
108	$\frac{3}{4}$	29 0 9
9	$\frac{1}{12}$	7 5 $2\frac{1}{4}$ } *Add.*
		12 1
117	=	£7 17 $3\frac{1}{4}$ *Anſ.*

38th. At 3*l.* 16*s.* 8*d.* per great groſs ; What coſt 7 great groſs, and 96 dozen ?

	£. s. d.
	3 16 8
	2
	3)7 13 4
Doz. 96 =	2 11 $1\frac{1}{4}$
Top line × 7 =	26 16 8
	£29 7 $9\frac{1}{4}$ *Anſ.*

VII. *To find the value of goods ſold by the thouſand.*

The price of 1 is given, to find the price of 1000.

RULE.

Multiply the given price, in pence, by 50, then divide the product by 12, and the quotient will be the anſwer in pounds, &c.

39th. At 6*d.* each ; What coſt 1000 ?

6
50
12)300
£25 *Anſ.*

Or, as 1000*s.* are 50*l.* take parts, for the pence out of 50.

6*d.*	$\frac{1}{2}$	50
	Anſ.	25

40th. What coſt 1000 at $2\frac{1}{4}$*d.* each.

2*d.*	$\frac{1}{6}$	50
$\frac{1}{4}$	$\frac{1}{8}$	8 6 8 } +
		1 0 10
		£9 7 6 *Anſ.*

To find the price of *one*, at ſo much per thouſand.

RULE.

Multiply the price by 12 ; divide the product by 50 ; then take the pounds for ſo many pence, and the ſhillings for ſuch a part of a penny as they are of a pound, which will be the anſwer.

41ſt. At

41st. At 5*l.* 4*s.* 2*d.* per 1000; What cost 1?

	£.	s.	d.
	5	4	2
			12
50 {	5)62	10	0
	10)12	10	
	£1	5	

Ans. 1¼*d.*

42d. At 354*l.* 3*s.* 4*d.* per 1000; What cost 1?

	£.	s.	d.
	354	3	4
			12
50 {	10)4250	0	0
	5)425		
	85		

Ans. 7*s.* 1*d.*

Or,

		£.	s.	d.
100	$\frac{1}{10}$	354	3	4
10	$\frac{1}{10}$	35	8	4
1	$\frac{1}{10}$	3	10	10
	Ans.	0	7	1

TABLE *of Aliquot Parts.* 1000 *the* Integer.

Also,

50 is $\frac{1}{20}$	200 is $\frac{1}{5}$	300 is $\frac{3}{10}$	700 is $\frac{7}{10}$
100 — $\frac{1}{10}$	250 — $\frac{1}{4}$	375 — $\frac{3}{8}$	750 — $\frac{3}{4}$
125 — $\frac{1}{8}$	500 — $\frac{1}{2}$	400 — $\frac{2}{5}$	800 — $\frac{4}{5}$
		600 — $\frac{3}{5}$	875 — $\frac{7}{8}$
		625 — $\frac{5}{8}$	900 — $\frac{9}{10}$

43d. At 1*l.* 17*s.* 9*d.* per 1000; What cost 115?

		£.	s.	d.	
100	$\frac{1}{10}$	1	17	9	
10	$\frac{1}{10}$	0	3	9¼	} *Add.*
		0	0	4½	
5	$\frac{1}{2}$	0	0	2¼	
115 =		£0	4	4	*Answer.*

44th. At

44th. At 2*l.* 1*s.* 8*d.* per 1000; What coſt 875?

£.	s.	d.
2	1	8
		7
8)14	11	8
£1	16	$5\frac{1}{2}$ *Anſ.*

45th. What coſt 33, at 24*s.* 8*d.* per 1000?

		£.	s.	d.	
50	$\frac{1}{20}$	1	4	8	
25	$\frac{1}{2}$	0	1	$2\frac{3}{4}$	
		0	0	$7\frac{1}{4}$	Add.
5	$\frac{1}{5}$	0	0	$1\frac{1}{4}$	
30	=	0	0	$8\frac{1}{2}$	Add.
3	$\frac{1}{10}$	0	0	$\frac{3}{4}$	
33	=	0	0	$9\frac{1}{4}$	*Anſwer.*

PRACTICE BY DECIMALS.

I. Since 2*s.* is $\frac{1}{10}$ of 1*l.* the decimal of 2*s.* is ,1: Wherefore any quantity being given at 2*s.* per ℔, yard, &c. the price is found in pounds and decimal parts of a pound, by ſeparating the unit figure of the given quantity from the reſt, for a decimal.

Let it be required to find the value of 356 yards at 2*s.* per yard?

By pointing off the unit figure 6 for a decimal, I find the amount to be £35,6, which is known to be equal to 35*l.* 12*s.* } £35,6

II. Conſequently, if the price be a multiple of 2*s.* (viz. any even number of ſhillings) the amount at 2*s.* being firſt found in pounds and decimal parts, as above, and that amount multiplied by the number which ſhews how often 2*s.* is contained in the given price, the product will be the amount required in pounds and decimal parts of a pound.

What coſt 427 gallons of wine, at 8*s.* per gallon?

£42,7 amount at 2*s.* per gallon.
4

Anſ. £170,8 *or* 170*l.* 16*s.*

The examples in Caſe 5th may be worked in this manner.
Likewiſe, if the price be pounds and even ſhillings.

754 yards at 1*l.* 8*s.*

75,4 amount at 2*s.*
14 × 2 = 28*s.*

3016
754

Anſ. £1055,6 = 1055*l.* 12*s.*

Or,

754 } Add.
75,4 × 4 = 301,6

£1055,6

III. *If*

III. *If the price be an aliquot part of 2s.*—Find the amount at 2*s.* and divide it by the denominator of the part, and the quotient will be the anſwer.

At 8*d.* per ℔ What coſt 976 ℔?

| 8*d.* | $\frac{1}{3}$ | 97,6

£32,533=£32 10 8 *Anſ.*

IV. *If the price be an aliquant (that is, uneven) part;* Divide it into aliquot parts.

7235 *yards, at* 7*d.*

4*d.*	$\frac{1}{6}$	723,5
3*d.*	$\frac{1}{8}$	120,583
		90,437

211,02=£211 0 4$\frac{3}{4}$ *Anſ.*

V. *If the price be pounds and ſhillings, or pounds, ſhillings and pence;* Reduce the ſhillings, &c. to the decimal of a pound, and multiply the quantity thereby, or the price by the quantity.

At 15*l.* 12*s.* 6*d.* *per Cwt.*; What coſt 75 *Cwt.*?

£15 12 6=£15,625
75
78125
109375
1171,875

£1171 17 6 *Anſ.*

VI. *If the quantity likewiſe be of divers denominations;* Reduce the leſs denominations to the decimal of that, whereof the price is given.

9℔ 10*oz.* of ſilk, at £4 5 9=£4,287
9℔ 10*oz.*=9,625

21435
8574
25722
38583
41,262375

£41 5 3 *Anſwer.*

Caſes 12th, and 13th, may be wrought in this manner.

Or,

Or, You may take parts for the lower denominations.

8*oz.*	$\frac{1}{2}$	4,287
2*oz.*	$\frac{1}{4}$	9
		38,583
		2,1435
		,535875
		41.262375
		£ 4: 5 2$\frac{3}{4}$

VII. When the price is any *odd* number of ſhillings : If it be required to know what quantity of any thing may be bought for any ſum of money, in pounds : Annex *two* cyphers to the money, and divide it by half the price.

Note. As half a ſhilling (or 6 pence) is ,5, therefore, to halve any odd number of ſhillings, is only to annex ,5 to half of the greateſt even number in the price.

1ſt. How many *yds.* at 7*s.* per *yd.* may I have for 435*l.* ?

Half=3,5)43500(1242$\frac{30}{35}$*yds. Anſ.*

```
35
 85
 70
 150
 140
 ---
  100
   70
  ---
   30
```

2d. How many pounds of tea, at 5*s.* per ℔ for 37*l.* ?

2,5)3700(148 ℔ *Anſ.*

```
25
120
100
---
 200
 200
 ---
```

3d. How many yards, at 9*s.* per yard, may I have for 540*l.* ?

Anſ. 1200 *yards.*

BILL *of* PARCELS.

Newburyport, June 1ſt, 1787.

Mr. Timothy Huckſter

Bought of *Samuel Merchant.*

25$\frac{1}{2}$℔ Bohea tea, at 3*s.* 6*d.* per ℔.
48℔ Cheeſe at 9*d.* per ℔.
15 Pair of worſted hoſe, at 5*s.* 8*d.* per pair.
4$\frac{1}{2}$ Dozen women's gloves, at 36*s.* 6*d.* per dozen.
19 Dozen knives and forks, at 5*s.* 9*d.* per dozen.
9 Grindſtones at 15*s.* 9*d.* per ſtone.
$\frac{1}{2}$*Cwt.* Brown ſugar, at 51*s.* per cwt.
31℔ Loaf ſugar, at 1*s.* 0$\frac{1}{2}$*d.* per ℔.

£34 3 3$\frac{1}{2}$

Received payment in full.

Samuel Merchant.

TARE

TARE AND TRET.

Tare and Tret are practical rules for deducting certain allowances, which are made by merchants and tradesmen in selling their goods by weight.

Tare is an allowance, made to the buyer, for the weight of the box, barrel, or bag, &c. which contains the goods bought, and is either at so much per box, &c. at so much per cwt. or at so much in the gross weight.

Tret is an allowance of 4lb in every 104lb for waste, dust, &c.

Cloff is an allowance of 2lb upon every 3 *Cwt.*

Gross weight is the whole weight of any sort of goods, together with the box, barrel, or bag, &c. which contains them.

Suttle is, when part of the allowance is deducted from the gross.

Neat weight is what remains after all allowances are made.

CASE I.*

When the tare is at so much per box, barrel or bag, &c.—Multiply the number of boxes, barrels, &c. by the tare, and subtract the product from the gross, and the remainder will be the neat weight required.

EXAMPLES.

1st. In 6 hogsheads of sugar, each weighing 9 *Cwt.* 2 *qrs.* 10lb gross, tare 25lb per hogshead; How much neat?

	Cwt.	*qr.*	lb		*Cwt.*	*qr.*	lb	
25×6=	1	1	10		9	2	10	*gross wt. of one hhd.*
							6	
					57	2	4	*gross.*
					1	1	10	*tare.*
				Ans.	56	0	22	*neat.*

2d. In 5 bags of cotton, marked with the gross weight as follows, tare 23lb per bag; What neat weight?

	Cwt.	*qr.*	lb
A =	7	1	19
B =	3	3	27
C =	5	1	12
D =	6	0	15
E =	8	1	0

Cwt. *qr.* lb
Ans. 30 0 14 *neat.*

3d. What is the neat weight of 15 hogsheads of tobacco, each 7 *Cwt.* 1 *qr.* 13lb. Tare 100lb per hogshead?

Ans. 97 *Cwt.* 0 *qr.* 11lb.

CASE

* This, as well as every other case in this rule, is only an application of the rules of Proportion and Practice.

CASE II.

When the tare is at ſo much per cwt.—Divide the groſs weight by the aliquot parts of a cwt. ſubtract the quotient from the groſs, and the remainder will be the neat weight.

EXAMPLES.

1ſt. In 129*cwt.* 3*qrs.* 16℔ groſs, tare 14℔ per cwt. What neat weight?

		Cwt.	qr.	℔	
14℔	$\frac{1}{8}$	129	3	16	*groſs.*
		16	0	$26\frac{1}{2}$	*tare.*
	Anſ.	113	2	$17\frac{1}{2}$	*neat.*

2d. In 97*cwt.* 1*qr.* 7℔ groſs, tare, 20℔ per cwt. What neat weight?

℔		Cwt.	qr.	℔	
16	$\frac{1}{7}$	97	1	7	*groſs.*
4	$\frac{1}{4}$	13	3	17	*Add.*
		3	1	25	
	Subtract	17	1	14	*tare.*
	Anſ.	79	3	21	*neat.*

3d. What is the neat weight of 9 barrels of potaſh, each weighing 305℔ groſs, tare 12℔ per cwt.?

Anſ. 2450℔ 14*oz.* 5*dr.*

4th. What is the value of the neat weight of 7*hhds.* of tobacco, at 5*l.* 7*s.* 6*d.* per cwt. each weighing 8*cwt.* 3*qrs.* 10℔ groſs, tare 21℔ per cwt.? *Anſ.* £270 4 $4\frac{1}{2}$ *reckoning the odd ozs.*

CASE III.

When tret is allowed with tare.—Divide the ſuttle weight by 26, and the quotient will be the tret, which ſubtract from the ſuttle, and the remainder will be the neat.

EXAMPLES.

1ſt. In 247*cwt.* 2*qrs.* 15℔ groſs, tare 28 per cwt. and tret 4℔ per 104℔, What neat weight?

℔		Cwt.	qr.	℔		
28	$\frac{1}{4}$	247	2	15		*groſs.*
		61	3	17	12	*tare, ſubtract.*
4	$\frac{1}{26}$	185	2	25	4	*ſuttle.*
		7	0	16	0	*tret, ſubtract.*
	Anſ.	178	2	9	4	*neat.*

2d. What is the neat weight of 4*hhds.* of tobacco, weighing as follow: The 1ſt. 5*cwt.* 1*qr.* 12℔ groſs, tare 65℔ per hogſhead; the 2d. 3*cwt.* 0*qr.* 19℔ groſs, tare 75℔; the 3d. 6*cwt.* 3*qrs.* groſs, tare,

tare 49 lb; and the 4th. 4*cwt.* 2*qrs.* 9 lb groſs, tare 35 lb and allowing tret to each as uſual? *Anſ.* 17*cwt.* 0*qr.* 19 lb +

CASE IV.

When tare, tret and cloff are allowed.—Deduct the tare and tret as before, and divide the ſuttle by 168, and the quotient will be the cloff, which ſubtract from the ſuttle, and the remainder will be the neat.

EXAMPLES.

1ſt. What is the neat weight of 1*hhd.* of tobacco, weighing 16*cwt.* 2*qrs.* 20 lb groſs, tare 14 lb per cwt. tret 4 lb per 104, and cloff 2 lb per 3*cwt.*?

	Cwt.	*qrs.*	lb	*oz.*	
14 lb is $\frac{1}{8}$)	16	2	20	0	*groſs.*
	2	0	9	8	*tare, ſubtract.*
4 lb is $\frac{1}{26}$)	14	2	10	8	
	0	2	6	13	*tret, ſubtract.*
2 lb is $\frac{1}{168}$)	14	0	3	11	*ſuttle.*
	0	0	9	5	*cloff, ſubtract.*
Anſ.	13	3	22	6	*neat.*

2d. If 9*hhds.* of tobacco, contain 85*cwt.* 0*qr.* 2 lb, tare 30 lb per hhd. tret and cloff as uſual, What will the neat weight come to at 6½*d.* per lb, after deducting for duties and other charges, 51*l.* 11*s.* 8*d.*? *Anſ.* 187*l.* 18*s.* 5*d.*

INVOLUTION, OR TO RAISE POWERS.

A power is the product ariſing from multiplying any given number into itſelf continually a certain number of times, thus:

$3\times3=9$ is the 2d. power, or ſquare of 3. $=3^2$

$3\times3\times3=27$ is the 3d. power, or cube of 3. $=3^3$

$3\times3\times3\times3=81$ is the 4th. power, or the biquadrate of 3, &c. $=3^4$

The number denoting the power is called the *index*, or the *exponent* of that power. Thus, the fourth power of 3 is 81, or 3^4; the ſecond power of 5 is 25, or 5^2, &c.

$2\times2=4$, the ſquare of 2; $4\times4=16=$4th power of 2; $16\times16=256=$8th power of 2, &c.

RULE.

Multiply the given number, root, or firſt power continually by itſelf, till the number of multiplications be 1 leſs than the index of the power to be found, and the laſt product will be the power required.

Note. Whence, becauſe fractions are multiplied by taking the products of their numerators, and of their denominators, they will be involved by raiſing each of their terms to the power required, and if a mixed number be propoſed, either reduce it to an improper

fraction, or reduce the vulgar fraction to a decimal, and proceed by the rule.

EXAMPLES.

1st. What is the 5th power of 9?

$$\begin{array}{rl} 9 & \\ 9 & \\ \hline 81 & = \textit{2d. power.} \\ 9 & \\ \hline 729 & = \textit{3d. power.} \\ 9 & \\ \hline 6561 & = \textit{4th. power.} \\ 9 & \\ \hline 59049 & = \textit{5th. power, or answer} = 9^5. \end{array}$$

2d. What is the 5th. power of $\frac{3}{5}$? *Anf.* $\frac{243}{3125}$.
3d. What is the 4th. power of ,045? *Anf.* ,000004100625.
Here we fee, that in raifing a fraction to a higher power, we decreafe its value.

EVOLUTION, OR THE EXTRACTION OF ROOTS.

The Root is a number whofe continual multiplication into itfelf produces the power, and is denominated the fquare, cube, biquadrate, or 2d. 3d. 4th. root, &c. accordingly as it is, when raifed to the 2d. 3d. 4th. &c. power, equal to that power. Thus, 4 is the fquare root of 16, becaufe $4\times4=16$, and 3 is the cube root of 27, becaufe $3\times3\times3=27$; and fo on.

Although there is no number of which we cannot find any power exactly, yet there are many numbers, of which precife roots can never be determined. But, by the help of decimals, we can approximate towards the root, to any affigned degree of exactnefs.

The roots, which approximate, are called *furd roots*, and thofe which are perfectly accurate, are called *rational roots.*

Roots are fometimes denoted by writing the character $\surd$ before the power, with the index of the root over it; thus the 3d. root of 36 is expreffed $\sqrt[3]{36}$, and the 2d. root of 36 is $\sqrt{36}$, the index 2 being omitted when the fquare root is defigned.

If the power be expreffed by feveral numbers, with the fign + or — between them, a line is drawn from the top of the fign over all the parts of it, Thus the 3d. root of 42 + 22 is $\sqrt[3]{47+22}$, and the 2d. root of 59 — 17 is $\sqrt{59-17}$, &c.

Sometimes roots are defigned like powers, with fractional indices. Thus, the fquare root of 15 is $15^{\frac{1}{2}}$, the cube root of 21 is $21^{\frac{1}{3}}$, and 4th, root of 37 — 20 is $\overline{37-20}^{\frac{1}{4}}$, &c.

A

A TABLE OF POWERS.

Roots, - - -	or 1ſt. Pow.	1	2	3	4	5	6	7	8	9
Squares, - -	or 2d. Pow.	1	4	9	16	25	36	49	64	81
Cubes, - - -	or 3d. Pow.	1	8	27	64	125	216	343	512	729
Biquadrates, -	or 4th. Pow.	1	16	81	256	625	1296	2401	4096	6561
Surſolids, - -	or 5th. Pow.	1	32	243	1024	3125	7776	16807	32768	59049
Square Cubes, -	or 6th. Pow.	1	64	729	4096	15625	46656	117649	262144	531441
Second Surſolids,	or 7th. Pow.	1	128	2187	16384	78125	279936	823543	2097152	4782969
Biquadrates Sqd.	or 8th. Pow.	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
Cubes Cubed, -	or 9th. Pow.	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
Surſolids Squared,	or 10th. Pow.	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	4386784401
Third Surſolids,	or 11th. Pow.	1	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
Square Cubes Sqd.	or 12th. Pow.	1	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481
Fourth Surſolids,	or 13th. Pow.	1	8192	1594323	67108864	1220703125	13060694016	96889010407	549755813888	2541865828329
2d. Surſolids Sqd.	or 14th. Pow.	1	16384	4782969	268435456	6103515625	78364164096	678223072849	4398046511104	22876792454961
Surſolids Cubed.	or 15th Pow.	1	32768	14348907	1073741824	30517578125	470184984576	4747561509943	35184372088832	205891132094649

The EXTRACTION *of the* SQUARE ROOT.

RULE.

*1. Diſtinguiſh the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and ſo on, which points ſhew the number of figures the root will conſiſt of.

2. Find the greateſt ſquare number in the firſt, or left hand, period, place the root of it at the right hand of the given number, (after the manner of a quotient in diviſion) for the firſt figure of the root, and the ſquare number, under the period, and ſubtract it therefrom, and to the remainder bring down the next period for a dividend.

3. Place the double of the root, already found, on the left hand of the dividend for a diviſor.

4. Seek how often the diviſor is contained in the dividend, (except the right hand figure) and place the anſwer in the root for the ſecond figure of it, and likewiſe on the right hand of the diviſor: Multiply the diviſor with the figure laſt annexed by the figure laſt placed in the root, and ſubtract the product from the dividend: To the remainder join the next period for a new dividend.

5. Double the figures already found in the root, for a new diviſor, (or, bring down your laſt diviſor for a new one, doubling the right hand figure of it) and from theſe, find the next figure in the root as laſt directed, and continue the operation, in the ſame manner, till you have brought down all the periods.

Note 1. If when the given power is pointed off as the power requires, the left hand period ſhould be deficient, it muſt nevertheleſs ſtand as the firſt period.

Note 2. If there be decimals in the given number, it muſt be pointed both ways from the place of units: If, when there are integers, the firſt period in the decimals be deficient, it may be completed

* In order to ſhew the reaſon of the rule, it will be proper to premiſe the following Lemma. The product of any two numbers can have, at moſt, but ſo many places of figures as are in both the factors, and at leaſt but one leſs.

Demonſtration. Take two numbers conſiſting of any number of places; but let them be the leaſt poſſible of thoſe places, viz. Unity with cyphers, as 100 and 10: Then their product will be 1 with ſo many cyphers annexed as are in both the numbers, viz. 1000; but 1000 has one place leſs than 100 and 10 together have: And ſince 100 and 10 were taken the leaſt poſſible, the product of any other two numbers, of the ſame number of places, will be greater than 1000; conſequently, the product of any two numbers can have, at leaſt, but one place leſs than both the factors.

Again, take two numbers, of any number of places, which ſhall be the greateſt poſſible of thoſe places, as 99 and 9. Now, 99×9 is leſs than 99×10; but 99×10 ($= 990$) contains only ſo many places of figures as are in 99 and 9; therefore, 99×9, or the product of any other two numbers, conſiſting of the ſame number of places, cannot have more places of figures, than are in both its factors.

Corollary 1. A ſquare number cannot have more places of figures than double the places of the root, and at leaſt but one leſs.

Corollary 2. A cube number cannot have more places of figures than triple the places of the root, and at leaſt but two leſs.

completed by annexing ſo many cyphers as the power requires: And the root muſt be made to conſiſt of ſo many whole numbers and decimals as there are periods belonging to each; and when the periods belonging to the given number are exhauſted, the operation may be continued at pleaſure by annexing cyphers.

EXAMPLES.

1ſt. Required the ſquare root of 30138696025?

$$\begin{array}{rl}
 & \dot{3}0\dot{1}3\dot{8}6\dot{9}6\dot{0}2\dot{5}(173605 \textit{ the root.} \\
 & 1 \\
\textit{1ſt. Diviſor} = 27) & 201 \\
 & 189 \\
\textit{2d. Diviſor} = 343) & 1238 \\
 & 1029 \\
\textit{3d. Diviſor} = 3466) & 20969 \\
 & 20796 \\
\textit{4th. Diviſor} = 347205) & 1736025 \\
 & 1736025
\end{array}$$

2d. Required the ſquare root of 575,5?

$$\begin{array}{rl}
 & \dot{5}7\dot{5},\dot{5}0(23,98+ \textit{ the root.} \\
 & 4 \\
43) & 175 \\
 & 129 \\
469) & 4650 \\
 & 4221 \\
4788) & 42900 \\
 & 38304 \\
 & 4596 \textit{ Remainder.}
\end{array}$$

3d. What is the ſquare root of 10342656? *Anſ.* 3216.
4th. What is the ſquare root of 964,5192360241? *Anſ.* 31,05671.
5th. What is the ſquare root of 234,09? *Anſ.* 15,3.
6th. What is the ſquare root of ,0000316969? *Anſ.* ,00563.
7th. What is the ſquare root of ,045369? *Anſ.* ,213.

RULES *for the* SQUARE ROOT *of* VULGAR FRACTIONS *and* MIXED NUMBERS.

After reducing the fraction to its loweſt terms, for this and all other roots; then,

1ſt. Extract

1ſt. Extract the *root* of the *numerator* for a *new numerator*, and the *root* of the *denominator* for a *new denominator*, which is the beſt method, provided the denominator be a complete power. But if it be not,

2d. Multiply the numerator and denominator together; and the root of this product being made the numerator to the denominator of the given fraction, or made the denominator to the numerator of it, will form the fractional part required.* *Or*,

3d. Reduce the vulgar fraction to a decimal, and extract its root.

4th. Mixed numbers may either be reduced to improper fractions, and extracted by the firſt or ſecond rule, or the vulgar fraction may be reduced to a decimal, then joined to the integer, and the root of the whole extracted.

EXAMPLES.

1ſt. What is the ſquare root of $\frac{144}{15129}$?

By Rule 1.

$\frac{144}{15129}=\frac{16}{1681}$

16(4 *root of the numerator.*
16

1681(41 *root of the denominator.*
16

81)81 *Therefore*, $\frac{4}{41}=$ *the root of the given fraction.*
81

By Rule 2.

$16\times 1681=26896$, and $\sqrt{26896}=164$. Then,

$\frac{164}{1681}=\frac{16}{164}=\frac{4}{41}=,09756+$

By Rule 3.

1681)16(,009518439+. And $\sqrt{,009518439}=,09756+$.

2d. What is the ſquare root of $\frac{2793}{8208}$? *Anſ.* $\frac{7}{12}$.

3d. What is the ſquare root of $42\frac{1}{4}$? *Anſ* $6\frac{1}{2}$.

Note. In extracting the ſquare or cube root of any ſurd number, there is always a remainder or fraction left, when the root is found. To find the value of which, the common method is, to annex *pairs* of cyphers to the reſolvend, for the ſquare, and *ternaries* of cyphers to that of the cube, which makes it tedious to diſcover the value of the remainder, eſpecially in the cube, whereas this trouble might be ſaved if the true denominator could be diſcovered.

Now,

* That is, ſuppoſe $a=7$, and $b=2$. the rule may be thus expreſſed: $\sqrt{\frac{a}{b}}=\frac{\sqrt{ab}}{b}=\frac{a}{\sqrt{ab}}$. Or, numerically thus: $\sqrt{\frac{7}{2}}=\frac{\sqrt{7\times 2}}{2}=\frac{7}{\sqrt{7\times 2}}=1,87+$, and this rule will ſerve whether the root be *finite* or *infinite*.

Now, all numbers whatever, which are to be extracted by the square or cube, contain in themselves their own fractions, and their own denominators; but neither of them can be known till an operation has taken place, when they are separated, and the fractions plainly appear. For, as in division the divisor is always the denominator to its own fraction, so likewise it is in the square and cube, each of their divisors being the denominators to their own particular fractions or numerators.

In the square, the quotient is always doubled for a new divisor; therefore, when the work is completed, the root doubled is the true divisor or denominator to its own fraction; as, if the root be 12, the denominator will be 24, to be placed under the remainder, which vulgar fraction, or its equivalent decimal, must be annexed to the quotient, or root, to complete it.*

If to the remainder, either of the square or cube, cyphers be annexed, and divided by their respective denominators, the quotient will produce the decimals belonging to the root.

APPLICATION AND USE OF THE SQUARE ROOT.

PROB. I. *To find a mean proportional between two numbers.*

RULE. Multiply the given numbers together, and extract the square root of the product; which root will be the mean proportional sought.

EXAMPLE.

What is the mean proportional between 24 and 96?

$\sqrt{96\times 24}=48$ *Answer.*

PROB. II. *To find the side of a* square *equal in* area *to any given* superficies *whatever.*

RULE. Find the *area*, and the square root is the side of the square sought.

EXAMPLES.

1st. If the area of a circle be 184,125, What is the side of a square equal in area thereto?

$\sqrt{184,125}=13,569+$ *Answer.*

2d. If the area of a triangle be 160, What is the side of a square equal in area thereto? $\sqrt{160}=12,649+$ *Answer.*

PROB. III. A certain general has an army of 5625 men; pray How many must he place in rank and file, to form them into a square? $\sqrt{5625}=75$ *Answer.*†

PROB. IV.

* Although these denominators give a small matter too much in the square root, and too little in the cube, yet they will be sufficient in common use, and are much more expeditious than the operation with cyphers.

† If you would have the number of men be double, triple, or quadruple, &c. as many in rank as in file, extract the square root of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. of the given number of men, and that will be the number of men in file, which double, triple, quadruple, &c, and the product will be the number in rank.

PROB. IV. Let 10952 men be ſo formed, as that the number in rank may be double the file.

$\sqrt{\frac{10952}{2}}=74$ in file, and $74\times2=148$ in rank.

PROB. V. If it be required to place 2016 men ſo as that there may be 56 in rank, and 36 in file, and to ſtand 4 feet diſtance in rank, and as much in file, How much ground do they ſtand on?

To anſwer this, or any of the kind, uſe the following proportion: As unity : to the diſtance :: ſo is the number in rank leſs by one : to a fourth number; next, do the ſame by the file, and multiply the two numbers together, found by the above proportion, and the product will be the anſwer.*

As 1 : 4 :: 56 — 1 : 220. And, as 1 : 4 :: 36 — 1 : 140. Then, $220\times140=30800$ ſquare feet, *the Anſwer.*

PROB. VI. Suppoſe I would ſet out an orchard of 600 trees, ſo that the length ſhall be to the breadth as 3 to 2, and the diſtance of each tree, one from the other, 7 yards; How many trees muſt it be in length, and how many in breadth? and, How many ſquare yards of ground do they ſtand on?

To reſolve any queſtion of this nature, ſay, as the ratio in length : is to the ratio in breadth :: ſo is the number of trees : to a fourth number, whoſe ſquare root is the number in breadth. And as the ratio in breadth : is to the ratio in length :: ſo is the number of trees : to a fourth, whoſe root is the number in length.

As 3 : 2 :: 600 : 400. And $\sqrt{400}=20=$ *number in breadth.*
As 2 : 3 :: 600 : 900. And $\sqrt{900}=30=$ *number in length.*
As 1 : 7 :: 30 — 1 : 203. And, as 1 : 7 :: 20 — 1 : to 133.
And $203\times133=26999$ ſquare yards, *the Anſwer.*

PROB. VII. Admit a leaden pipe $\frac{3}{4}$ inch diameter will fill a ciſtern in 3 hours; I demand the diameter of another pipe which will fill the ſame ciſtern in 1 hour.

RULE.† As the given time is to the ſquare of the given diameter, ſo is the required time : to the ſquare of the required diameter. $\frac{3}{4}=,75$; and $,75\times,75=,5625$. Then, as 3*h.* : ,5625 :: 1*h.* : 1,6875 *inverſely*, and $\sqrt{1,6875}=1,3$ *inch nearly, Anſ.*

PROB. VIII. If a pipe, whoſe diameter is 1,5 inch, fill a ciſtern in 5 hours, In what time will a pipe, whoſe diameter is 3,5 inches fill the ſame?

1.5

* The above rule will be found uſeful in planting trees, having the diſtance of ground between each given.

† The areas of circles are to one another as the ſquares of their diameters.

The area of a circle is the ſame as the ſuperficial content.

The circumference and periphery are the ſame thing.

The diameter of a circle is a line drawn through the centre from one ſide of the circumference to the other, and is the longeſt line that can be drawn acroſs a circle.

The centre of a circle is the point round which the circle is deſcribed.

Note. For the meaſuring of a circle and its parts, ſee Menſuration of Superficies, from Art. 12th to Art. 26th and the Note.

1,5×1,5=2,25; and 3,5×3,5=12,25. Then, as 2,25 : 5 :: 12,25 : ,9183 *hour, inversely*, =55 *min.* 5 *sec. Answer.*

PROB. IX. If a pipe 6 inches bore, will be 4 hours in running off a certain quantity of water, In what time will 3 pipes, each 4 inches bore, be in discharging double the quantity?

6×6=36. 4×4=16, and 16×3=48. Then, as 36 : 4*h.* :: 48 : 3*h.* inversely, and as 1*w.* : 3*h.* :: 2*w.* : 6*h. Answer.*

PROB. X. Given the diameter of a circle to make another circle, which shall be 2, 3, 4, &c. times greater or less than the given circle.

RULE. Square the given diameter, and if the required circle be greater, multiply the square of the diameter by the given proportion, and the root of the product will be the required diameter. But if the required circle be less, divide the square of the diameter by the given proportion, and the root of the quotient will be the diameter required.

There is a circle whose diameter is 4 inches; I demand the diameter of a circle 3 times as large?

4×4=16; and 16×3=48; and $\sqrt{48}$=6,928+ *inches, Ans.*

PROB. XI. To find the diameter of a circle, equal in area, to an ellipsis, (or oval) whose transverse and conjugate diameters are given.*

RULE. Multiply the two diameters of the ellipsis together, and the square root of that product will be the diameter of a circle, equal to the ellipsis.

Let the transverse diameter of an ellipsis be 48, and the conjugate, 36; What is the diameter of an equal circle?

48×36=1728, and $\sqrt{1728}$=41,569+ *the Ans.*

Note. The square of the hypothenuse, or the longest side of a right angled triangle, (by 47th B. 1. *Euc.*) is equal to the sum of the squares of the other two sides; and consequently the difference of the squares of the hypothenuse and either of the other sides is the square of the remaining side.

PROB. XII. A line 36 yards long will exactly reach from the top of a fort to the opposite bank of a river, known to be 24 yards broad. The height of the wall is required?

36×36=1296; and 24×24=576. Then, 1296—576=720, and $\sqrt{720}$=26,83+ *yards, the Answer.*

PROB. XIII. The height of a tree, growing in the centre of a circular island 44 feet in diameter, is 75 feet, and a line stretched from the top of it over to the hither edge of the water, is 256 feet. What is the breadth of the stream, provided the land on each side of the water be level?

256

* The transverse and conjugate are the longest and shortest diameters of an ellipsis; they pass through the centre, and cross each other at right angles.

$256\times256=65536$; *and* $75\times75=5625$: *Then* $65536-5625=59911$ *and* $\sqrt{59911}=24476+$ *and* $24476-\frac{44}{2}=22276$ *feet, Anſ.*

PROB. XIV. Suppoſe a ladder 60 feet long be ſo planted as to reach a window 37 feet from the ground, on one ſide of the ſtreet, and without moving it at the foot, will reach a window 23 feet high on the other ſide; I demand the breadth of the ſtreet?

$60\times60=3600$. $37\times37=1369$. $23\times23=529$: *Then*, $3600-1369=2231$, *and* $\sqrt{2231}=47,23+$, *and* $3600-529=3071$, *and* $\sqrt{3071}=55,41+$, *then*, $47,23+55,41=102,64$ *feet, the anſwer.*

PROB. XV. Two ſhips ſail from the ſame port; one goes due north 45 leagues, and the other due weſt 76 leagues: How far are they aſunder?*

$45\times45=2025$. $76\times76=5776$. *Then*, $5776+2025=7801$ and $\sqrt{7801}=88,32$ *leagues, the anſwer.*

EXTRACTION *of the* CUBE ROOT.

A Cube is any number multiplied by its *ſquare.* To extract the cube root, is to find a number which, being multiplied into its ſquare, ſhall produce the given number.

FIRST METHOD.

RULE.

†1. Separate the given number into periods of three figures each, by putting a point over the unit figure, and every third figure beyond the place of units.

2. Find the greateſt cube in the left hand period, and put its root in the quotient.

3. Subtract the cube, thus found, from the ſaid period, and to the remainder bring down the next period, and call this the *dividend.*

4. Multiply the ſquare of the quotient by 300, calling it the triple ſquare, and the quotient by 30, calling it the triple quotient, and the ſum of theſe call the *diviſor.*

5. Seek how often the diviſor may be had in the dividend, and place the reſult in the quotient.

6. Multiply the triple ſquare by the laſt quotient figure, and write the product under the dividend; multiply the ſquare of the laſt

* The ſquare root may in the ſame manner be applied to navigation; and, when deprived of other means of ſolving problems of that nature, the following proportion will ſerve to find the courſe.

As the ſum of the hypothenuſe (or diſtance) and half the greater leg (whether difference of latitude or departure) is to the leſs leg; ſo is 86, to the angle oppoſite the leſs leg.

† The reaſon of pointing the given number, as directed in the rule, is obvious from *Coroll.* 2, to the *Lemma* made uſe of in demonſtrating the ſquare root.

laſt quotient figure by the triple-quotient, and place this product under the laſt; under all, ſet the cube of the laſt quotient figure and call their ſum the *ſubtrahend*.

7. Subtract the ſubtrahend from the dividend, and to the remainder bring down the next period for a new dividend with which proceed as before, and ſo on till the whole be finiſhed.

Note. The ſame rule muſt be obſerved for continuing the operation, and pointing for decimals, as in the ſquare root.

EXAMPLES.

1ſt. Required the cube root of 436036824287?

436036824287(7583 *root.* 7×7×300 = 14700=1*ſt. Trip.ſq.*
343 7×30 = 210=1*ſt. do. quo.*

1*ſt. Diviſ.*=14910)93036=1*ſt. Dividend.* 14910=1*ſt. Diviſor.*

73500 14700×5 = 73500
5250 5×5×210 = 5250
125 5×5×5 = 125

78875=1*ſt. Subtrahend.* 78875=1*ſt. Subtra.*

2*d. Div.*=1689750)1416824=2*d. Divid.* 75×75×300= 1687500=2*d. Tri. ſq.*
75×30 = 2250=2*d. do. quo.*
13500000
144000 1689750=2*d. Diviſ.*
512
1687500×8= 13500000
13644512=2*d. Subtra.* 2250×8×8= 144000
8×8×8 = 512
3*d. Div.*=172391940)517312287=3*d. Divid.*
13644512=2*d. Subtra.*
517107600
204660 758×758×300=172369200=3*d. Trip.ſq.*
27 758×30 22740=3*d. do. quo.*

517312287=3*d. Subtrah.* 172391940=3*d. Diviſor.*

.... 172369200×3=517107600
22740×3×3 = 204660
3×3×3 = 27

517312287=3*d. Subtra.*

2d. What is the cube root of 34965783? *Anſ.* 327.
3d. What is the cube root of 84,604519? *Anſ.* 4,39.
4th. What is the cube root of ,008649? *Anſ.* ,2052+.
5th. What is the cube root of $\frac{125}{343}$? *Anſ.* $\frac{5}{7}$.

To find the true donominator, to be placed under the remainder, after the operation is finiſhed.

In the extraction of the cube root, the quotient is ſaid to be ſquared and tripled for a new diviſor; but is not really ſo, till the triple number of the quotient be added to it, therefore when the operation is finiſhed, it is but ſquaring the quotient, or root, then

then multiplying it by 3, and to that number adding the triple number of the root, when it will become the divifor, or true denominator to its own fraction, which fraction muft be annexed to the quotient, to complete the root.

Suppofe the root to be 12, when fquared it will be 144, and multiplied by 3, it makes 432, to which add 36, the triple number of the root, and it produces 468 for a denominator.*

SECOND METHOD.

Rule.

1. Having pointed the given number into periods of three figures each, find the greateft cube in the left hand period, fubtracting it therefrom and placing its root in the quotient; to the remainder bring down the next period and call it the *dividend.*

2. Under this dividend write the triple fquare of the root, fo that units in the latter may ftand under the place of hundreds in the former; and under the faid triple fquare, write the triple root, removed one place to the right hand, and call the fum of thefe the *divifor.*

3. Seek how often the divifor may be had in the dividend, exclufive of the place of units, and write the refult in the quotient.

4. Under the divifor write the product of the triple fquare of the root by the laft quotient-figure, fetting down the unit's place of this line, under the place of tens in the divifor; under this line, write the product of the triple root by the fquare of the laft quotient-figure, fo as to be removed one place beyond the right hand figure of the former; and, under this line, removed one place forward to the right hand, write down the cube of the laft quotient-figure, and call their fum the *Subtrahend.*

5. Subtract the fubtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and fo on until the whole be finifhed.

Example.

* It may not be amifs to remark here, that the denominators, both of the fquare and cube, fhew how many numbers they are denominators to, that is, what numbers are contained between any fquare or cube number and the next fucceeding fquare or cube number, exclufive of both numbers, for a complete number, of either, leaves no fraction, when the root is extracted, and confequently has no ufe for a denominator, but all the numbers contained between them have occafion for it:—Suppofe the fquare root to be 12, then its fquare is 144, and the denominator 24, which will be a denominator to all the fucceding numbers, until we come to the next fquare number, *viz.* 169, whofe root is 13, with which it has nothing to do, for between the fquare numbers 144 and 169 are contained 24 numbers, excluding both the fquare numbers. It is the fame in the cube; for, fuppofe the root to be 6, the cube number is 216, and its denominator 126 will be a denominator to all the fucceeding numbers, until we come to the next cube number, *viz.* 343, whofe root is 7, with which it has nothing to do, as ceafing then to be a denominator; for between the cube 343 and 216 are 126 numbers, excluding both cubes. And fo it is with all other denominators, either in the fquare or cube.

EXAMPLE.

Required the Cube Root of 16194277?

16194277(253=Root.
8

8194	=	First dividend.
12	=	Triple square of 2.
6	=	Triple of 2.
126	=	First divisor.
60	=	Triple square of 2 multiplied by 5.
150	=	Triple of 2 multiplied by the square of 5.
125	=	Cube of 5.
7625	=	First Subtrahend.
569277	=	Second dividend.
1875	=	Triple square of 25.
75	=	Triple of 25.
18825	=	Second divisor.
5625	=	Triple square of 25 multiplied by 3.
675	=	Triple of 25 multiplied by the square of 3.
27	=	Cube of 3.
569277	=	Second Subtrahend.

FIRST METHOD *by* APPROXIMATION.

RULE.

1. Find, by trial, a cube near to the given number, and call it the *supposed* cube.

2. Then, as twice the *supposed* cube, added to the given number, is to twice the given number, added to the *supposed* cube, so is the root of the *supposed* cube, to the true root, or an approximation to it.

3. By taking the cube of the root, thus found, for the *supposed* cube, and repeating the operation, the root will be had to a greater degree of exactness.

EXAMPLES.

It is required to find the cube root of 54854153?

Let

Let 64000000 = ſuppoſed cube, whoſe root is 400;

Then, 64000000	54854153
2	2
128000000	109708306
54854153	64000000
As 182854153 :	173708306 :: 400
	400

182854153)69483322400(379 = *root nearly.*

Again, let 54439939 = ſuppoſed cube, whoſe root is 379.

Then, 54439939	54854153
2	2
108879878	109708306
54854153	54439939
As 163734031 :	164148245 :: 379
	379
	1477334205
	1149037715
	492444735

163734031)62212184855(379,958793+ = *root corrected.*

SECOND METHOD *by* APPROXIMATION.

RULE.

1. Divide the reſolvend by three times the aſſumed root, and reſerve the quotient.

2. Subtract one twelfth part of the ſquare of the aſſumed root from the quotient.

3. Extract the ſquare root of the remainder.

4. To this root add one half of the aſſumed root, and the ſum will be the true root, or an approximation to it; take this approximation as the aſſumed root, and, by repeating the proceſs, a root farther approximated will be found, which operation may be farther repeated, as often as neceſſary, and the root diſcovered to any aſſigned exactneſs.

Note. In order to find the value of the firſt aſſumed root, in this or any other power, divide the reſolvend into periods by beginning at the place of units, and including in each period, ſo many figures as there are units in the exponent of the root; viz. 3 figures in the cube root; 4 for the biquadrate, and ſo on; then, by a table of powers, or otherwiſe, find a figure, which (being involved to the power whoſe exponent is the ſame with that of

of the required root) is the nearest to the value of the first period of the resolvend at the left hand, and to that figure annex so many cyphers as there are periods remaining in the integral part of the resolvend; this figure, with the cyphers annexed, will be the assumed root, and equal to r in the theorem; and it is of no importance whether the figure thus chosen be, when involved, greater or less than the left hand period, as the theorem is the same in both cases.

1st. What is the cube root of $\dot{4}36\dot{0}36\dot{8}24\dot{2}8\dot{7}$?

$$7000 = \text{assumed root.}$$
$$3$$

$$21{,}000)436036824287(20763658{,}2994$$

Subtract $7000 \times 7000 \div 12 = 4083333{,}3333$

$$\sqrt{16680324{,}9661} = 4084{,}15$$

Add $\frac{1}{2}$ *the assumed root* $= 3500$

And it gives the approximatad root $= 7584{,}15$

For the second operation, use the approximated root as the assumed one, and proceed as above.

THIRD METHOD *by* APPROXIMATION.

1. Assume the root in the usual way, then multiply the square of the assumed root by 3, and divide the resolvend by this product; to this quotient add $\frac{2}{3}$ of the assumed root, and the sum will be the true root, or an approximation to it.

2. For each succeeding operation let the last approximated root be the assumed root, and, proceeding in this manner, the root may be extracted to any assigned exactness.

1st. What is the cube root of 7?

Let the assumed root be 2. Then, $2 \times 2 \times 3 = 12$ the divisor.

$12)7{,}0(,583$ to this add $\frac{2}{3}$ of $2 = 1{,}333$, &c. that is, $,583 + 1{,}333 = 1{,}916$ approximated root.

Now assume 1,916 for the root. Then, by the second process, the root is $\frac{7}{3 \times \overline{1{,}916}^2} + \frac{2}{3} \times 1{,}916 = 1{,}9126$, &c.

2d. What is the cube root of 9? Let 2 be the assumed root as before. Then, $\frac{9}{12} + \frac{2}{3} \times 2 = 2{,}08$ the approximated root. Now assume 2,08. Then, $\frac{9}{3 \times \overline{2{,}08}^2} + \frac{2}{3} \times 2{,}08 = 2{,}08008$, &c.

3d. What is the cube root of 282?——Let 6 be the assumed root. Then, $6 \times 6 \times 3 = 108)282(2{,}611$, &c. and $2{,}611 + \frac{2}{3}$ of $6 = 6{,}611$ approximated root. Now assume 6,611, and it will be $6{,}611 \times 6{,}611 \times 3 = 131{,}116)282(2{,}1507$, &c. and $2{,}1507 + \frac{2}{3}$ of $6{,}611 = 6{,}558$ a farther approximated root,

4th. What

4th. What is the cube root of 1728?——Here the aſſumed root is 10. Then, $10\times10\times3=300$)1728(5,76, and $5,76+\frac{2}{3}$ of $10=12,426$.———Now aſſume 12,426, then $12,426\times12,426\times3=463,216428$)1728(3,732, and $3,732+\frac{2}{3}$ of $12,426=12,014$ a farther approximated root, and ſo on.

APPLICATION *and* USE *of the* CUBE ROOT.

1. To find *two* mean proportionals between any two given numbers.

Rule.—1. Divide the *greater* by the *leſs*, and extract the cube root of the quotient.

2. Multiply the root, ſo found, by the leaſt of the given numbers, and the product will be the *leaſt*.

3. Multiply this product by the ſame root, and it will give the greateſt.

E x a m p l e s.

1ſt. What are the two mean proportionals between 6 and 750?

$750\div6=125$, and $\sqrt[3]{125}=5$. Then, $5\times6=30=$leaſt, and $30\times5=150=$greateſt. *Anſ.* 30 and 150.

Proof. As 6 : 30 :: 150 : 750.

2d. What are the two mean proportionals between 56 and 12096? *Anſ.* 336 and 2016.

Note. The ſolid contents of ſimilar figures are in proportion to each other, as the cubes of their ſimilar ſides or diameters.

3d. If a bullet 6 inches diameter weigh 32℔, What will a bullet of the ſame metal weigh, whoſe diameter is 3 inches?

$6\times6\times6=216$. $3\times3\times3=27$. As 216 : 32℔ :: 27 : 4℔ *Anſ.*

4th. If a globe of ſilver of 3 inches diameter, be worth £45, What is the value of another globe, of a foot diameter?

$3\times3\times3=27$. $12\times12\times12=1728$. As 27 : 45 :: 1728 : £2880 *Anſ.*

The ſide of a cube being given, to find the ſide of that cube which ſhall be double, triple, &c. in quantity to the given cube.

Rule.—Cube your given ſide, and multiply it by the given proportion between the given and required cube, and the Cube root of the product will be the ſide ſought.

5th. If a cube of ſilver, whoſe ſide is 4 inches, be worth £50, I demand the ſide of a cube of the like ſilver, whoſe value ſhall be 4 times as much?

$4\times4\times4=64$, and $64\times4=256$. $\sqrt[3]{256}=6,349+$ *inches, Anſ.*

6th. There is a cubical veſſel, whoſe ſide is 2 feet, I demand the ſide of a veſſel, which ſhall contain three times as much?

$2\times2\times2=8$, and $8\times3=24$. $\sqrt[3]{24}=2,884=2$ *ft.* $10\frac{3}{5}$*inch. Anſ.*

7th. The diameter of a buſhel meaſure being $18\frac{1}{2}$ inches, and the height 8 inches, I demand the ſide of a cubic box, which ſhall contain that quantity? *Anſ.* 12,908 *inches.*

8th. Suppoſe

8th. Suppoſe a ſhip of 500 tons has 89 feet keel, 36 feet beam, and is 16 feet deep in the hold; What are the dimenſions of a ſhip of 200 tons, of the ſame mould and ſhape?

$89 \times 89 \times 89 = 704969 =$ cubed keel.

As $500 : 200 :: 704969 : 281987,6$ cube of the required keel.

$\sqrt[3]{281987,6} = 65,57$ feet the required keel.

As $89 : 65,57 :: 36 : 26,522 = 26\frac{1}{2}$ feet, beam, nearly.

As $89 : 65,57 :: 16 : 11,7$ feet, depth of the hold.

9th. From the proof of any cable to find the ſtrength of any other.

RULE.—The ſtrength of cables, and conſequently the weights of their anchors, are as the cubes of their peripheries.

If a cable, 12 inches about, require an anchor of 18 *Cwt.* Of what weight muſt an anchor be, for a 15 inch cable?

As $12 \times 12 \times 12 : 18$ *Cwt.* $:: 15 \times 15 \times 15 : 35,15625$ *Cwt.* *Anſ.*

10th. If a 15 inch cable require an anchor 35,15625 *Cwt.* :: What muſt the circumference of a cable be, for an anchor of 18 *Cwt.*?

As $35,15625 : 15 \times 15 \times 15 :: 18 : 1728$, and $\sqrt[3]{1728} = 12$ *Anſ.*

EXTRACTION *of the* BIQUADRATE ROOT.

RULE.

Extract the ſquare root of the reſolvend, and then, the ſquare root of that root, and you will have the biquadrate root.

What is the biquadrate root of 20736?

```
  . . .                        . .
  20736(144                    144(12 root required.
  1                            1
  ---                          --
24)107                       22)44
   96                           44
   ---                          --
284)1136                        ..
    1136
```

Two Methods of extracting the Biquadrate Root by Approximation, according to the two General Theorems *for extracting the roots of all powers, in pages* 214 *and* 216.

RULE I.

1. Divide the reſolvend by *ſix* times the ſquare of the aſſumed root, and from the quotient ſubtract $\frac{1}{18}$ part of the ſquare of the aſſumed root.
2. Extract the ſquare root of the remainder.
3. Add $\frac{2}{3}$ of the aſſumed root to the ſquare root, and the ſum will be the true root, or an approximation to it.

4. For

4. For every ſucceeding operation) either in this or the following method) proceed in the ſame manner, as in the firſt, each time uſing the laſt approximated root for the aſſumed root,

The biquadrate root of 20736 is required.
Here 10 *is the aſſumed root.*
$10\times10\times6=600)20736(34{,}56$
Subtract. $10\times10\div18=\underline{5{,}5555}$
$\sqrt{29{,}0044}=5{,}38$
Add $\frac{2}{3}$ *of* $10=6{,}66$

Approximated root 12,04, *To be made the aſſumed root for the next operation.*

RULE II.

Divide the reſolvend by *four* times the cube of the aſſumed root: To the quotient add *three fourths* of the aſſumed root, and the ſum will be the true root, or an approximation to it.

Let the biquadrate of 20736 be required, as before?
The aſſumed root is 10.
$10\times10\times10\times4=4000)20736(5{,}184$
Add $\frac{3}{4}$ *of* $10=7{,}5$

Approximated root 12,684, *To be made the aſſumed root for the next operation.*

EXTRACTION *of the* SURSOLID ROOT *by* APPROXIMATION.

A particular RULE.*

1. Divide the reſolvend by *five* times the aſſumed root, and to the quotient add *one twentieth* part of the *fourth* power of the ſame root.
2. From the ſquare root of this ſum ſubtract *one fourth* part of the ſquare of the aſſumed root.
3. To the ſquare root of the remainder add *one half* of the aſſumed root, and the ſum is the root required, or an approximation to it.

Note. This Rule will give the root true to *five* places, at the leaſt, (and generally to eight or nine places) at the firſt proceſs.

Required

$$* \; r\pm e=\sqrt{\sqrt{\frac{G}{5r}+\frac{r^4}{20}}-\frac{rr}{4}}+\frac{r}{2}$$

Required the surfolid root of 281950621875?

$$\begin{array}{rl} & 200 = \textit{assumed root,} \\ & \underline{\quad 5 \quad} \\ & 1000) \; 281950621,875 \textit{ quotient.} \\ \textit{Add } 200\times200\times200\times200\div20 & = \underline{80000000} \\ & \sqrt{361950621,875} = 19025 \textit{ nearly.} \\ \textit{Subtract} & 200\times200\div4 = \underline{10000} \\ & \sqrt{9025} = 95 \\ \textit{Add half the assumed root} & = \underline{100} \\ & \textit{Required root } 195 \end{array}$$

A General RULE *for* EXTRACTING *the* ROOTS *of all* POWERS.

RULE.

*1. Prepare the given number, for extraction, by pointing off from the unit's place, as the required root directs.

2. Find the first figure of the root by trial, or by inspection into the table of powers, and subtract its power from the left hand period.

3. To the remainder bring down the first figure in the next period, and call it the *dividend.*

4. Involve the root to the next inferior power to that which is given, and multiply it by the number, denoting the given power, for a *divisor.*

5. Find how many times the divisor may be had in the dividend, and the quotient will be another figure of the root.

6. Involve the whole root to the given power, and subtract it from the *given number*, as before.

7. Bring down the first figure of the next period to the remainder for a new dividend, to which find a new divisor, as before, and, in like manner, proceed till the whole be finished.

EXAMPLES.

* The extracting of roots of very high powers will, by this rule, be a tedious operation :—The following method, when practicable, will be much more convenient.

When the index of the power, whose root is to be extracted, is a composite number, take any two or more *indices*, whose product is equal to the given *index*, and extract out of the given number a root answering to another of the indices, and so on to the last.

Thus, the fourth root = square root of the square root ;—the sixth root = square root of the cube root ;—the eighth root = square root of the fourth root ;—the ninth root = the cube root of the cube root ;—the tenth root = square root of the fifth root ;—the twelfth root = cube root of the fourth, &c.

EXAMPLES.

1st. What is the cube root of 20346417?

$\dot{2}0\dot{3}46\dot{4}17(273$	$2\times2\times2=8$ *root of the 1st. period, or 1st. Subtrahend.*
$2^3 = 8 =$ *1st. Subtrah.*	$2\times2=4$ (*=next inferior power,) and,*
	4×3 (*the index of the given pow.*) $=12$ *1st. divisor.*
$2^2\times3=12)123=$ *Dividend.*	$27\times27\times27=19683=$ *2d. Subtrahend.*
	$27\times27=729$ (*next inferior power) and,*
$27^3 = 19683=$ *2d. Subtr.*	729×3 (*=index of the given power*) $=2187=$ *2d. divisor.*
$27^2\times3=2187)6634=$ *2d. Div.*	$273\times273\times273=20346417=$ *3d. Subtrahend.*
$273^3 = 20346417=$ *3d. Subt.*	
....	

2d. What is the biquadrate root of 348279998976? *Ans.* 431,9+.

3d. Extract the sursolid, or fifth root of 281950621875? *Ans.* 195.

4th. Extract the square cubed, or sixth root of 1178420166015625? *Ans.* 325

A General RULE *for* EXTARCTING ROOTS *by* APPROXIMATION.

RULE.*

1. Subtract *one* from the exponent of the root required, and multiply half of the remainder by the same exponent, and farther

* The general theorem for the extraction of all roots, by approximation, from whence the rule was taken, and the Theorems deducible from it, as high as the twelfth power. Let G = resolvend whose root is to be extracted. § $r\pm e$ = root required; r being assumed, as near the true root, and m = exponent of the power—then the equation will stand thus,

$$r\pm e=\sqrt{\frac{G}{m\times\frac{m-1}{2}r^{m-2}}-\frac{m\times\overline{m-3}+2}{m\times\overline{m-1}\;3}rr}:+\frac{m-2}{m-1}r. \text{ Hence,}$$

Theorem for the cube root. $$r\pm e=\sqrt{\frac{G}{3r}-\frac{rr}{12}}+\frac{r}{2}$$

For the Biquadrate - - $$\sqrt{\frac{G}{6rr}-\frac{rr}{18}}+\frac{2r}{3}$$

For the Sursolid - - - $$\sqrt{\frac{G}{10r^3}-\frac{3rr}{80}}+\frac{3r}{4}$$

For the squared cube root $$\sqrt{\frac{G}{15r^4}-\frac{2rr}{75}}+\frac{4r}{5}$$ (See next page.)

§ *Or, by the following Theorem in which the fraction is obtained in numbers to the lowest terms in all the odd powers; and in the even powers only by halving the numerator and denominator found by this equation.*

$$r\pm e=\sqrt{\frac{G}{\frac{m-1}{2}mr^{m-2}}-\frac{\overline{m-2}\,r^2}{m\,\overline{m-1}^{\,2}}}+\frac{\overline{m-2}}{m-1}r.$$

ther multiply this product by that power of the aſſumed root, whoſe exponent is *two* leſs than the exponent of the root required, and with this laſt product divide the reſolvend and reſerve the quotient.

2. From this quotient is to be ſubtracted the following number, viz. the ſquare of the aſſumed root multiplied by a fraction whoſe numerator is found by ſubtracting *three* from the exponent of the root, multiplying the remainder by that exponent, and adding *two* to the product; and the denominator is found by ſubtracting *one* from that exponent, and multiplying the cube of the remainder by the ſame exponent.

3. After this ſubtraction is made, extract the ſquare root of the remainder.

4. Multiply the aſſumed root by a fraction whoſe numerator is the remainder after *two* is ſubtracted from the exponent, and the denominator is the remainder after *one* is ſubtracted from the exponent: Add this product to that ſquare root and the ſum is the root required, or an approximation to it.

EXAMPLE.

What is the ſquare cubed root of 1178420166015625?

Let the aſſumed root = 300.

Exponent of the required root is 6. Then, $\frac{\overline{6-1}}{2}\times 6 = 15$.

$300^4 = 8100000000$ and this multiplied by $15 = 121500000000$.

$1178420166015625 \div 121500000000 = 9698,9314$, from this

Subtract $\frac{\overline{6-3}\times 6+2}{\overline{6-1}\times\overline{6-1}\times\overline{6-1}\times 6}\times 90000 = 2400$

And $\sqrt{7298,9314} = 85,43$

Add $\frac{6-2}{6-1}\times 300 = 240$

Approximated root $= 325,43$

For the 2d. operation, let $325,43$ = aſſumed root.

Another

For the ſecond ſurſolid	$\sqrt{\frac{G}{21r^5}-\frac{5rr}{252}}+\frac{5r}{6}$
For the ſquared Biquadrate	$\sqrt{\frac{G}{28r^6}-\frac{3rr}{196}}+\frac{6r}{7}$
For the cubed cube - - - -	$\sqrt{\frac{G}{36r^7}-\frac{7rr}{576}}+\frac{7r}{8}$
For the ſquared ſurſolid - - -	$\sqrt{\frac{G}{45r^8}-\frac{4rr}{405}}+\frac{8r}{9}$
For the third ſurſolid - - -	$\sqrt{\frac{G}{55r^9}-\frac{9rr}{1100}}+\frac{9r}{10}$
For the ſquared ſquare cube -	$\sqrt{\frac{G}{66r^{10}}-\frac{5rr}{726}}+\frac{10r}{11}$ &c.

Another METHOD *by* APPROXIMATION.*

RULE.

1. Having aſſumed the root in the uſual way, involve it to that power denoted by the exponent leſs 1.

2. Multiply this power by the exponent.

3. Divide the reſolvend by this product, and reſerve the quotient.

4. Divide the exponent of the given power, leſs 1, by the exponent, and multiply the aſſumed root by the quotient.

5. Add this product to the reſerved quotient, and the ſum will be the true root, or an approximation.

6. For every ſucceeding operation, let the root, laſt found, be the aſſumed root.

EXAMPLE.

What is the ſquare cubed root of 1178420166015625?

The exponent is 6. Let the aſſumed root be 300.

Then, $300^5 \times 6 = 14580000000000$.

14580000000000)1178420166015625(80,824.

Add $\frac{5}{6} \times 300 = 250$

330,824 = *approximated root.*

For the next operation, let 330,824 be the aſſumed root.

* A rational formula for extracting the root of any pure power by approximation.

Let the reſolvend be called G, and let $r \pm e$ be the required root, r being aſſumed in the uſual way.

Let $G^{\frac{1}{m}}$ be required; then $r \pm e = \frac{G}{mr^{m-1}} + \frac{m-1}{m}r$, the general Theorem.

Hence, For the cube root $r \pm e = \frac{G}{3r^2} + \frac{2}{3}r$.

For the biquadrate - - $\frac{G}{4r^3} + \frac{3}{4}r$.

For the ſurſolid - - $\frac{G}{5r^4} + \frac{4}{5}r$.

For cube cubed - - - $\frac{G}{6r^5} + \frac{5}{6}r$.

For the ſeventh root - - $\frac{G}{7r^6} + \frac{6}{7}r$.

For the eighth - - $\frac{G}{8r^7} + \frac{7}{8}r$.

For the ninth - - - $\frac{G}{9r^8} + \frac{8}{9}r$.

For the tenth - - $\frac{G}{10r^9} + \frac{9}{10}r$.

For the eleventh - - $\frac{G}{11r^{10}} + \frac{10}{11}r$.

For the twelfth - - $\frac{G}{12r^{11}} + \frac{11}{12}r$. &c.

Of PROPORTION in GENERAL.

Numbers are compared together to diſcover the relations they have to each other.

There muſt be two numbers to form a compariſon: The number, which is compared, being written firſt, is called the *antecedent*; and that, to which it is compared, the *conſequent.*

Numbers are compared with each other two different ways: The one compariſon conſiders the *difference* of the two numbers, and is called arithmetical relation, the difference being ſometimes named the arithmetical ratio; and the other conſiders their *quotient*, which is termed geometrical relation, and the quotient, the geometrical ratio. Thus, of the numbers 12 and 4, the difference, or arithmetical ratio, is $12-4=8$; and the geometrical ratio is $\frac{12}{4}=3$.*

If two, or more, couplets of numbers have equal ratios, or differences, the equality is termed proportion; and their terms ſimilarly poſited, that is, either all the greater, or all the leſs taken as antecedents, and the reſt as conſequents, are called proportionals. So the two couplets 2, 4, and 6, 8, taken thus, 2, 4, 6, 8, or thus, 4, 2, 8, 6, are arithmetical proportionals; and the two couplets, 2, 4, and 8, 16, taken thus, 2, 4, 8, 16, or thus, 4, 2, 16, 8, are geometrical proportionals.†

Proportion is diſtinguiſhed into continual and diſcontinual. If, of ſeveral couplets of proportionals, written down in a ſeries, the difference or ratio of each conſequent, and the antecedent of the next following couplet, be the ſame as the common difference or ratio of the couplets, the proportion is ſaid to be continual, and the numbers themſelves, a ſeries of continual proportionals, or an arithmetical or geometrical proportion. So 2, 4, 6, 8, form an arithmetical proportion; for $4-2=6-4=8-6=2$; and 2, 4, 8, 16, a geometrical proportion; for $\frac{4}{2}=\frac{8}{4}=\frac{16}{8}=2$.

But

* Ratios are, here, always conſidered as the reſult of the greater term of compariſon diminiſhed, or divided, by the leſs; not regarding which of them be the antecedent.

† To denote numbers as being geometrically proportional, the couplets are ſeparated by a double colon, and a colon is written between the terms of each couplet; we may, alſo, denote arithmetical proportionals by ſeparating the couplets by a double colon, and writing a colon turned horizontally between the terms of each couplet. So the above arithmeticals may be written thus, $2 \cdot\cdot 4 :: 6 \cdot\cdot 8$, and $4 \cdot\cdot 2 :: 8 \cdot\cdot 6$; where the firſt antecedent is leſs or greater than its conſequent by juſt ſo much as the ſecond antecedent is leſs or greater than its conſequent: And the geometricals thus, $2 : 4 :: 8 : 16$, and $4 : 2 :: 16 : 8$; where the firſt antecedent is contained in, or contains its conſequent, juſt ſo often, as the ſecond is contained in, or contains its conſequent.

Four numbers are ſaid to be *reciprocally* or *inverſely* proportional, when the fourth is leſs than the ſecond, by as many times, as the third is greater than the firſt, or when the firſt is to the third, as the fourth to the ſecond, and vice verſa. Thus 2, 9, 6 and 3, are reciprocal proportionals.

Note. It is common to read the geometricals $2 : 4 :: 8 : 16$, thus, 2 is to 4 as 8 to 16, or, As 2 to 4 ſo is 8 to 16.

Harmonical

But if the difference or ratio of the consequent of one couplet, and the antecedent of the next couplet be not the same as the common difference or ratio of the couplets, the proportion is said to be discontinual. So 4, 2, 8, 6, are in discontinued arithmetical proportion; for $4-2=8-6=2=$ common difference of the couplets, $8-2=6=$ difference of the consequent of one couplet and the antecedent of the next; also, 4, 2, 16, 8, are in discontinued geometrical proportion; for $\frac{4}{2}=\frac{16}{8}=2=$ common ratio of the couplets, and $\frac{16}{2}=8=$ ratio of the consequent of one couplet and the antecedent of the next.

ARITHMETICAL PROPORTION.

Theorem I.

If any four quantities a, b, c, d, (2, 4, 6, 8) be in Arithmetical Proportion,

Harmonical Proportion is that, which is between those numbers which assign the lengths of musical intervals, or the lengths of strings sounding musical notes; and of *three numbers* it is, when *the first is to the third, as the difference between the first and second is to the difference between the second and third*, as the numbers 3, 4, 6. Thus, if the lengths of strings be as these numbers, they will sound an octave 3 to 6, a fifth 2 to 3, and a fourth 3 to 4.

Again, between four numbers, when *the first is to the fourth, as the difference between the first and second is to the difference between the third and fourth*, as in the numbers 5, 6, 8, 10: For strings of such lengths will sound an octave 5 to 10; a sixth greater 6 to 10; a third greater 8 to 10; a third less 5 to 6; a sixth less 5 to 8; and a fourth 6 to 8.

A series of numbers in harmonical proportion is, reciprocally, as another series in arithmetical proportion.

As $\left\{\begin{array}{l}\text{Harmonical } 10 \cdot\cdot 12 \cdot\cdot 15 \cdot\cdot 20 \cdot\cdot 30 \cdot\cdot 60 \\ \text{Arithmetical } 6 \cdot\cdot 5 \cdot\cdot 4 \cdot\cdot 3 \cdot\cdot 2 \cdot\cdot 1\end{array}\right\}$ for here $10:12::5:6$; and $12:15::4:5$, and so of all the rest. Whence those series have an obvious relation to, and dependence on, each other.

1. Let a, b, c be the three numbers in musical proportion. Then, because we have $a:c::a-b:b-c$; *therefore*, $ab-ac=ac-bc$; whence, if any two of the three be given, the other may be found by the following Theorems.

I. $\frac{ab}{2a-b}=c$. II. $\frac{2ac}{2a-b}=b$. III. $\frac{cb}{2c-b}=a$.

For example, Suppose you would find a musical mean proportional between the *monochord* $50=a$, and the *octave* $25=c$; then, by Theor. II. $\frac{2ac}{a+c}=b=\frac{2500}{75}=$ 33,3, which is the length of that chord, called a *fifth*.

2. If there be four numbers in musical proportion, as a, b, c, d; then, since it is $a:d::a-b:c-d$, we have $ac-ad=ad-db$. From which equation we have the following Theorems.

I. $\frac{db}{2d-c}=a$. II. $\frac{a}{d}\times\overline{2d-c}=b$. III. $\frac{2ad-db}{a}=c$. IV. $\frac{ac}{2a-b}=d$.

Hence, when any three of those numbers are given, the fourth may be found.

Thus, let 10, 8, 6 be given to find a fourth harmonical proportion.

$\frac{a\times c}{2a-b}=\frac{10\times 6}{20-8}=\frac{60}{12}=5$, *the octave*.

This harmonical theory may be carried much farther. See *Martin's Newtonian Philosophy*, Vol. II, page 123.

Proportion,* the ſum of the two means is equal to the ſum of the two extremes.†

And if any three quantities, a, b, c, (2, 4, 6) be in Arithmetical Proportion, the double of the mean is equal to the ſum of the extremes.

THEOREM 2.

In any continued Arithmetical Proportion (1, 3, 5, 7, 9, 11) the ſum of the two extremes, and that of every other two terms, equally diſtant from them, are equal. Thus, $1+11=3+9=5+7$.‡

When the number of terms is odd, as in the proportion 3. 8. 13. 18. 23, then, the ſum of the two extremes being double to the mean or middle term, the ſum of any other two terms, equally remote from the extremes, muſt likewiſe be double to the mean.

THEOREM 3.

In any continued Arithmetical Proportion, a, $a+b$, $a+2b$, $a+3b$, $a+4b$, &c. (4, $4+2$, $4+4$, $4+6$, $4+8$, &c.) the laſt or greateſt term is equal to the firſt or leaſt more the common difference of the terms drawn into the number of all the terms after the firſt, or into the whole number of the terms, leſs one.§

THEOREM 4.

The ſum of any rank, or ſeries, of quantities in continued Arithmetical Proportion (1. 3. 5. 7. 9. 11) is equal to the ſum of the two extremes multiplied into half the number of terms.‖

ARITHMETICAL

* Although, in the compariſon of quantities according to their differences, the term *proportion* is uſed; yet the word, *progreſſion*, is frequently ſubſtituted in its room, and is indeed more proper; the former from being, in the common acceptation of it, ſynonymous with ratio, which is only uſed in the other kind of compariſon.

† For, ſince $b-a$ $(4-2)=d-c$ $(8-6,)$ therefore, $b+c$ $(4+6)=a+d$ $(2+8.)$

‡ Since, by the nature of progreſſionals, the ſecond term exceeds the firſt by juſt ſo much as its correſponding term, the laſt but one, wants of the laſt, it is evident that when theſe correſponding terms are added, the exceſs of the one will make good the defect of the other, and ſo their ſum be exactly the ſame with that of the two extremes, and in the ſame manner it will appear that the ſum of any two other correſponding terms muſt be equal to that of the two extremes.

§ For ſince each term, after the firſt, exceeds that preceding it by the common difference, it is plain that the laſt muſt exceed the firſt by ſo many times the common difference as there are terms after the firſt; and therefore muſt be equal to the firſt, and the common difference repeated that number of times.

‖ For, becauſe (by the ſecond Theorem) the ſum of the two extremes, and that of every other two terms, equally remote from them, are equal, the whole ſeries, conſiſting of half ſo many ſuch equal ſums as there are terms, will therefore be equal to the ſum of the two extremes, repeated half as many times as there are terms.

The ſame thing alſo holds, when the number of terms is odd, as in the ſeries 4, 8, 12, 16, 20; for then, the mean, or middle term, being equal to half the ſum of any two terms, equally diſtant from it on contrary ſides, it is obvious that the value of the whole ſeries is the ſame as if every term thereof were equal to the mean, and therefore is equal to the mean (or half the ſum of the two extremes) multiplied by the whole number of terms; or to the ſum of the extremes multiplied by half the number of terms.

The

ARITHMETICAL PROGRESSION.

Any rank of numbers, more than two, increaſing by a common exceſs, or decreaſing by a common difference, is ſaid to be in Arithmetical Progreſſion.

If the ſucceeding terms of a progreſſion exceed each other, it is called an aſcending ſeries or progreſſion ; if the contrary, a deſcending ſeries.

So { 0. 2. 4. 6. 8. 10, &c. *is an aſcending arithmetical ſeries.*
1. 2. 4. 8. 16. 32, &c. *is an aſcending geometrical ſeries.*

And { 10. 8. 6. 4. 2. 0, &c. *is a deſcending arithmetical ſeries.*
32. 16. 8. 4. 2. 1, &c. *is a deſcending geometrical ſeries.*

The numbers which form the ſeries, are called the *terms* of the progreſſion.

Note.—The firſt and laſt terms of a progreſſion are called the extremes, and the other terms the means.

Any three of the five following things being given, the other two may be eaſily found.

1. The *firſt* term.
2. The *laſt* term.
3. The *number* of terms.
4. The *common difference.*
5. The *ſum* of all the terms.

PROBLEM I.

The firſt term, the laſt term, and the number of terms being given, to find the common difference.

RULE.*

Divide the difference of the extremes by the number of terms leſs 1, and the quotient will be the common difference ſought.

EXAMPLES.

The ſum of any number of terms (x) of the arithmetical ſeries of odd numbers 1, 3, 5, 7, 9, &c. is equal to the ſquare (x^2) of that number.

For, $0+1$ *or the ſum of* 1 *term* $= 1^2$ *or* 1
$1+3$ *or the ſum of* 2 *terms* $= 2^2$ *or* 4
$4+5$ *or the ſum of* 3 *terms* $= 3^2$ *or* 9
$9+7$ *or the ſum of* 4 *terms* $= 4^2$ *or* 16
$16+9$ *or the ſum of* 5 *terms* $= 5^2$ *or* 25, *&c.*

Whence, it is plain, that, let x be any number whatever, the ſum of x terms will be x^2.

EXAMPLE.

The firſt term, the ratio, and number of terms given, to find the ſum of the ſeries.

A gentleman travelled 29 days, the firſt day he went but 1 mile, and increaſed every day's travel 2 miles ; How far did he travel? $29\times29=841$ *miles, Anſ.*

* The difference of the firſt and laſt term evidently ſhews the increaſe of the firſt term by all the ſubſequent additions, till it becomes equal to the laſt ; and as the number of thoſe additions was one leſs than the number of terms, and the increaſe, by every addition, equal, it is plain that the total increaſe, divided by the number of additions, muſt give the difference of every one ſeparately ; whence the rule is manifeſt.

EXAMPLES.

1ſt. The extremes are 3 and 39, and the number of terms is 19; What is the common difference?

$$\left.\begin{array}{r}39\\-\ 3\end{array}\right\}\textit{Extremes.}$$

Divide by the number of terms leſs 1 = 19 − 1 = 18)36(2 *Anſ.*

$$\begin{array}{r}36\\ \hline\end{array}$$

Or, $\frac{39-3}{19-1} = 2.$

2d. A man had 10 ſons, whoſe ſeveral ages differed alike; the youngeſt was 3 years old, and the eldeſt 48; What was the common difference of their ages?

$$\frac{48-3}{10-1} = 5 \textit{ Anſ.}$$

3d. A man is to travel from Boſton to a certain place in 9 days, and to go but 5 miles the firſt day, increaſing every day by an equal exceſs, ſo that the laſt day's journey may be 37 miles; Required the daily increaſe?

$$\frac{37-5}{9-1} = 4 \textit{ Anſ.}$$

PROBLEM II.

The firſt term, the laſt term, and the number of terms being given, to find the ſum of all the terms.

RULE.*—Multiply the ſum of the extremes by the number of terms, and half the product will be the anſwer.

EXAMPLES.

1ſt. The extremes of an arithmetical ſeries are 3 and 39, and the number of terms 19; Required the ſum of the ſeries?

39

* Suppoſe another ſeries of the ſame kind with the given one be placed under it in an inverſe order; then will the ſum of every two correſponding terms be the ſame as that of the firſt and laſt; conſequently, any one of thoſe ſums, multiplied by the number of terms, muſt give the whole ſum of the two ſeries.

Let 1. 2. 3. 4. 5. 6. 7. 8, *be the given ſeries.*
And 8. 7. 6. 5. 4. 3. 2. 1, *the ſame inverted.*
Then, $9+9+9+9+9+9+9+9=9\times8=72$, *and*

$$1+2+3+4+5+6+7+8=\frac{72}{2}=36.$$

$$\left.\begin{array}{r} 39 \\ +\ 3 \end{array}\right\} \textit{Extremes.}$$

$$\begin{array}{rr} \textit{Sum} = & 42 \\ \textit{Number of terms} = & \times\ 19 \\ \hline & 378 \\ & 42 \\ \hline & 2)798 \\ \hline & 399\ \textit{Ans.} \end{array}$$

Or, $\frac{\overline{39+3} \times 19}{2} = 399.$

2d. It is required to find how many ſtrokes the hammer of a clock would ſtrike in a week, or 168 hours, provided it increaſed 1 at each hour ?

$$\frac{\overline{168+1} \times 168}{2} = 14196\ \textit{Anſ.}$$

3d. Suppoſe a number of ſtones were laid a yard diſtant from each other for the ſpace of a mile, and the firſt a yard from a baſket ; What length of ground will that man travel over, who gathers them up ſingly, returning with them one by one to the baſket ?

$$\frac{\overline{3520+2} \times 1760}{2} = 3099360\ \textit{yards} = 1761\ \textit{miles, Anſ.}$$

N. B. In this queſtion, there being 1760 yards in a mile, and the man returning with each ſtone to the baſket, his travel will be doubled ; therefore the firſt term will be 2, and the laſt 1760×2, and the number of terms 1760.

4th. A man bought 25 yards of linen in Arithmetical Progreſſion ; for the 4th yard he gave 12 ſhillings, and for the laſt yard 3*l.* 15*s.* What did the whole amount to, and what did it average per yard ?

$\frac{75-12}{22-1} = 3$ *the common difference by which the firſt term is found to be* 3.

Then $\frac{\overline{75+3} \times 25}{2} =$ 48*l.* 15*s. and the average price is* 1*l.* 19*s. per yd.*

5th. Required the ſum of the firſt 1000 numbers in their natural order ?

$$\frac{\overline{1000+1} \times 1000}{2} = 500500\ \textit{Anſ.}$$

Problem III.

Given the extremes and the common difference, to find the number of terms.

Rule.

RULE.*—Divide the difference of the extremes by the common difference, and the quotient increaſed by 1 will be the number of terms required.

EXAMPLES.

1ſt. The extremes are 3 and 39, and the common difference 2; What is the number of terms?

$$\left.\begin{array}{r}39\\-\ 3\end{array}\right\}\textit{Extremes.}$$

$$\textit{Common difference} = 2)36$$

$$\textit{Quotient} = 18$$

$$\textit{Add}\ \ 1$$

$$19\ \textit{Anſ.}$$

Or, $\frac{39-3}{2}+1=19.$

2d. A man, going a journey, travelled the firſt day 7 miles, the laſt day 51 miles, and each day increaſed his journey by 4 miles; How many days did he travel, and how far?

$$\frac{51-7}{4}+1=12\ \textit{days, and}\ \frac{\overline{51+7}\times 12}{2}=348\ \textit{miles, Anſ.}$$

PROBLEM IV.

The extremes and common difference given, to find the ſum of all the ſeries.

RULE.—Multiply the ſum of the extremes by their difference increaſed by the common difference, and the product divided by twice the common difference, will give the ſum.

EXAMPLES.

1ſt. If the extremes are 3 and 39, and the common difference 2; What is the ſum of the ſeries?

$39+3=42$ *ſum of the extremes.*

$39-3=36=$ *difference of extremes.*

$36+2=38=$ *difference of extremes increaſed by the common difference.*

42

* By the firſt Problem, the difference of the extremes, divided by the number of terms leſs 1, gave the common difference; conſequently, the ſame divided by the common difference, muſt give the number of terms leſs 1; hence, this quotient, augme[illegible] by 1, muſt be the anſwer to the queſtion.

$$\begin{array}{r} 42 \\ \times\ 38 \\ \hline 336 \\ 126 \\ \hline \end{array}$$

Twice the common diff. $= 4)1596$

399 *Anſwer.*

Or, $\dfrac{\overline{39+3}\times\overline{39-3+2}}{2\times 2} = 399.$

2d. A owes B a certain ſum, to be diſcharged in a year, by paying 6*d.* the firſt week, 18*d.* the ſecond, and thus to increaſe every weekly payment by a ſhilling, till the laſt payment be 2*l.* 11*s.* 6*d.* ; What is the debt ?

$$\frac{\overline{51,5+,5}\times\overline{51,5-,5+1}}{1\times 2} = £67\ 12s.\ \textit{Anſ.}$$

PROBLEM V.

The extremes and the ſum of the ſeries given, to find the common difference.

RULE. Divide the product of the ſum and difference of the extremes, by the difference of twice the ſum of the ſeries, and the ſum of the extremes, and the quotient will be the common difference.

EXAMPLES.

1ſt. Let the extremes be 3 and 39, and the ſum 399 ; What is the common difference ?

Sum of the extremes $= 39 + 3 = \ 42$
Diff. of the extremes $= 39 - 3 = \times\ 36$

$$\begin{array}{r} 252 \\ 126 \\ \hline \end{array}$$

$\overline{399\times 2} - 42 = 756)1512(2$ *Anſ.*
1512

Or, $\dfrac{\overline{39+3}\times\overline{39-3}}{\overline{399\times 2}-\overline{39+3}} = 2.$

2d. A owes B 67*l.* 12*s.* to be diſcharged in a year, by weekly payments ; the firſt payment to be 6*d.* and the laſt, 2*l.* 11*s.* 6*d.* ; What is the common difference of the payments, and what will each payment be ?

$\dfrac{\overline{51,5+,5}\times\overline{51,5-,5}}{\overline{1352\times 2}-\overline{51,5+,5}} = 1s.$ and $6d. + 1s. = 1s.\ 6d. =$ 2*d. payment*, 1*s.* 6*d.* + 1*s.* = 2*s.* 6*d.* = 3*d. payment, &c.*

PROBLEM VI.

PROBLEM VI.

The extremes and ſum of the ſeries given, to find the number of terms.

RULE.—Twice the ſum of the ſeries, divided by the ſum of the extremes, will give the number of terms.

EXAMPLES.

1ſt. Let the extremes be 3 and 39, and the ſum of the ſeries 399; What is the number of terms?

Sum of the ſeries = 399
× 2

Sum of extremes =39+3=42)798(19 *Anſ.*
42
—
378
378

Or, $\frac{399\times 2}{39+3}=19.$

2d. A owes B 67*l.* 12*s.* to be paid weekly in Arithmetical Progreſſion, the firſt payment to be 6*d.* and the laſt 51*s.* 6*d.*; How many payments will there be? and, How long will he be in diſcharging the debt?

$\frac{1352\times 2}{51,5+,5}=$ 52 *payments, and as many weeks, Anſ.*

PROBLEM VII.

The firſt term, the common difference, and ſum of the ſeries given, to find the number of terms.

RULE.—To the ſquare of the difference of twice the firſt term and the common difference, add the rectangle (or product) of the ſum and the common difference multiplied by 8, and extract the ſquare root of the ſum, from which root take twice the firſt term leſs the common difference; divide the remainder by twice the common difference, and the quotient will be the number of terms.

EXAMPLE.

If the firſt term be 3, the common difference 2, and the ſum of the ſeries 399; Required the number of terms?

$3\times 2=6=$ Twice the ſum of the firſt term.
$6-2=4=$ Diff. of twice the firſt term and the common diff.
$4\times 4=16=$ Square of the ſaid difference.
$399\times 2\times 8=6384=$ Rect. of the ſum and com. diff. mult. by 8.
$6384+16=6400=$ Sum of the ſaid eightfold rectangle and the ſquare of the aforeſaid difference.
$\sqrt{6400}=80=$ Square root of the laſt mentioned ſum.

$80-4=76=$ Difference of the ſaid root and twice the firſt term leſs the common difference.

$\frac{76}{4}=19$ The number of terms.

$$Or,\ \frac{\sqrt{\overline{3\times 2-2}|^2+399\times 2\times 8}-3\times 2-2}{2\times 2}=19.$$

PROBLEM VIII.

The firſt term, the common difference, and the ſum of the ſeries given, to find the laſt term.

RULE.—To the ſquare of the difference of twice the firſt term and the common difference, add the rectangle of the ſum and the common difference, and extract the ſquare root of their ſum, from which root take the common difference; and the remainder, divided by 2, will be the laſt term.

EXAMPLE.

If the firſt term be 3, the common difference 2, and the ſum of the ſeries 399; What is the laſt term?

$3\times 2=6.$ $6-2=4.$ $4\times 4=16.$ $399\times 2\times 8=6384.$ $6384+16=6400.$ $\sqrt{6400}=80.$ $80-2=78.$ And $78\div 2=39$ *the Anſwer.*

$$Or,\ \frac{\sqrt{\overline{3\times 2-2}|^2+399\times 2\times 8}-2}{2}=39.$$

PROBLEM IX.

The firſt term, the common difference, and the number of terms given, to find the laſt term.

RULE.—The number of terms leſs 1, multiplied by the common difference, and the firſt term added to the product, will give the laſt term.

EXAMPLES.

1ſt. If the firſt term be 3, the common difference 2, and the number of terms 19; What is the laſt term?

Number of terms =	19
	− 1
Number of terms leſs 1 =	18
Common difference =	× 2
	36
Firſt term =	+ 3
	39 *the Anſ.*

Or, $\overline{19-1}\times 2+3=39.$

2d. A owes B a certain ſum, to be paid in Arithmetical Progreſſion; the firſt payment is 6*d*, the number of payments 52, and the common

common difference of the payments is 12*d.*; What is the last payment? $\overline{52-1}\times 12+6=618d.=2l.\ 11s.\ 6d.$ *Ans.*

PROBLEM X.

The first term, common difference, and number of terms given, to find the sum of the series.

RULE.—To the first term add the product of the number of terms less 1 by half the common difference, and their sum, multiplied by the number of terms, will give the sum of the progression.

EXAMPLES.

1st. If the first term be 3, the common difference 2, and number of terms 19; What is the sum of the series?

First term $= 3$

Add the product of the number of terms less 1 by $\frac{1}{2}$ com. diff. $= \overline{19-1}\times 1 = 18$

Their sum 21

Multiply by the number of terms $= 19$

189

21

Or, $19\times\overline{3+\overline{19-1}\times 1}=399$ *Ans.* $= 399$

2d. Sixteen persons gave charity to a poor man; the first gave 5*d.* and the second 9*d.* and so on in Arithmetical Progression; I demand what sum the last person gave, and how much the poor man received in all?

Answer $\overline{16-1}\times 4+5=65d.=5s.\ 5d.$ the last gave.

And $16\times\overline{5+\overline{16-1}\times\frac{4}{2}}=560d.=2l.\ 6s.\ 8d.$ the whole sum.

PROBLEM XI.

Given the first term, the number of terms, and the sum of the series, to find the common difference.

RULE.—From the sum subtract the rectangle of the first term and number of terms; twice the remainder, divided by the product of the number of terms and number of terms less 1, will give the common difference.

EXAMPLE.

If the first term be 3, the number of terms 19, and the sum 399; What is the common difference?

Sum of the series $= 399$

Subtract the product of the first term and number of terms $= 3\times 19 = 57$

Remainder $= 342$

Multiplied by 2

Divide by the product of the number of terms and number of terms less 1 $\} = 19\times 18 = 342)684(2$ *Ans.*

684

...

Or, $\frac{2\times\overline{399-3\times 19}}{\overline{19-1}\times 19}=2$

PROBLEM XII.

Given the first term, number of terms, and the sum of the series, to find the last term.

RULE

RULE.—Divide twice the ſum by the number of terms; from the quotient take the firſt term, and the remainder will be the laſt.

EXAMPLES.

1ſt. If the firſt term be 3, the number of terms 19, and the ſum 399; What is the laſt term?

Sum of the terms = 399
Multiply by 2

Divide by the number of terms = 19)798

Quotient = 42
Subtract the firſt term = 3

Anſwer = 39

Or, $\frac{399\times2}{19}-3=39.$

2d. A merchant being indebted to 12 creditors £2460, ordered his clerk to pay the firſt £40, and the reſt increaſing in Arithmetical Progreſſion; I demand the difference of the payments, and the laſt payment?

Anſ. $\frac{\overline{2\times2460}-\overline{40\times12}}{\overline{12-1}\times12}=30l.=$ *diff.* & $\frac{2460\times2}{12}-40=370l.$ *laſt paym.*

PROBLEM XIII.

The common difference, the laſt term, and ſum of the progreſſion given, to find the firſt term.

RULE.—From the ſquare of twice the laſt term plus the common difference, take 8 times the rectangle of the ſum and common difference, and extract the ſquare root of the remainder, which (root) either add to, or ſubtract from the common difference, (as the caſe may require) and half the ſum or difference will be the firſt term.

EXAMPLES.

1ſt. If the common difference be 2, the laſt term 39, and the ſum of the terms 399; Required the firſt term?

Laſt term 39
Multiplied by 2

Product = 78
Add the common difference = 2

80
Multiplied by 80

From the ſquare of twice the laſt term plus the common difference = 6400
Take 8 times the rectangle of the ſum and common diff. $=399\times2\times8=6384$

Remainder = 16
Square root of 16 = 4
Sum of the common difference and the ſquare root of 16 $=2+4=6$
And half the ſum $=\frac{6}{2}=3$ *Anſ.*

Or,

Or, $\frac{2+\sqrt{\overline{39\times2+2}|^2-399\times2\times8}}{2}=3.$

2d. A merchant being indebted to ſeveral perſons 1080*l.* he ordered his clerk to pay the greateſt creditor 142*l.* the greateſt but one 132*l.* and ſo on, to decreaſe in Arithmetical Progreſſion; What did the leaſt creditor receive?

Anſ. $\frac{10-\sqrt{\overline{142\times2+10}|^2-1080\times10\times8}}{2}=£\,2.$

Problem XIV.

Given the common difference, the laſt term, and ſum of the ſeries, to find the number of terms.

Rule.—From the ſquare of twice the laſt term plus the common difference take 8 times the rectangle of the ſum and common difference, and extract the ſquare root of the remainder, which (root) either ſubtract from, or add to, twice the laſt term plus the common difference (as the caſe may require) and the remainder or ſum, divided by twice the common difference, will give the number of terms.

Examples.

1ſt. If the common difference be 2, the laſt term 39, and the ſum of the terms 399; I demand the number of terms.

Laſt term 39
Multiply by 2
78
Add the common difference = 2
80
80

Square of twice the laſt term plus the com. diff. = 6400
Sub. 8 times the rect. of the ſum & com. diff. $=399\times2\times8=6384$
16

Square root of 16 = 4
Sum of twice the laſt term plus the com. diff. $=39\times2+2=80$

Sum of twice the laſt term and com. diff. minus the ſquare root of $16=80-4$ } = 76

Which remainder divided by twice the com. diff. $=\frac{76}{4}=19$ *Anſ.*

Or, $\frac{39\times2+2-\sqrt{\overline{39\times2+2}|^2-399\times2\times8}}{2\times2}=19$

2d. A merchant being indebted to ſeveral perſons 1080*l.* he ordered his clerk to pay the greateſt creditor 142*l.*; the greateſt but one 132*l.* and ſo on, to decreaſe in Arithmetical Progreſſion. How many creditors had he?

Anſ. $\frac{142\times2+10+\sqrt{\overline{142\times2+10}|^2-1080\times10\times8}}{10\times2}=15$ *Creditors.*

Problem

PROBLEM XV.

Given the laſt term, the number of terms, and the ſum of the terms, to find the firſt term.

RULE.—Divide twice the ſum by the number of terms; from the quotient ſubtract the laſt term, and the remainder will be the firſt.

EXAMPLES.

1. If the laſt term be 39, the number of terms 19, and the ſum of the ſeries 399; What is the firſt term?

Sum of the ſeries = 399
Multiply by 2

Divide by the number of terms = 19)798

Quotient = 42
From the quotient take the laſt term = 39
Remainder = 3 *Anſ.*

Or, $\frac{399 \times 2}{19} - 39 = 3.$

2. A man had 10 ſons, whoſe ſeveral ages differed alike; the eldeſt was 48 years old, and the ſum of all their ages was 255; What was the age of the youngeſt?

$$\frac{255 \times 2}{10} - 48 = 3 \text{ years, } Anſ.$$

PROBLEM XVI.

Given the laſt term, the number of terms, and the ſum of the ſeries, to find the common difference.

RULE.—Double the rectangle of the number of terms and the laſt term minus the ſum of the ſeries; divide the product by the rectangle of the number of terms and the number of terms minus 1, and the quotient will be the common difference.

EXAMPLES.

1. If the laſt term be 39, the number of terms 19, and the ſum of the ſeries 399; What is the common difference?

Number of terms = 19
Multiply by the laſt term = 39

171
57

Rectangle of the number of terms, and the laſt term = 19 × 39 = 741
Subtract the ſum of the ſeries = 399

Remainder = 342
Multiply by 2

Divide by the rectangle of the number of terms, and number of terms minus 1 = 19 × 18 = 342)684(2 *Anſ.*
684

Or,

$$Or, \frac{2\times\overline{19\times39-399}}{\overline{19-1}\times19}=2.$$

2. Sixteen perſons gave charity to a poor man in ſuch proportion as to form an arithmetical ſeries: The laſt gave 5*s.* 5*d.* and the whole ſum amounted to 2*l.* 6*s.* 8*d.* What did each give leſs than the other from the laſt down to the firſt?

$$\frac{2\times\overline{16\times65-560}}{\overline{16-1}\times16}=4d.\ Anſ.$$

PROBLEM XVII.

The common difference, number of terms, and the laſt term given, to find the firſt term.

RULE.—From the laſt term ſubtract the product of the terms leſs 1 by the common difference, and the remainder will be the firſt term.

EXAMPLES.

1. If the common difference be 2, the number of terms 19, and the laſt term 39; What is the firſt?

Laſt term $=39$

Subtract the number of terms leſs 1 multiplied by the common difference $\}=\overline{19-1}\times2=36$

Remains 3 *Anſ.*

Or, $39-\overline{19-1}\times2=3.$

2. A man travelled 6 days, each day going 4 miles farther than on the preceding day, till the laſt day's journey was 40 miles; How far did he ride the firſt day?

$40-\overline{6-1}\times4=20$ *miles, Anſ.*

PROBLEM XVIII.

The common difference, the number of terms, and laſt term given, to find the ſum of the ſeries.

RULE.—From the laſt term take the number of terms minus 1, multiplied by half the common difference, and the remainder multiplied by the number of terms will give the ſum.

EXAMPLES.

1. If the common difference be 2, number of terms 19, and the laſt term 39; What is the ſum of the ſeries?

Laſt term $=39$

Subtract the number of terms leſs 1 multiplied by $\frac{1}{2}$ the common difference $\}=\overline{19-1}\times1=18$

Remainder $=21$

Multiply by the number of terms $=19$

189
21

Anſwer, 399.

Or,

Or, $19\times39-\overline{\overline{19-1}\times1}=399$.

2. A man performed a journey in 6 days, and, each day, travelled 4 miles farther than on the preceding day, till his laſt day's travel was 40 miles: How far did he travel in the whole?

Anſ. $6\times40-\overline{\overline{6-1}\times\frac{4}{2}}=180$ *miles.*

PROBLEM XIX.

The ſum of the terms, the number of terms, and the common difference given, to find the firſt term.

RULE.—Divide the ſum by the number of terms; from the quotient take half the product of the number of terms, minus unity, by the common difference, and the remainder will be the firſt term.

EXAMPLES.

1. If the ſum of the ſeries be 399, the number of terms 19, and the common difference 2; What is the firſt term?

Number of terms $=19)399=$ *ſum.*

Quotient $=21$

Subtract $\frac{1}{2}$ the product of the number of terms, leſs 1, by the common difference, $\Big\}=\frac{\overline{19-1}\times2}{2}=18$

Anſ. 3

Or, $\frac{399}{19}-\frac{2\times\overline{19-1}}{2}=3$.

2. A man travelled 180 miles in 6 days; he increaſed his journey, each day, by 4 miles: How far did he travel the firſt day?

$\frac{180}{6}-\frac{4\times\overline{6-1}}{2}=20$ *miles, Anſ.*

PROBLEM XX.

The ſum of the terms, number of terms, and the common difference given, to find the laſt term.

RULE.—Divide the ſum of the ſeries by the number of terms; to the quotient add half the product of the number of terms, minus unity by the common difference, and the ſum will be the laſt term.

EXAMPLES.

1. If the ſum of the ſeries be 399, the number of terms 19, and the common difference 2; What is the laſt term?

Divide by the number of terms $=19)399$ *ſum*

Quotient $=21$

Add $\frac{1}{2}$ the product of the number of terms, leſs 1, by the common difference $\Big\}=\frac{\overline{19-1}\times2}{2}=18$

Anſ. $=39$

Or, $\frac{399}{19}+\frac{2\times\overline{19-1}}{2}=39$.

2. A

9. A perſon bought a farm for 510*l.* to be paid monthly in Arithmetical Progreſſion, and to be completed in a year, each payment

The following Table contains a ſummary of the whole doctrine of Arithmetical Progreſſion.

Caſe	Given	Required	Solution.
CASES of ARITHMETICAL PROGRESSION.			
1.	*aln*	*d*	$\frac{l-a}{n-1}$
		s	$\frac{\overline{a+l}\times n}{2}$
2.	*ald*	*n*	$\frac{l-a}{d}+1$
		s	$\frac{\overline{l+a}\times\overline{l-a+d}}{2d}$
3.	*als*	*d*	$\frac{\overline{l+a}\times\overline{l-a}}{2s-\overline{l+a}}$
		n	$\frac{2s}{a+l}$
4.	*ads*	*n*	$\frac{\sqrt{\overline{2a-d}\vert^2+8ds}-\overline{2a-d}}{2d}$
		l	$\frac{\sqrt{\overline{2a-d}\vert^2+8ds}-d}{2}$
5.	*adn*	*l*	$\overline{n-1}\times d+a$
		s	$n\times\overline{a+\overline{n-1}\times\frac{d}{2}}$

payment to exceed that preceding by 5*l*. What were the first and last payments?

Ans. $\frac{510}{12} - \frac{5 \times \overline{12-1}}{2} = 15l.$ the first payment, and

$\frac{510}{12} + \frac{5 \times \overline{12-1}}{2} = 70l.$ the last payment.

GEOMETRICAL

Case	Given	Required	Solution.
6.	*ans*	*d*	$\frac{2 \times \overline{s-an}}{\overline{n-1} \times n}$
		l	$\frac{2s}{n} - a$
7.	*lds*	*a*	$\frac{d \pm \sqrt{\overline{2l+d}\vert^2 - 8ds}}{2}$
		n	$\frac{2l+d \mp \sqrt{\overline{2l+d}\vert^2 - 8ds}}{2d}$
8.	*lns*	*a*	$\frac{2s}{n} - l$
		d	$\frac{2 \times \overline{nl-s}}{\overline{n-1} \times n}$
9.	*lnd*	*a*	$l - \overline{n-1} \times d$
		s	$n \times \overline{l - \overline{n-1} \times \frac{d}{2}}$
10.	*dns*	*a*	$\frac{s}{n} - \frac{d \times \overline{n-1}}{2}$
		l	$\frac{s}{n} + \frac{d \times \overline{n-1}}{2}$

Here $\begin{cases} a = \text{first term.} \\ l = \text{last term.} \\ n = \text{number of terms.} \\ d = \text{common difference.} \\ s = \text{sum of all the terms.} \end{cases}$

GEOMETRICAL PROPORTION.

THEOREM 1.

If four quantities, *a. b. c. d.* (2. 6. 4. 12.) be in Geometrical Proportion, the product of the two means, *bc* (6×4) will be equal to that of the two extremes, *ad* (2×12) whether they are continued, or diſcontinued,* and, if three quantities, *a. b. c.* (2. 4. 8.) the ſquare of the mean is equal to the product of the two extremes.

THEOREM 2.

If four quantities, *a. b. c. d.* (2. 6. 4. 12.) are ſuch, that the product of two of them, *ad* (2×12) is equal to the product of the other two, *bc* (6×4,) then are thoſe quantities proportional.†

THEOREM 3.

If four quantities, *a. b. c. d.* (2. 6. 4. 12.) are proportional, the rectangle of the means, divided by either extreme, will give the other extreme.‡

THEOREM 4.

The products of the correſponding terms of two Geometrical Proportions are alſo proportional.

That is, if $a : b :: c : d$ ($2 : 6 :: 4 : 12$,) and $e : f :: g : h$ ($2 : 4 :: 5 : 10$,) then will $ae : bf :: cg : dh$ ($2\times2 : 6\times4 :: 4\times5 : 12\times10$.)§

THEOREM

* For ſince the ratio of *a* to *b* (2 to 6,) or the part, which *a* is of *b* (2 is of 6) is expreſſed by $\frac{a}{b}$ $\left(\frac{2}{6}\right)$ and the ratio of *c* to *d* (4 to 12,) in like manner, by $\frac{c}{d}$ $\left(\frac{4}{12}\right)$; and ſince, by ſuppoſition, the two ratios are equal, let them both be multiplied by *bd*, (6×12) and the products $\frac{a}{b}\times bd$ $\left(\frac{2}{6}\times6\times12\right)$ and $\frac{c}{d}\times bd$ $\left(\frac{4}{12}\times6\times12\right)$ will likewiſe be equal; that is, $\frac{abd}{b}=\frac{cbd}{d}$ or $ad=cb$ $\left(\frac{2\times6\times12}{6}=\frac{4\times6\times12}{12}\right.$, or, $\left.2\times12=6\times4.\right)$

† For ſince, by ſuppoſition, the products *ad* (2×12) and *bc* (6×4) are equal, let both be divided by *bd* (6×12) and the quotients $\frac{ad}{bd}$ $\left(\frac{a}{b}\right)$ and $\frac{bc}{bd}$ $\left(\frac{c}{d}\right)$ $\frac{2\times12}{6\times12}$ $\left(\frac{2}{6}\right)$ $\frac{6\times4}{6\times12}$ $\left(\frac{4}{12}\right)$ will alſo be equal; and therefore $a : b :: c : d$.

‡ For, by the ſecond Theorem, $ad=bc$ ($2\times12=6\times4$) whence dividing both ſides of the equation by *a* (2) we have $d=\frac{bc}{a}$ $\left(12=\frac{6\times4}{2}\right)$ Hence, if the two means and one extreme be given, the other extreme may be found.

§ For $\frac{a}{b}=\frac{c}{d}$ $\left(\frac{2}{6}=\frac{4}{12},\right)$ and $\frac{e}{f}=\frac{g}{h}$ $\left(\frac{2}{4}=\frac{5}{10}\right)$ by ſuppoſition; whence, $\frac{a}{b}\times\frac{e}{f}=\frac{c}{d}\times\frac{g}{h}$ $\left(\frac{2}{6}\times\frac{2}{4}=\frac{4}{12}\times\frac{5}{10}\right)$ by equal multiplication; and conſequently $\frac{ae}{bf}=\frac{cg}{dh}$ $\left(\frac{2\times2}{6\times4}=\frac{4\times5}{12\times10}\right)$ that is, $ae : bf :: cg : dh$ ($2\times2 : 6\times4 :: 4\times5 : 12\times10$:) Hence it follows, that if any quantities be proportional, their ſquares, cubes, &c. will likewiſe be proportional.

THEOREM 5.

If four quantities, $a.\ b.\ c.\ d.$ (2. 6. 4. 12.) are directly proportional.

Then,	1. *Directly*,	$a : b :: c : d$	$(2 : 6 :: 4 : 12)$
	2. *Inversely*,	$b : a :: d : c$	$(6 : 2 :: 12 : 4)$
	3. *Alternately*,	$a : c :: b : d$	$(2 : 4 :: 6 : 12)$
	4. *Compoundedly*,	$a : a+b :: c : c+d$	$(2 : 8 :: 4 : 16)$
	5. *Dividedly*,	$a : b-a :: c : d-c$	$(2 : 4 :: 4 : 8)$
	6. *Mixtly*,	$b+a : b-a :: d+c : d-c$	$(8 : 4 :: 16 : 8)$
	7. *By Multiplication*,	$ra : rb :: c : d$	$(2r : 6r :: 4 : 12)$
	8. *By Division*,	$\frac{a}{r} : \frac{b}{r} :: c : d$	$(\frac{2}{r} : \frac{6}{r} :: 4 : 12)$

Because the product of the means, in each case, is equal to that of the extremes, and therefore the quantities are proportional by Theorem 2.

THEOREM 6.

If three numbers, $a.\ b.\ c.$ (2. 4. 8.) be in continued proportion, the square of the first will be to *that* of the second, as the first number to the third; that is, $a^2 : b^2 :: a : c$ $(2\times 2 : 4\times 4 :: 2 : 8.)$*

THEOREM 7.

In any continued Geometrical Proportion (1. 3. 9. 27. 81. &c.) the product of the two extremes, and *that* of every other two terms, equally distant from them, are equal.†

THEOREM 8.

The sum of any number of quantities, in continued Geometrical Proportion, is equal to the difference of the rectangle of the second and last terms and the square of the first, divided by the difference of the first and second terms.‡

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* For since $a : b :: b : c$ $(2 : 4 :: 4 : 8)$ thence will $ac=bb$ $(2\times 8=4\times 4)$ by Theorem 1; and therefore $aac=abb$ $(2\times 2\times 8=2\times 4\times 4)$ by equal multiplication; consequently, $a^2 : b^2 :: a : c$ $(2\times 2 : 4\times 4 :: 2 : 8)$ by Theorem 2.

In like manner it may be proved that, of four quantities continually proportional, the *cube* of the first is to *that* of the second, as the first quantity to the fourth.

† For, the ratio of the first term to the second being the same as *that* of the last but one to the last, these four terms are in proportion; and therefore, by Theorem 1, the rectangle of the extremes is equal to *that* of their two adjacent terms; and after the same manner, it will appear that the rectangle of the third and last but two is equal to *that* of their two adjacent terms, the second and last but one, and so of the rest; whence the truth of the proposition is manifest.

‡ For, let the first term of the proportion be denoted by a, the common ratio by r, the number of terms by n, and the sum of the whole series by s; then, it is plain that the second term will be expressed by $a\times r$, or, ar; the third by $ar\times r$, or ar^2; the fourth by $ar^2\times r$, or, ar^3; and the nth, or last, term by ar^{n-1}; and therefore the proportion will stand thus, $a+ar+ar^2+ar^3 \cdots\cdots + ar^{u-2} + ar^{r-1} = s$; which equation multiplied by r, gives $ar+ar^2+ar^3+ar^4 \ldots\ldots + ar^{n-1} + ar^n = rs$; from the

GEOMETRICAL PROGRESSION.

A Geometrical Progreſſion is, when a *rank*, or *ſeries*, of numbers increaſes, or decreaſes, by the continual multiplication, or diviſion, of ſome equal number.

PROBLEM I.

Given one of the extremes, the ratio, and the number of the terms of a geometrical ſeries, to find the other extreme.

RULE.—Multiply, or divide, (as the caſe may require) the given extreme by ſuch power of the ratio, whoſe exponent* is equal to the *number* of terms leſs 1, and the product, or quotient, will be the other extreme.

EXAMPLES.

the firſt equation being ſubtracted, there will remain $-a+ar^n=rs-s$: Whence, $s=\left(\frac{ar^n-a}{r-1}=\frac{r\times ar^{n-1}-a}{r-1}\right)=\frac{ar\times ar^{n-1}-aa}{ar-a}$: (Or, take any ſeries of numbers whatever, as 2. 6. 18. 54. 162. 486. and their ſum will be 2+6+18+54+162+486=728 : This equation multiplied by the ratio, will ſtand thus, 6+18+54+162+486+1458=2184 ; now it is plain that the ſum of the ſecond ſeries will be ſo many times the firſt, as is expreſſed by the ratio ; ſubtract the firſt ſeries from the ſecond, and it will give 1458−2=2184−728, which is evidently ſo many times the ſum of the firſt ſeries, as is expreſſed by the ratio leſs 1 ; whence $\frac{1458-2}{3-1}=\frac{2916-4}{6-2}$, as was to be demonſtrated.)

* As the *laſt term*, or any term near the laſt, is very tedious to be found by continual multiplication, it will often be neceſſary, in order to aſcertain them, to have a ſeries of numbers in Arithmetical Proportion, called *indices*, or *exponents*, beginning either with a cypher, or an unit, whoſe common difference is *one*.

When the *firſt* term of the ſeries and the *ratio* are *equal*, the *indices* muſt begin with an unit, and, in this caſe, the product of any two terms is equal to that term, ſignified by the *ſum* of their *indices*.

Thus, $\begin{cases} 1.\ 2.\ 3.\ 4.\ 5.\ 6, \text{\&c. indices, or arithmetical series.} \\ 2.\ 4.\ 8.\ 16.\ 32.\ 64, \text{\&c. geometrical series (leading terms.)} \end{cases}$

Now, 6+6=12=the index of the twelfth term, and

64×64=4096=the twelfth term.

But, when the *firſt* term of the ſeries and the *ratio* are *different*, the *indices* muſt begin with a cypher, and the ſum of the *indices*, made choice of, muſt be *one leſs* than the *number of terms*, given in the queſtion ; becauſe 1 in the *indices* ſtands over the *ſecond term*, and 2, in the *indices*, over the *third* term, &c. And, in this caſe, the *product* of any *two* terms, divided by the *firſt*, is equal to that term *beyond* the firſt, ſignified by the *ſum* of their *indices*.

Thus, $\begin{cases} 0.\ 1.\ 2.\ 3.\ 4.\ 5.\ 6, \text{\&c. indices.} \\ 1.\ 3.\ 9.\ 27.\ 81.\ 243.\ 729, \text{\&c. geometrical series.} \end{cases}$

Here, 6 + 5 = 11 the index of the 12th term.

729×243=177147 the 12th term, becauſe the firſt term of the ſeries and the ratio are different, by which mean a cypher ſtands over the firſt term.

Thus, by the help of theſe *indices*, and a few of the firſt terms in any geometrical ſeries, any term, whoſe diſtance from the firſt term is aſſigned, though it were ever ſo remote, may be obtained without producing all the terms.

Note. If the *ratio* of any geometrical ſeries be *double*, the *difference* of the *greateſt* and *leaſt* terms is equal to the ſum of all the terms, except the greateſt ; if the *ratio* be *triple*, the *difference* is *double* the ſum of all but the greateſt ; if the *ratio* be *quadruple*, the difference is *triple* the ſum of all but the greateſt, &c.

In any geometrical ſeries decreaſing, and continued *ad infinitum*, *half* the *greateſt* term is equal to the *ſum* of all the remaining terms, *ad infinitum*.

EXAMPLES.

1. If the firſt term be 4, the ratio 4, and the number of terms 9; What is the laſt term?

1. 2. 3. 4.+ 4= 8
4. 16. 64. 256.×256=65536= power of the ratio, whoſe exponent is leſs by 1, than the number of terms.

65536=8th power of the ratio.
Multiply by 4=firſt term.

262144=laſt term.

Or, $4\times4^8=262144=$ *the Anſwer.*

2. If the laſt term be 262144, the ratio 4, and the number of terms 9, What is the firſt term?

Laſt term.
8th power of the ratio $4^8=65536$)262144(4=the firſt term.

Or, $\frac{262144}{4^8}=4$ the firſt term.

Again, Given the firſt term, and the ratio, to find any other term aſſigned

RULE I.

When the indices begin with an unit.

1. Write down a few of the leading terms of the ſeries, and place their indices over them.

2. Add together ſuch indices, whoſe ſum ſhall make up the entire index to the term required.

3. Multiply the terms of the geometrical ſeries, belonging to thoſe indices, together, and the product will be the term ſought.

EXAMPLES.

1. If the firſt term be 2, and the ratio 2, What is the 13th term?

1. 2. 3. 4. 5+ 5+3= 13
2. 4. 8. 16. 32×32×8=8192 *Anſ.* *Or*, $2\times2^{12}=8192$.

2. A merchant wanting to purchaſe a cargo of horſes for the Weſtindies, a jockey told him he would take all the trouble and expenſe, upon himſelf, of collecting and purchaſing 30 horſes for the voyage, if he would give him what the laſt horſe would come to by doubling the whole number by a half penny, that is, two farthings for the firſt, four for the ſecond, eight for the third, &c. to which the merchant, thinking he had made a very good bargain, readily agreed: Pray, what did the laſt horſe come to, and what did the horſes, one with another, coſt the merchant?

1. 2. 3. 4. 5. 6+ 6=12th. 12+ 12+ 6=laſt term.
2. 4. 8. 16. 32. 64×64=4096, and 4096×4096×64=
1073741824 *qrs.*=£1118481 1*s.* 4*d.* and their average price was £37282 14*s.* $0\frac{1}{2}$*d.* apiece.

RULE

RULE II.

When the indices begin with a cypher.

1. Write down a few of the leading terms of the ſeries, as before, and place their indices over them.

2. Add together the moſt convenient indices to make an index, leſs by 1 than the number expreſſing the place of the term ſought.

3. Multiply the terms of the geometrical ſeries together, belonging to thoſe indices, and make the product a dividend.

4. Raiſe the firſt term to a power, whoſe index is one leſs than the *number* of terms multiplied, and make the reſult a diviſor, by which divide the dividend, and the quotient will be that term *beyond the firſt*, ſignified by the ſum of thoſe indices, or the term ſought.

3. If the firſt term be 5, and the ratio 3; What is the 7th term?

0. 1. 2. $3+2+1=$ $6=$index to 6th term beyond the 1ſt or 7th.
5. 15. 45. $135\times45\times15=91125=$dividend.

The number of terms, multiplied, is 3 (viz. $135\times45\times15$,) and $3-1=2$ is the power to which the term 5 is to be raiſed; but the 2d power of 5 is $5\times5=25$, and therefore $91125\div25=3645$ the 7th term required.

PROBLEM II.

Given the firſt term, the ratio, and number of terms, to find the ſum of the ſeries.

RULE.—Raiſe the ratio to a power, whoſe index ſhall be equal to the number of terms, from which ſubtract 1; divide the remainder by the ratio, leſs 1, and the quotient, multiplied by the firſt term, will give the ſum of the ſeries.

EXAMPLES.

1. If the firſt term be 5, the ratio 3, and the number of terms 7; What is the ſum of the ſeries?

Ratio$=3\times3\times3\times3\times3\times3\times3=2187=$7th power of the ratio.
Subtract 1

Divide by the ratio leſs $1=3-1=2$)2186

Quotient$=1093$
Multiply by the firſt term$=$ 5

Sum of the ſeries$=5465$

$$Or, \frac{3^7-1}{3-1}\times5=5465 \textit{ Anſ.}$$

2. A ſhopkeeper ſold 13 yards of cloth, on the following terms, viz. 2*d.* for the firſt yard, 4*d.* for the ſecond, 8*d.* for the third, &c. I demand the price of the cloth.

$$\frac{2^{13}-1}{2-1}\times2=16382d.=68l.\ 5s.\ 2d.\ \textit{Anſ.}$$

3. A

3. A gentleman, whose daughter was married on a new year's day, gave her a guinea, promising to triple it on the first day of each month in the year; Pray what did her portion amount to?

$$\frac{3^{12}-1}{3-1}\times 1 = 265720 \text{ guineas, } Ans.$$

4. What debt can be discharged in a year, by paying 1 shilling the first month, 10s. the second, and so on, each month in a tenfold proportion?

$10^{12}-1 \div 10-1 \times 1 = 111111111111s. = 5555555555l.\ 11s.$ *Ans.*

5. A man threshed wheat 9 days for a farmer, and agreed to receive but 8 wheat corns for the first day's work, 64 for the second, and so on in an eightfold proportion; I demand what his 9 days' labour amounted to, rating the wheat at 5s. per bushel?*

$$\frac{8^9-1}{8-1}\times 8 = 153391688 \text{ corns. Amount} = 78l.\ 0s.\ 5\tfrac{1}{2}d.\ Ans.$$

6. An ignorant fop wanting to purchase an elegant house, a facetious gentleman told him he had one which he would sell him on these moderate terms, viz. that he should give him a penny for the first door, 2d. for the second, 4d. for the third, and so on, doubling at every door, which were 36 in all: It is a bargain, cried the simpleton, and here is a guinea to bind it; Pray, what would the house have cost him?

$$\frac{2^{36}-1}{2-1}\times 1 = 68719476735d. = £286331153\ 1s.\ 3d.\ Ans.$$

7. A young fellow, well skilled in numbers, agreed with a rich farmer to serve him 10 years, without any other reward, but the produce of one wheat corn for the first year, and that produce to be sowed from year to year, till the end of the time, allowing the increase but in a tenfold proportion; What is the sum of the whole produce, and what will it amount to, at 5s. per bushel?

$$\frac{10^{10}-1}{10-1}\times 10 = 1111111110 \text{ corns. Amount} = £5651\ 8/0\tfrac{9}{10}\ Ans.$$

8. Suppose one farthing had been put out, at 6 per cent. per annum, Compound Interest,† at the commencement of the Christian Æra; What would it have amounted to in 1784 years; and suppose the amount to be in standard gold, allowing a cubic inch to be worth 53l. 2s. 8d. How large would the mass have been?

Ans. $\frac{2^{150}-1}{2-1}\times 1 = £1486716346568748209435714551509890767065361\ 11\ 3\tfrac{3}{4}.$

$= 2798085972212123041597957123293359421 0766$ cubic inches of gold.

As $355 : 113 :: \overline{360\times 69,5} : 7964$ earth's diameter. $360\times 69,5\times 7964\times 1327,33$

$= 264482820122$ cubic miles in the globe,

$= 672733373088547413 68832000$ cubic inches in the globe. Then,

$27980859722121230415979571232933594210766 \div 67273337308854741368832000$

$= 41593089984 0288,8$, which, however incredible it may appear to some, is more

* *Note*, 7680 wheat or barley corns are supposed to make a pint.

† Any sum, at 6l. per cent, per annum compound interest, will double in eleven years

more than four hundred and fifteen millions of millions, nine hundred and thirty thousand, eight hundred and ninety nine million, eight hundred and forty thousand, two hundred and eighty eight times larger than the globe we inhabit.*

PROBLEM III.

The first term, the last term (or the extremes) and the ratio given, to find the sum of the series.

RULE.—Divide the difference of the extremes by the ratio less by 1: Add the greater extreme to the quotient, and the result will be the sum of all the terms.

Or, Multiply the greatest term by the ratio, from the product subtract the least term; then divide the remainder by the ratio, less by 1, and the quotient will be the sum of all the terms.

Or, When all the terms are given, then, from the product of the *second* and *last* terms subtract the *square* of the *first* term; this remainder being divided by the *second* term *less* the *first*, will give the sum of the series.

EXAMPLES.

1. If the series be 2. 6. 18. 54. 162. 486. 1458. 4374. What is its sum total?

First method.

From the greatest term $=4374$
Subtract the least $=\quad 2$

Divide by the ratio, less $1=3-1=2)4372$ diff. of extremes.

Quotient $=2186$
Add the greater extreme $=4374$

6560

$$Or,\ \frac{4374-2}{3-1}+4374=6560\ \textit{Ans.}$$

Second method.

Greatest term $=4374$
Multiply by the ratio $=\quad 3$

Product $=13122$
Subtract the least term $=\quad 2$

Divide by the ratio, less by $1\times 3-1=2)13120$

6560 *Ans.*

$$Or,\ \frac{\overline{4374\times 3}-2}{3-1}=6560$$

Third

years and three hundred and twenty five days, or 11,889 years, or 11,89 is near enough, then, if you divide 1784 by 11,89, it will give the number of terms in this case equal to 150; the ratio will be 2, and the first term 1.

* To find the solid content of a globe. See Art. 34th of Mensuration of Solids. *Note,* that ,523598 is two thirds of ,785398 the area of a circle, whose diameter is 1.

Third method.

Greatest term $=4374$
Multiply by the second term $=6$

Product $=26244$
Subtract the square of the first term $=2\times 2=4$

Divide the remainder by the 2d term less the 1st $=6-2=4)26240$

Ans. 6560

Or, $\dfrac{\overline{4374\times 6}-4}{6-2}=6560.$

2. A man travelled 6 days, the first day he went 4 miles, and each day doubling his day's travel; his last day's ride was 128 miles; How far did he go in the whole?

$\dfrac{128-4}{2-1}+128=252$ *miles, Ans.*

3. A gentleman, dying, left 5 sons, to whom he bequeathed his estate as follows, viz. to his youngest son 1000*l.*; to the eldest 5062*l.* 10*s.* and ordered that each son should exceed the next younger by the equal ratio of $1\frac{1}{2}$; What did the several legacies amount to?

$\dfrac{5062{,}5-1000}{1{,}5-1}+5062{,}5=$ £13187 10*s.* *Ans.*

PROBLEM IV.

Given the extremes and ratio, to find the number of terms.

RULE.—Divide the greatest term by the least; find what power of the ratio is equal to the quotient, then, add 1 to the index of that power, and the sum will be the number of terms.

Or, Subtract the logarithm* of the least term from *that* of the greatest; divide the remainder by the logarithm of the ratio, and add 1 to the quotient.

EXAMPLES.

1. If the least term be 2, the greatest term 4374, and the ratio 3; What is the number of terms?

Divide by the least term $=2)4374=$ greatest term.

$3\times 3\times 3\times 3\times 3\times 3\times 3=$ quotient, $2187=$ 7th power, then $7+1=8$ *answer.*

Or,

* Logarithms are artificial numbers, the addition of which answers to multiplication of whole numbers, and subtraction, to division.

Or, From the logar. of the greatest term=3.64088
Subtract the logarithm of the least term=0.30103

Divide the remainder by the logarithm of the ratio } =,4771 2)3.33985(7+1=8, *Ans.*
3 33984

1

2. A gentleman travelled 252 miles; the first day he rode 4 miles; the last 128, and each day's journey was double to the preceding one; How many days was he in performing the journey?

PROBLEM V.

Given the least term, the ratio, and the sum of the series, to find the last term.

RULE.—Multiply the sum of the series by the ratio, less 1, to that product add the first term, and the result, divided by the ratio, will give the last term.

EXAMPLES.

1. If the first term be 2, the ratio 3, and the sum of the series 6560; What is the last term?

Sum of the series=6560
Multiply by the ratio less 1= 2

Product=13120
Add the least term= 2

Divide their sum by their ratio=3)13122

4374 *Ans.*

Or, $\frac{\overline{3-1}\times 6560+2}{3}=4374$ *Ans.*

2. A gentleman performed a journey of 252 miles; the first day he rode 4 miles, and each day after the first, twice so far as the day before; How far did he ride the last day?

$\frac{\overline{2-1}\times 252+4}{2}=128$ *miles, Ans.*

PROBLEM VI.

Given the least term, the ratio, and the sum of the series, to find the number of terms.

RULE.—To the product of the sum of the series and the ratio minus 1, add the first term; which sum, divided by the first term, will give that power of the ratio signified by the number of terms.

Or, From the logarithm of the sum of the series plus the first term, multiplied by the ratio minus unity, take the logarithm of the first term; the remainder, divided by the logarithm of the ratio, will give the number of terms.

EXAMPLE.

EXAMPLE.

If the first term be 2, the ratio 3, and the sum of the series 80; What is the number of terms?

Sum=80
Multiply by the ratio less 1=3—1= 2

160
Add the first term= 2

Divide by the first term=2)162

81 which, found in the Table of Power, is the 4th power of the ratio, therefore, the number of terms is 4.

By Logarithms.

Sum=80
Add the first term= 2

82
Multiply by the ratio less 1=3—1= 2

Logarithm of 164=2.21484
Subtract the logarithm of the first term= .30103

Divide by the logarithm of the ratio=.47712)1.91381(4 *Ans.*
1 90848

533

PROBLEM VII.

Given the extremes, and the sum of the series, to find the ratio.

RULE.—From the sum of the series subtract the least term; divide the remainder by the sum of the series minus the greatest term, and the quotient will be the ratio.

EXAMPLES.

1. If the least term be 2, the greatest term 4374, and the sum of the series 6560; What is the ratio?

Sum of the series=6560
Subtract the least term= 2

Divide the rem. by the sum of the series, minus greatest term } =6560—4374=2186)6558(3 *Ans.*
6558

2. A debt of 252*l.* was paid in Geometrical Progreſſion, the firſt payment was 4*l.* and the laſt 128*l.* In what ratio did the payments exceed each other?

Anſ. $\frac{252-4}{252-128}=2$, viz. a double ratio.

PROBLEM VIII.

Given the extremes, and the ſum of the ſeries, to find the number of terms.

RULE 1.—From the logarithm of the laſt term ſubtract the logarithm of the firſt, and make the remainder a dividend.

2. Subtract the logarithm of the ſum minus the laſt term from the logarithm of the ſum minus the firſt term, and make the remainder a diviſor.

3. Divide the dividend by the diviſor, and the quotient plus 1, will be equal to the number of terms.

EXAMPLE.

If the leaſt term be 2, the greateſt term 4374, and the ſum of the ſeries 6560; What is the number of terms?

From the logarithm of the greateſt term=3.64088
Take the logarithm of the leaſt term=0.30103

Dividend=3.33985

From the logarithm of the ſum minus the firſt term=3.81677
Take the logarithm of the ſum minus the laſt term=3.33965

Diviſor= .47712

Then, .47712)3.33985(7+1=8 *Anſ.*
3 33984

1

$$Or, \frac{L.4374-L.2}{L.6558-L.2186}+1=8.$$

PROBLEM IX.

The firſt term, the number of terms, and the laſt term given, to find the ratio.

RULE.—Divide the greater extreme by the leſs, and extract ſuch root of the quotient, whoſe index is equal to the number of terms, leſs 1. Or, find the quotient in the Table of Powers, the root of which is the anſwer.

EXAMPLES.

1. Given the extremes 2 and 4374, and the number of terms 8; What is the ratio?

Divide

Divide by the leaſt term $=2)4374=$greateſt term.

$$\sqrt[7]{2187}=3$$

$$Or, \sqrt[8-1]{\frac{4374}{2}}=3, \textit{Anſ.}$$

PROBLEM X.

The extremes and number of terms given, to find the ſum of the ſeries.

RULE 1.—Subtract the leaſt term from the greateſt, and make the difference a dividend.

2. Divide the greateſt term by the leaſt, and extract ſuch root of the quotient, whoſe index is equal to the number of terms leſs 1; take 1 from the ſaid root, and make the remainder a diviſor. (Or find the quotient in the table of powers, which will ſhew the root, from which ſubtract 1.)

3. Divide the dividend by the diviſor, and the greateſt term, added to the quotient, will give the ſum of the ſeries.

EXAMPLE.

Given the extremes 2 and 4374, and the number of terms 8; What is the ſum of the ſeries?

From the greateſt term $=4374$
Take the leaſt $=2$

Make this remainder a dividend 4372

Divide the greateſt term by the leaſt 2)4374

And extract the 7th root of the quotient, $\sqrt[7]{2187}=3$: Then,
$3-1=2)4372$

Quotient $=2186$
Add the greateſt term $=4374$

6560 *Anſ.*

$$Or, 4374+\frac{4374-2}{\sqrt[8-1]{\frac{4374}{2}}-1}=6560$$

PROBLEM XI.

Given the ratio, the number of terms, and the greateſt term, to find the leaſt term.

RULE.—Divide the greateſt term by ſuch power of the ratio, whoſe index is equal to the number of terms, leſs 1, and the quotient will be the leaſt term.

EXAMPLE.

EXAMPLE.

If the ratio be 2, the number of terms 6, and the greateſt term 128; What is the leaſt?

Divide the laſt term by $2\times2\times2\times2\times2$ $=$5th power of the ratio $\}=32)128(4$ *Anſ.*

$$\begin{array}{r}128\\ \hline\end{array}$$

$$Or, \frac{128}{2^{6-1}}=4.$$

PROBLEM XII.

Given the ratio, the number of terms, and the greateſt term, to find the ſum of the ſeries.

RULE 1.—Divide the greateſt term by ſuch power of the ratio, whoſe index is equal to the number of terms leſs 1; take the quotient from the laſt term, and make the remainder a dividend.

2. Divide the dividend by the ratio leſs 1, and the quotient, added to the greateſt term, will give the ſum of the ſeries.

EXAMPLE.

If the ratio be 4, the number of terms 6, and the greateſt term 3072; What is the ſum of the ſeries?

Divide the laſt term by the 5th power of the ratio $\}=4\times4\times4\times4\times4=1024)3072(3$

3072

From the laſt term $=3072$
Take the quotient $=3$

Divide by the ratio leſs $1=4-1=3)3069$

Quotient $=1023$
Add the greateſt term $=3072$

Anſ. $=4095$

$$Or,\ 3072+\frac{3072-\frac{3072}{4^{6-1}}}{4-1}=4095.$$

PROBLEM XIII.

Given the ratio, the number of terms, and the ſum of the ſeries, to find the leaſt term.

RULE.—Divide the ratio, leſs 1, by ſuch power, leſs 1, of the ratio, whoſe index is equal to the number of terms, and the quotient, multiplied by the ſum of the ſeries, will give the leaſt term.

EXAMPLE.

EXAMPLE.

If the ratio be 4, the number of terms 6, and the ſum of the ſeries 4095 ; What is the leaſt term ?

$4\times4\times4\times4\times4\times4=4096$, and $4096-1=4095$, then, the ratio leſs 1, divided by 4095, is $\frac{3}{4095}$, and $\frac{3}{4095}\times\frac{4095}{1}=\frac{12285}{4095}=3$ *Anſwer.*

$Or, \frac{4-1}{4^6-1}\times4095=3.$

PROBLEM XIV.

Given the ratio, the number of terms, and the ſum of the ſeries, to find the greateſt term.

RULE 1.—Subtract that power of the ratio, which is equal to the number of terms leſs 1, from that power of it, which is equal to the whole number of terms.

2. Divide the remainder by that power of the ratio minus unity which is equal to the number of terms, and the quotient, multiplied by the ſum of the ſeries, will give the greateſt term.

EXAMPLE.

If the ratio be 4, the number of terms 6, and the ſum of the ſeries 4095 ; What is the greateſt term ?

From $4\times4\times4\times4\times4\times4=4^6=4096$

Subtract $4\times4\times4\times4\times4 \quad =4^5=1024$

Divide by $4^6-1=4095)3072=\frac{3072}{4095}$ which, multiplied by the ſum, is $\frac{3072}{4095}\times\frac{4095}{1}=\frac{12579840}{4095}=3072$ *Anſ.*

$Or, \frac{4^6-4^{6-1}}{4^6-1}\times4095=3072.$

The two laſt problems may be ſolved by one ſhort operation thus : Divide the ſum by the ratio, and the remainder after the operation will be the leaſt term ; then take the quotient from the ſum of the ſeries, and the remainder will be the greateſt term.

For the leaſt term.

```
4)4095(1023 quotient.
  4
  --
  09
   8
  --
   15
   12
   --
    3 Anſ.
```

For the greateſt term.

From the ſum $=4095$

Subtract the quotient $=1023$

Anſ. $=3072$

PROBLEM

PROBLEM XV.

Given the ratio, the laſt term, and the ſum of the ſeries, to find the firſt term.

RULE.—From the ſum of the ſeries take the laſt term, and multiply the remainder by the ratio; then take this product from the ſum of the ſeries, and the remainder will be the firſt term.

EXAMPLE.

If the ratio be 4, the laſt term 3072, and the ſum of the ſeries 4095; What is the firſt term?

From the ſum=4095
Take the laſt term=3072

Remainder=1023
Multiply by the ratio= 4

Subtract 4092 from the ſum.

And the remainder 3 is the *Anſwer.*

PROBLEM XVI.

Given the ratio, the laſt term, and the ſum of the ſeries, to find the number of terms.

RULE 1.—Multiply the difference between the ſum and the laſt term by the ratio, and note the product.

2. Subtract this product from the ſum, and note the remainder.

3. From the logarithm of the laſt term ſubtract the logarithm of the remainder.

4. Divide this laſt remainder by the logarithm of the ratio, and the quotient, plus unity, will give the number of terms.

EXAMPLE.

If the ratio be 3, the laſt term 54, and the ſum of the ſeries 80; What is the number of terms?

From the ſum=80
Take the laſt term=54

Remainder=26
Multiply by the ratio= 3

Product=78
From the ſum=80
Take the product=78

Remainder= 2
From the logarithm of 54=1.73239
Take the logarithm of the remainder= .30103

Divide by the logar. of the ratio=.47712)1.43136(3+1=4 *Anſ.*

PROBLEM

PROBLEM XVII. and XVIII.

Given the number of terms, the laſt term, and the ſum of the ſeries, to find the firſt term and the ratio.

The ſolution of theſe two Problems being very tedious by the Theorems, they may be ſolved by a very ſhort operation; thus, Divide the ſum of the ſeries by the difference between the ſum and the laſt term; the quotient will give the ratio, and the remainder, after the operation, the firſt term. EXAMPLE.

The following Table exhibits a ſummary view of the doctrine of Geometrical Progreſſion.

CASES *of* GEOMETRICAL PROGRESSION.			
Caſe	*Given*	*Required*	*Solution.*
1.	*arn*	l	ar^{n-1}
		s	$\frac{r^{n}-1}{r-1}\times a$
2.	*arl*	s	$l+\frac{l-a}{r-1}$
		n	$\frac{L.l-L.a}{L.r}+1$
3.	*ars*	l	$\frac{\overline{r-1}\times s\times a}{r}$
		n	$\frac{L.\overline{\overline{r-1}\times s+a}-L.a}{L.r}$
4.	*als*	r	$\frac{s-a}{s-l}$
		n	$\frac{L.l-L.a}{L.\overline{s-a}-L.\overline{s-l}}+1$
5.	*ans*	r	$\frac{rs}{a}-r^{n-1}=\frac{s-a}{a}$
		l	$l\times\overline{s-l}\vert^{n-1}=a\times\overline{s-a}\vert^{n-1}$

EXAMPLE.

If the number of terms be 4, the laſt term 54, and the ſum of of the ſeries 80; Required the firſt term and the ratio?

From the ſum=80
Take the laſt term=54
Divide by the difference=26)80(3 the ratio.
78
The firſt term=2

SIMPLE

Caſe	Given	Required	Solution.
6.	anl	r	$\overline{\frac{l}{a}}\Big\vert^{\frac{1}{n-1}}$
		s	$l+\dfrac{l-a}{\overline{\frac{l}{a}}\Big\vert^{\frac{1}{n-1}}-1}$
7.	rnl	a	$\dfrac{l}{r^{n-1}}$
		s	$l+\dfrac{l-\frac{l}{r^{n-1}}}{r-1}$
8.	rns	a	$\dfrac{r-1}{r^{n}-1}\times s$
		l	$\dfrac{r^{n}-r^{n-1}}{r^{n}-1}\times s$
9.	rls	a	$s-r\times\overline{s-l}$
		n	$\dfrac{L.l-L.\overline{s-r\times\overline{s-l}}}{L.r}+s$
10.	nls	a	$\overline{a\times s-a}\Big\vert^{n-1}=\overline{l\times s-l}\Big\vert^{n-1}$
		r	$r^{n}+\dfrac{s}{l-s}r^{n-1}=\dfrac{l}{l-s}$

Here $\begin{cases} a = \text{firſt or leaſt term.} \\ l = \text{laſt or greateſt term.} \\ s = \text{ſum of all the terms.} \\ n = \text{number of terms.} \\ r = \text{ratio.} \\ L = \text{logarithm.} \end{cases}$

SIMPLE INTEREST.

Intereſt is a premium allowed by the borrower of any ſum of money to the lender, according to a certain rate per cent. agreed on, which by law is ſtated at 6*l.* that is, 6*l.* for the uſe of 100*l.* for one year, &c.

Principal is the money lent.

Rate is the ſum per cent. agreed on.

Amount is the principal and intereſt added together.

Intereſt is of two ſorts, *ſimple* and *compound.*

Simple Intereſt is that, which is allowed for the principal lent only.

Note. The rules for Simple Intereſt ſerve alſo to calculate commiſſion, brokerage, inſurance, purchaſing ſtocks, or any thing elſe rated at ſo much per cent.

General Rule.

1. Multiply the principal by the rate, and divide the product by 100 (or, which is the ſame, cut off the two right hand figures in the pounds, which muſt be reduced to the loweſt denomination, each time cutting off as at firſt) and the quotient will be the anſwer for one year.

2. For more years than one, multiply the intereſt of one year by the given number of years, and the product will be the anſwer for that time.

3. If there be parts of a year, as months, or days, work for the months by the aliquot parts of a year, and for the days, by the Rule of Three Direct, or (which is ſufficiently exact for common uſe,) allowing 30 days to the month, take aliquot parts of the ſame.

Note. When the rate per cent. per annum, is	£	multiply the principal by		of the given number of months, cutting off, as before directed, and you will have the intereſt for the given time.
	9		$\frac{3}{4}$	
	8		$\frac{2}{3}$	
	6		$\frac{1}{2}$	
	4		$\frac{1}{3}$	
	3		$\frac{1}{4}$	
	2		$\frac{1}{6}$	

Examples.

1. What is the intereſt of 573*l.* 13*s.* 9½*d.* for one year, at 6*l.* per cent. per ann.?

£.	s.	d.
573	13	9½
		6
34|42	2	9
20		
8|42		
12		
5|13		
4		
,52 *Anſ.* £34 8*s.* 5*d.*		

What

2. What is the intereſt of 329*l.* 17*s.* 6½*d.* for 3 years, 7 months, and 12 days, at 5*l.* per cent. per annum?

£.	s.	d.
329	17	6½
		5
16|49	7	8½
20		
9|87		
12		
10|52		
4		
2|10		

Or thus:

		£.	s.	d.
£5	$\frac{1}{20}$	329	17	6½
		16	9	10½
				3
6 months	½	49	9	7½
1 month	$\frac{1}{6}$	8	4	11¼
10 days	⅓	1	7	5¾
2 days	⅕		9	1¼
			1	9¼
		£59	13 *Anſ.*	

Then,

6 months	½	16	9	10½	intereſt of 1 year
				3	
		49	9	7½	ditto of 3 years
1 month	$\frac{1}{6}$	8	4	11¼	ditto of 6 months
10 days	⅓	1	7	5¾	ditto of 1 month
2 days	⅕		9	1¼	ditto of 10 days
			1	9¼	ditto of 2 days.
					Y. m. d.
		59	13		ditto of 3 7 12

3. What is the intereſt of 439*l.* 12*s.* 9½*d.* at 6*l.* per cent. per ann. for 16 months?

	£.	s.	d.	
	439	12	9½	
			8	= ½ the number of months.
Anſ.	£35|17	2	4	
	20			
s.	3|42			
	12			
d.	5|08			

4. What

4. What is the intereſt of 591*l.* 15*s.* 9$\frac{1}{4}$*d.* for 15 months, at 8*l.* per cent. per ann.? 10 = $\frac{2}{3}$ the number of months.

£59|17 17 8$\frac{1}{2}$
20

s. 3|57
12

d. 6|92
4

qrs. 3|70

5. What is the intereſt of 347*l.* 7*s.* 5$\frac{1}{2}$*d.* at 4*l.* per cent. per ann. for 15 months? 5 = $\frac{1}{3}$ the number of months.

£17|36 17 3$\frac{1}{2}$
20

s. 7|37
12

d. 4|47
4

qr. 1|90

6. What is the intereſt of 517*l.* 15*s.* 4*d.* for 1 month, at 6*l.* per cent. per ann.? *Anſ.* £2 11 9$\frac{1}{4}$.

7. Of 457*l.* 12*s.* 8$\frac{1}{2}$*d.* for 2 months, at 6*l.* per cent. per ann.? *Anſ.* £4 11 6$\frac{1}{4}$.

8. Of 347*l.* 5*s.* 9*d.* for 8 months, at 6*l.* per cent. per ann.? *Anſ.* £5 4 2.

9. Of 397*l.* 19*s.* for 4 months, at 6*l.* per cent. per ann.? *Anſ.* £7 19 2.

10. 508*l.* 10*s.* 5$\frac{1}{2}$*d.* for 5 months, at 6*l.* per cent. per ann.? *Anſ.* £12 14 3.

11. 719*l.* 19*s.* 4*d.* for 6 months, at 6*l.* per cent. per ann.? *Anſ.* £21 11 11$\frac{3}{4}$.

12. 396*l.* 5*s.* 10*d.* for 7 months, at 6*l.* per cent. per ann.? *Anſ.* £13 17 4$\frac{3}{4}$.

13. 517*l.* 11*s.* 11$\frac{1}{2}$*d.* for 8 months, at 6*l.* per cent. per ann.? *Anſ.* £20 14 0$\frac{3}{4}$.

14. 245*l.* 5*s.* 8$\frac{3}{4}$*d.* for 9 months, at 6*l.* per cent. per ann.? *Anſ.* £11 0 9.

15. 195*l.* 15*s.* 5$\frac{1}{2}$*d.* for 10 months, at 6*l.* per cent. per ann.? *Anſ.* £9 15 9$\frac{1}{4}$.

16. 148*l.*

16. 148*l.* 12*s.* 6½*d.* for 11 months, at 6*l.* per cent. per ann. ?
Anſ. £8 3 5¾.

17. 509*l.* 9*s.* 2*d.* for 13 months, at 6*l.* per cent. per ann. ?
Anſ. £33 2 3½.

18. 317*l.* 17*s.* 8½*d.* for 14 months, at 6*l.* per cent. per ann. ?
Anſ. £22 5 0¼.

19. 443*l.* 10*s.* 3*d.* for 15 months, at 6*l.* per cent. per ann. ?
Anſ. £33 5 3.

20. 293*l.* 7*s.* 9*d.* for 16 months, at 6*l.* per cent. per ann. ?
Anſ. £23 9 5.

21. 333*l.* 13*s.* 3¾*d.* for 17 months, at 6*l.* per cent. per ann. ?
Anſ. £28 7 2¾.

22. 517*l.* 6*s.* 6*d.* for 18 months, at 6*l.* per cent. per ann. ?
Anſ. £46 11 2.

23. 347*l.* 11*s.* 7½*d.* for 19 months, at 6*l.* per cent. per ann. ?
Anſ. £33 0 4¾.

24. 419*l.* 12*s.* 5*d.* for 20 months, at 6*l.* per cent. per ann. ?
Anſ. £41 19 2¾.

25. 537*l.* 13*s.* 5½*d.* for 16 months, at 9*l.* per cent. per ann. ?
Anſ. £64 10 4¼.

26. 197*l.* 19*s.* 1½*d.* for 15 months, at 8*l.* per cent. per ann. ?
Anſ. £19 15 10¾.

27. 217*l.* 10*s.* 4½*d.* for 18 months, at 4*l.* per cent. per ann. ?
Anſ. £13 1.

28. 327*l.* 15*s.* 9*d.* for 16 months, at 3*l.* per cent. per ann. ?
Anſ. £13 2 2¾.

29. 487*l.* 16*s.* 4½*d.* for 30 months, at 2*l.* per cent. per ann. ?
Anſ. £24 7 9¾.

30. 1*l.* for 1 year, at 6 per cent. per ann. ?
Anſ. 1*s.* 2*d.* $1\frac{6}{10}$*qr.*

31. 517*l.* 12*s.* 8½*d.* for 5 years, 11 months and 25 days, at 6*l.* per cent. per ann. ?
Anſ. £185 18 4.

A TABLE

A TABLE of RATIOS, from *one* pound, &c. to *ten* pounds.

Rate per cent.	*Ratios.*	*Rate per cent.*	*Ratios.*	*Rate per cent.*	*Ratios.*
1	,01	4	,04	7	,07
$1\frac{1}{4}$	,0125	$4\frac{1}{4}$	,0425	$7\frac{1}{4}$	,0725
$1\frac{1}{2}$	,015	$4\frac{1}{2}$	,045	$7\frac{1}{2}$	,075
$1\frac{3}{4}$	,0175	$4\frac{3}{4}$	,0475	$7\frac{3}{4}$	,0775
2	,02	5	,05	8	,08
$2\frac{1}{4}$	,0225	$5\frac{1}{4}$	,0525	$8\frac{1}{4}$	,0825
$2\frac{1}{2}$	,025	$5\frac{1}{2}$	,055	$8\frac{1}{2}$	,085
$2\frac{3}{4}$	,0275	$5\frac{3}{4}$	,0575	$8\frac{3}{4}$	,0875
3	.03	6	,06	9	,09
$3\frac{1}{4}$	,0325	$6\frac{1}{4}$	,0625	$9\frac{1}{4}$	,0925
$3\frac{1}{2}$	,035	$6\frac{1}{2}$	,065	$9\frac{1}{2}$	,095
$3\frac{3}{4}$	,0375	$6\frac{3}{4}$	,0675	$9\frac{3}{4}$	,0975
				10	1,

Ratio is the Simple Intereſt of 1*l.* for 1 year, at the rate per cent. agreed on.

A TABLE *for the ready finding the decimal parts of a year, equal to any number of* days, *or* quarters *of a* year.

Days.	*Decimal parts.*	*Days.*	*Decimal parts.*	*Days.*	*Decimal parts.*
1	,00274	10	,027397	100	,273973
2	,005479	20	,054794	200	,547945
3	,008219	30	,082192	300	,821918
4	,010959	40	,109589	365	1,000000
5	,013699	50	,136986	$\frac{1}{4}$ *of a year*=,25	
6	,016438	60	,164383	$\frac{1}{2}$ *of a year*=,5	
7	,019178	70	,191781	$\frac{3}{4}$ *of a year*=,75	
8	,021918	80	,219178		
9	,024657	90	,246575		

CASE I.*

The principal, time, and ratio are given, to find the intereſt and amount.

RULE.—Multiply the principal, time and ratio continually together, and the laſt product will be the intereſt, commiſſion, brokage, &c. to which add the principal, and the ſum will be the amount.

EXAMPLES.

1. Required the amount of 537*l.* 10*s.* at 6*l.* per cent. per ann. for 5 years? Principal

* The following Theorems will ſhew all the poſſible caſes of Simple Intereſt; Where p=principal, t=time, r=ratio, and a=amount.

I. $ptr+p=a$. II. $\frac{a}{tr+1}=p$. III. $\frac{a-p}{tp}=r$. IV. $\frac{a-p}{rp}=t$.

Principal 537,5
Multiply by the ratio= ,06

Product 32,250
Multiply by the time= 5

Interest=161,250
Add the principal=537,5

Amount=£698,75
20

15,00 *Anſ.* £698 15*s.*

Or, 537,5×,06×5+537,5=£698 15*s.*

2. What is the ſimple intereſt of 917*l.* 16*s.* at 5*l.* per cent. per ann. for 7 years? *Anſ.* £321 4 7.

3. What is the amount of 391*l.* 17*s.* at 4½*l.* per cent. per ann. for 3¼ years? *Anſ.* £449 3 1¾.

4. What is the amount of 235*l.* 3*s.* 9*d.* at 5¼*l.* per cent. per ann. from March 5th, 1784, to November 23d, 1784? *Anſ.* £244 0 8½.

5. If my correſpondent is to have 2½*l.* per cent.; What will his commiſſion on 785*l.* 15*s.* amount to? *Anſ.* £19 12 10½.

6. What will be the intereſt and amount of 445*l.* 10*s.* in 3 years and 129 days, at 8½*l.* per cent. per annum?
Anſ. Intereſt, £126 19 8½, and the amount=£572 9 8½.

7. If a broker diſpoſes of a cargo for me, to the amount of 637*l.* 10*s.* on commiſſion at 1¼*l.* per cent. and procures me another cargo of the value of 817*l.* 15*s.* on commiſſion at 1¾*l.* per cent.; What will his commiſſion, on both cargoes, amount to? *Anſ.* £22 5 7.

CASE II.

The amount, time, and ratio given, to find the principal.

RULE.—Multiply the ratio by the time; add unity to the product for a diviſor, by which ſum divide the amount, and the quotient will be the principal.

EXAMPLES.

1. What principal will amount to 1045*l.* 14*s.* in 7 years, at 6*l.* per cent. per annum?

Ratio=,06
Multiply by the time= 7

Product=,42
Add 1

Diviſor=1,42)1045,7(736,4084+ =£736 8 2.

 Or,

Or, $\frac{1045,7}{,06 \times 7+1}=£736\ 8\ 2$ *Ans.*

2. What principal will amount to 3810*l.* in 6 years, at $4\frac{1}{2}$*l.* per cent. per annum? *Ans.* £3000.

3. What principal will amount to 666*l.* 9*s.* $0\frac{1}{4}$*d.* in $3\frac{1}{2}$ years, at $5\frac{1}{4}$*l.* per cent. per annum? *Ans.* £563.

4. What principal will amount to 335*l.* 7*s.* 3*d.* in 3 years and 97 days, at $9\frac{1}{2}$*l.* per cent. per annum? *Ans.* £255 19 $0\frac{3}{4}$.

CASE III.

The amount, principal, and time given, to find the ratio.

RULE.—Subtract the principal from the amount; divide the remainder by the product of the time and principal, and the quotient will be the ratio.

EXAMPLES.

1. At what rate per cent. will 543*l.* amount to 705*l.* 18*s.* in 5 years?

From the amount=705,9
Take the principal=543

Divide by 543×5=2715)162,90(,06
162 90

Or, $\frac{705,9-543}{543\times 5}=,06=£6$ *Ans.*

2. At what rate per cent. will 391*l.* 17*s.* amount to 449*l.* 3*s.* $1\frac{1}{4}$*d.* ,74*qr.* in $3\frac{1}{4}$ years? *Ans.* £$4\frac{1}{2}$.

3. At what rate per cent. will 413*l.* 12*s.* 6*d.* amount to 546*l.* 4*s.* $10\frac{1}{4}$*d.* in $4\frac{3}{4}$ years? *Ans.* £$6\frac{3}{4}$.

4. At what rate per cent. will 3000*l.* amount to 3810*l.* in 6 years? *Ans.* £$4\frac{1}{2}$.

CASE IV.

The amount, principal, and rate per cent. given, to find the time.

RULE.—Subtract the principal from the amount; divide the remainder by the product of the ratio and principal; and the quotient will be the time.

EXAMPLES.

1. In what time will 543*l.* amount to 705*l.* 18*s.* at 6*l.* per cent. per annum?

From the amount=705,9
Take the principal=543

Divide by 543×,06=32,58)162,9(5 *years, Ans.*
162 9

2. In

2. In what time will 3000*l.* amount to 3810*l.* at $4\frac{1}{2}$ per cent. per annum? *Anf.* 6 *years.*

3. In what time will 391*l.* 17*s.* amount to 449*l.* 3*s.* $1\frac{3}{4}$*d.* at $4\frac{1}{2}$*l.* per cent. per annum? *Anf.* $3\frac{1}{4}$ *years.*

To find the Intereft of any Sum, at 6 per cent. per ann. for any number of Months.

Rule.—If the months be an even number, multiply the principal by half that number; and if the months be uneven, halve the even months, to which annex $\frac{5}{10}$; thus, the half of 19 is 9,5; and multiply the principal as before, cutting off two figures more at the right hand, than there are decimals in both factors, which reduce to farthings, each time cutting off as at firft.

4. What is the intereft of 345*l.* 16*s.* 6*d.* for 9 years and 11 months, at 6 per cent. per annum?

```
                      Y.  m.
                       9  11
                      12
                      ---
                    2)119 months.
                      -----
         345,825      59,5 = ½ No. of months.
            59,5
        --------
         1729125
        3112425
       1729125
      ----------
     £205,765875 = £205 15 3¾ Anf.
        Principal = £345 16 6
                    ------------
         Amount = £551 11 9¾
```

A Table *of decimal parts for every day in the twelfth part of a year, which confifts of* $365\frac{1}{4}$ *days.*

Days	dec.pts.	Days	dec.pts.	Days	dec.pts.	Days	dec.pts.	Days	dec.pts.
1	,033	7	,23	13	,427	19	,624	25	,821
2	,066	8	,263	14	,46	20	,657	26	,854
3	,098	9	,296	15	,493	21	,69	27	,887
4	,131	10	,328	16	,526	22	,723	28	,92
5	,164	11	,361	17	,558	23	,756	29	,953
6	,197	12	,394	18	,591	24	,788	30	,986

To find the Intereft of any Sum, either for Months, or Months and Days, at 6 per cent. per annum.

Rule.

Multiply the principal by the number of months (or months and parts, anfwering to the given number of days in the table) and

and cut off one figure at the right hand of the product more than is required by the rule in decimals, and the product will be the interest for the given time, in shillings and decimal parts of a shilling.

1. What is the interest of 100*l.* for a year?

Principal = 100
Mult. by the months = 12

Ans. *s.* 120|0 = 6*l.*

Note. This Table may also be used for the *parts of a year*, in Compound Interest, after having worked for whole years.

2. What is the interest of 250*l.* 10*s.* for 19 months and 7 days?

Principal = £250,5
Time = 19,23

7515
5010
22545
2505

Ans. *s.* 481,7115
= £24 1 $8\frac{1}{2}$.

Another Method of calculating INTEREST *for Months, at 6l. per cent. per annum.*

RULE.

If the principal consist of pounds only, cut off the unit figure, and, as it then stands, it will be the interest for one month in shillings and decimal parts:—If it consist of pounds, shillings, &c. reduce the shillings, &c. to decimals, which, with the unit figure of the pounds, will be decimal parts of a shilling.

EXAMPLES.

1. What is the interest of 175*l.* for 5 months?

£175 = 17,5 *shill.* = interest for 1 month.

17,5
Mult. by the time = 5

2|0|87,5

Ans. = £4 7 6

2. What is the interest of 255*l.* 16*s.* for 7 months?

Shill.
£255 16 = 25,58 int. for 1 mo.
7

2,0|179,06

£8 19 $0\frac{1}{2}$ *Ans.*

SIMPLE INTEREST IN FEDERAL MONEY.

PROBLEM I.

When the principal is given in lawful money, and the interest is required in federal money, at 6 per cent. per annum.

RULE.—Reduce the shillings, &c. to their equivalent decimal, by inspection, divide the whole by 5, and the quotient is the annual interest: Or, multiply the principal by 2, and the product (having the unit figure of the pounds cut off) will be the interest as before.

EXAMPLES.

EXAMPLES.

1. Required the annual intereſt of 517*l.* 3*s.* 7½*d.* at 6 per cent. ?

3*s.* = ,15
1½*d.* = 30
Exceſs of 12 = 1
———
,181

5)517,181 *Dols. c. m.*
103,436 = 103,43,6 *Anſ.*

Or, 517,181
2
——— *Dols. c. m.*
103,4362 = 103,43,6$\frac{2}{10}$

2. Required the annual intereſt of 1*l.* in dollars ?

5)1,00
20 *cents, Anſ.*

PROBLEM II.

When the principal is given in lawful money, and the intereſt and amount are required in federal money, at 6 per cent.

RULE.—Reduce the lawful money to federal, then divide the principal by 20 and that quotient by 5 ; add thoſe quotients together, and they are the intereſt ; or add them to the principal, and their ſum is the amount.

EXAMPLES.

1. Required the amount of 425*l.* 16*s.* 8½*d.* for one year, at 6 per cent. ?

,8
34
1
———
,835

3)425,835
———
20)1419,450
5)70,9725
14,1945
——— *Dols. c. m.*
1504,6170 = 1504,61,7 *Anſ.*

2. Required the amount of 112*l.* 4*s.* 6*d.* for 1 year ?

,2
24
1
———
,225

3)112,225
———
20)374,083
5)18,7041
3,7408
——— *Dols. c. m.*
396,52,79 = 396,52,7,9 *Anſ.*

PROBLEM III.

When the principal is lawful money, and the monthly intereſt is required in federal money.

RULE.—Reduce the ſhillings, &c. to decimals, by inſpection, then ſeparate the right hand figure of the pounds with the decimals, divide by 6, and the quotient is the anſwer in dollars, cents, &c.

EXAMPLE

EXAMPLE.

Required the monthly intereſt of 425*l.* 16*s.* 8½*d.* in federal money?

```
 8
 34          6)42,5835 Dols.c. m.
  1            7,09725=7,09,7¼ Anſ.
 ---
 835
```

PROBLEM IV.

When the principal is federal money, and the intereſt is required in the ſame.

RULE.—Work according to the general rule in ſimple intereſt, that is, multiply by the rate of intereſt, ſeparate the two right hand figures of the dollars in the product, and it will give the intereſt in dollars, cents, &c.

N. B. The figures, which are more than three places to the right hand of the comma, are of no account.

EXAMPLES.

1. What is the annual intereſt of 537*D.* 24*c.* 6*m.* at 6 per cent. ?

```
D. c. m.
537,24,6
       6
---------  D. c. m.
32,23476=32,23,4 Anſ.
```

2. What is the intereſt of 1465*D.* 46*c.* 6*m.* for 16 months, at 6 per cent. per annum?

```
D. c. m.
1465,46,6
        8=half the number of months.
---------  D. c. m.
117,237280=117,23,7 Anſ.
```

3. What is the intereſt of 537*D.* 34*c.* 7*m.* for 19 months, at 6 per cent. per annum?

```
D. c. m.
537,34,7
     9,5=half the number of months.
--------
 2686735
4836123
---------  D. c. m.
51,047965=51,04,7 Anſ.
```

N. B. Becauſe there are 4 decimals in the multiplicand and multiplier, I cut off 4 figures for them, and two more according to the rule.

PROBLEM V.

When the principal is federal money, and the monthly intereſt is required in the ſame, at 6 per cent. per annum.

RULE.—Separate the two right hand figures of the dollars, and you then have the intereſt of two months; half of which is the monthly intereſt in dollars, cents, &c. If there is but one place, or figure, of dollars, a cypher muſt be prefixed to the left hand.

EXAMPLES.

1. What is the monthly intereſt of 9*D*. 59*c*. 7*m*. at 6 per cent. per annum?

2),09597 *c. m.*
,047985=4,8 nearly, *Anſ.*

2. What is the monthly intereſt of 100*D*. 50*c*. 5*m*.?

2)1,00505 *c. m.*
,50252=50,2½, *Anſ.*

Rules for calculating Intereſt for Days.

RULE I.

Multiply the given principal by the given number of days, and that product by the rate on the pound: Divide the laſt product by 365 (the number of days in a year) and it will give the intereſt.

EXAMPLE.

What is the intereſt of 360*l*. 10*s*. for 175 days, at 6 per cent.?

$$\frac{360{,}5\times175\times{,}06}{365}=£10{,}37=£10\ 7s.\ 4\tfrac{3}{4}d.$$

RULE II.

Multiply the given principal by the given number of days, and divide the product by 6083, for 6*l*. per cent.; (the number of days in which any ſum will double, at that rate) the quotient will give the anſwer.

EXAMPLE.

What is the intereſt of 327*l*. 10*s*. at 6 per cent. per annum, for 210 days?

$$\frac{327{,}5\times210}{6083}=£11{,}306=£11\ 6\ 1\tfrac{1}{4}\ \textit{Anſ.}$$

Rule for making a Diviſor for any Rate per Cent.

Multiply 365 by 100, and divide the product by the rate.

Thus, for 6 per cent. $\frac{365\times100}{6}=6083$ diviſor.

For 5 per cent. $\frac{365\times100}{5}=7300$ diviſor, &c.

Perhaps

Perhaps the moſt convenient way to calculate at 6 per cent. is firſt to do it for 5 and then add one fifth of the quotient to itſelf; becauſe, by cutting off the two cyphers in the diviſor, you have to divide only by 73.

Hence, when intereſt is to be calculated on caſh accounts, accounts current, or any other accounts, where partial payments are made, or partial debts are contracted; multiply the ſeveral balances into the days they are at intereſt, and the ſum of theſe products, divided as above, will give the intereſt at 5*l.* or 6*l.* per cent. and for any other rate, make the proper addition or deduction; or find a diviſor, as before directed.

EXAMPLE.

On the 1ſt of January I lent 450*l.* 10*s.* 6*d.* which I received back in the following partial payments, viz. on the 14th of January, 57*l.* 11*s.* 9*d.*; on the 7th of February, 39*l.* 3*s.* 10*d.*; on the 19th of March, 63*l.* 5*s.* 2*d.*; on the 4th of April, 45*l.*; on the 26th of April, 19*l.* 12*s.* 6*d.*; on the 12th of May, 100*l.*; on the 10th of June, 60*l.* 7*s.* 3*d.*; and on the 1ſt of Auguſt, 65*l* 10*s.*; What intereſt is due at 6 per cent.?

Dates.			£.	s.	d.	Days.	Products.
January	1	Lent on demand - - - -	450	10	6	13	5856 16 6
	14	Received in part - - - -	57	11	9		
		Balance	392	18	9	24	9430 10 0
February	7	Received in part - - - -	39	3	10		
		Bal.	353	14	11	40	14149 16 8
March	19	Received in part - - - -	63	5	2		
		Bal.	290	9	9	16	4647 16 0
April	4	Received in part - - - -	45	0	0		
		Bal.	245	9	9	22	5400 14 6
Ditto	26	Received in part - - - -	19	12	6		
		Bal.	225	17	3	16	3613 16 0
May	12	Received in part - - - -	100	0	0		
		Bal.	125	17	3	29	3650 0 3
June	10	Received in part - - - -	60	7	3		
		Bal.	65	10	0	52	3406 0 0
Auguſt	1	Received in full of the principal	65	10	0		50155 9 11

I have

(5)

73,00)501,55 9 11(6 17 $4\frac{3}{4}$ intereſt at 5 per cent.
438   1 7 $5\frac{3}{4}$

6355   £8 4 $10\frac{1}{2}$ ditto at 6 per cent.
20

)1271,09
73

541
511

3009
12

)361,19
292

6919
4

)276,76
219

5776

I have given Peter Truſty a caſh credit for 1000*l.* in conſequence of which, on the 12th of May, I paid his bill for 250*l.*; ditto 27th, paid his draught for 280*l.*; June 1ſt, he gave me a a bill on the Maſſachuſetts bank at ſight for 290*l.*; July 17th, he paid me per receipt 70*l.*; Auguſt 20th, he drew for 750*l.* at ſight; ditto 31ſt, he paid me per receipt, 500*l.*; Sept. 15th, he drew at ſight for 135*l.*; and 3d of October, for 175*l.*; October 29th, he paid me per recept 250*l.*; and November 3d, 125*l.*; November 12th, he drew at ſight for 375*l.*; and ditto 18th, for 125*l.*; January 1ſt, he paid me per receipt 290*l.*; and 20th ditto, 210*l.* On the 1ſt of March following he demands a ſettlement; What is then due to me, intereſt at 6 per cent.?

Dates.			£.	Days.	Products.
May	12	Paid his bill - - - -	250	15	3750
Ditto	27	Paid his draught - - -	280		
		Balance	530	5	2650
June	1	Received in part - - -	290		
		Bal.	240	46	11040
July	17	Received in part - - -	70		
		Bal.	170	34	5780
Auguſt	20	Paid - - - - - - -	750		
		Bal.	920	11	10120
Ditto	31	Received in part - - -	500		
		Bal.	420	15	6300
September	15	Paid - - - - - - -	135		
		Bal.	555	18	9990
October	3	Paid - - - - - - -	175		
		Bal.	730	26	18980
Ditto	29	Received in part - - -	250		
		Bal.	480	5	2400 — 71010
November	3	Received in part - - -	125		
		Carried over *Bal.*	355		

Dates.				£.	Days.	Products.
						71010 brought over.
			Bal.	355	9	3195
November	12	Paid - - - - - - -		375		
			Bal.	730	6	4380
Ditto	18	Paid - - - - - - -		125		
			Bal.	855	44	37620
January	1	Received in part - -		290		
			Bal.	565	19	10735
Ditto	20	Received in part - -		210		
March	1		Bal.	355	40	14200
						141140

Then, 73,00)141140(19 6 8 interest at 5 per cent.
(5) 3 17 4

£23 4 0 interest at 6 per cent.
355 0 0

£378 4 0 balance in my favour.

When cash credits are given, a balance should be made upon every transaction, which should be multiplied into the days the first leisure minute; then, when the time of settlement comes, you will only have to add up the products, and divide as above, and the account will be finished.

A owes B the following sums, with the interest on them, at 6 per cent. per annum, as follows; viz. 60*l.* for 7 months, 150*l.* for 15 months, 75*l.* 10*s.* for 9 months, 145*l.* 15*s.* for 27 months, and 397*l.* 12*s.* for 45½ months; What is the amount of principal and interest?

£.		Months.		
60	×	7	=	420
150	×	15	=	2250
75,5	×	9	=	679,5
145,75	×	27	=	3935,25
397,6	×	45,5	=	18090,8

£
200)25375,55(126,877 interest.
828,85 principal.

£955,727 amount, *Answer.*

Note. I divide by 200, the number of months, in which any sum will double at 6 per cent. per annum, and it gives the interest.

When partial payments are made upon notes, bonds, &c. at any interval *greater* than a year, the interest is calculated in a progressive manner, by adding the interest to the principal at the time of the first payment, and from the sum deducting the payment, &c.

ANNUITIES.

ANNUITIES *or* PENSIONS *in* ARREARS, *at* SIMPLE INTEREST.

An Annuity is a ſum of money, payable every year, half year, or quarter, for a certain number of years, or forever.

When the debtor keeps the annuity in his own hands, beyond the time of payment, it is ſaid to be in *arrears.*

The ſum of all the annuities, for the time they have been forborne, together with the intereſt due upon each, is called the *amount.*

If an annuity is to be bought off, or paid all at once, at the beginning of the firſt year, the price, which ought to be given for it, is called the *preſent worth.*

C A S E.

When the annuity, time, and rate of intereſt are given, to find the amount.

R U L E.*

1. Find the ſum of the natural ſeries of numbers 1, 2, 3, 4, &c. to the number of years leſs one.

2. Multiply this ſum by one year's intereſt of the annuity, and the product will be the whole intereſt due upon the annuity.

3. To this product add the product of the annuity and time, and the ſum will be the amount ſought.

E X A M P L E S.

1. What is the amount of an annuity of 90*l.* for 6 years, allowing ſimple intereſt, at 6 per cent. per annum?

$1+2+3+4+5=15=3\times 5=$ ſum of the number of years leſs 1.

$$\begin{array}{rl} £. & s. \\ 5 & 8 = 1 \text{ year's intereſt of the annuity.} \\ & 3 \\ \hline 16 & 4 \\ & 5 \\ \hline 81 & = \text{whole intereſt due.} \end{array}$$

Add $540=90\times 6=$ annuity multiplied by the time.

621 = amount required.

2. If

* Whatever the time may be, there is due upon the firſt year's annuity ſo many years' intereſt as the whole number of years leſs one, and gradually one leſs upon every ſucceeding year to the laſt but one, and none upon the laſt; therefore, in the whole, there is due ſo many years' intereſt of the annuity, as the ſum of the ſeries 1, 2, 3, 4, &c. to the number of years leſs one; conſequently, one year's intereſt, multiplied by this ſum, muſt be the whole intereſt due; to which, if all the annuities be added, the ſum is plainly the amount.

Let r=ratio, n=annuity, t=time, and a=amount: Then will the following theorems give the ſolution of all the different caſes.

I.

2. If a houſe be let upon a leaſe for 8 years, at 36*l.* per annum, I demand the amount for that time, at 5 per cent. per annum ?

Anſ. £338 8*s.*

3. If a ſalary of 120*l.* payable every half year, remain unpaid 7 years ; What would it amount to in that time, at 6*l.* per cent. per annum ?* *Anſ.* £1003 16*s.*

4. If a ſalary of 120*l.* payable every quarter, remain unpaid 7 years ; What would it amount to in that time, at 6*l.* per cent. per annum ? *Anſ.* £1010 2*s.*

PRESENT WORTH *of* ANNUITIES *at* SIMPLE INTEREST.

CASE.

The annuity, rate, and time given, to find the preſent worth.

RULE.†

Find the preſent worth of each year, by itſelf, diſcounting from the time it falls due, and the ſum of all theſe will be the preſent worth required.

EXAMPLES.

I. $\frac{2a-2tn}{t^2n-tn}=a.$ II. $\frac{2a}{t^2r+2t-tr}=n.$ III. $\frac{2a-2tn}{t^2n-tn}=r.$

IV. $\left|\frac{2a}{rn}+\frac{d^2}{4}\right|^{\frac{1}{2}}-\frac{d}{2}=t.$ *In the laſt Theorem,* $d=\frac{2n-rn}{rn}$

And in Theorem firſt, if a ſum cannot be found equal to the amount, the problem is impoſſible in whole years.

* In this Caſe, when the annuities, &c. are to be paid half yearly, or quarterly, then for half yearly payments, take $\frac{1}{2}$ of the ratio, $\frac{1}{2}$ the annuity, &c. and twice the number of years.

For quarterly payments, take $\frac{1}{4}$ of the ratio, $\frac{1}{4}$ of the annuity, &c. and 4 times the number of years, and work as before.

† If we grant the condition of allowing ſimple intereſt to be conſiſtent, the reaſon of this rule will be evident from the nature of diſcount ; for all the annuities may be conſidered ſeparately as ſo many ſingle and independent debts, due after 1, 2, 3, &c. years ; ſo that the preſent worth of each being found, their ſum muſt be the preſent worth of the whole,

But the purchaſing of annuities by ſimple intereſt is unjuſt and abſurd, which may eaſily be made to appear by one inſtance only. The price of an annuity of 100*l.* to continue 30 years, diſcounting at 6 per cent. will amount to 2000*l.* nearly, the intereſt of which for one year only exceeds the annuity : Would it not therefore be highly abſurd to give a ſum, which would yield me nearly 120*l.* yearly, forever, for an annuity of 100*l.* to continue only 30 years ?

Let p=preſent worth, and the other letters as before.

$$\text{Then,}\left\{\begin{array}{l} n\times\frac{1}{1+r}+\frac{1}{1+2r}+\frac{1}{1+3r}\text{ \&c. to }\frac{1}{1+tr}=p \\ p\div\frac{1}{1+r}+\frac{1}{1+2r}+\frac{1}{1+3r}\text{ \&c. to }\frac{1}{1+tr}=n \end{array}\right.$$

Moſt

EXAMPLES.

1. What is the present worth of 400*l.* per annum, to continue 6 years, at 6 per cent. per annum?

$$\left.\begin{array}{l}106\\112\\118\\124\\130\\136\end{array}\right\}:100::400:\left\{\begin{array}{lllll}377{,}35849= & \text{present worth of} & & \text{one} & \text{year.}\\357{,}14285= & — & — & \text{2d} & \text{year.}\\338{,}98305= & — & — & \text{3d} & \text{year.}\\322{,}58064= & — & — & \text{4th} & \text{year.}\\307{,}6923= & — & — & \text{5th} & \text{year.}\\294{,}11764= & — & — & \text{6th} & \text{year.}\end{array}\right.$$

1997,87497 = £1997 17*s.* $5\frac{3}{4}$*d.* = pres. worth.

2. What is the yearly rent of a house of 36*l.* to continue 8 years, worth, in ready money, at 5 per cent. per annum?

Ans. £237 3*s.* $9\frac{1}{2}$*d.*

DISCOUNT

Is an allowance made for the payment of any sum of money, before it becomes due, and is the difference between that sum, due some time hence, and its present worth.

The *present worth* of any sum, or debt, due some time hence, is such a sum, as if put to interest, would in that time and at the rate per cent. for which the discount is to be made, amount to the sum or debt, then due.

RULE I.*

As the amount of 100*l.* for the given rate and time, is to 100*l.* So is the given sum, or debt, to the present worth.

Subtract

Most authors give the following Theorems, viz.

I. $\dfrac{t^2r-tr+2t}{2tr+2}\times n=p.$ II. $\dfrac{tr+1}{t^2r-tr+2t}\times 2p=n.$

III. $\dfrac{\overline{2nt-p}\times 2}{2pt-nt^2-nt}=r.$ IV. $\dfrac{2}{r}-\dfrac{2p}{n}-1=x.$ Then, $\sqrt{\dfrac{2p}{nr}+\dfrac{x^2}{4}}-\dfrac{x}{2}=t.$

Note. The same thing is to be observed in this Case as in the Case of Annuities in Arrears, respecting half yearly and quarterly payments.

I have shewn the method of computing Annuities by Simple Interest, more for the gratification of the curious than for real utility, it being not only customary, but also most equitable, to allow compound interest.

It may be observed that the Theorem $\dfrac{t^2r-tr+2t}{2tr+2}$ gives the answer to the first question = 2028*l.* which therefore appears to be erroneous.

* That an allowance ought to be made for paying money before it becomes due, which is supposed to bear no interest till after it is due, is very reasonable; for if I keep the money in my own hands till the debt shall become due, it is plain I may make an advantage of it by putting it out to interest for that time; but if I pay it

Subtract the present worth from the given sum, and the remainder will be the discount required.

Or,

As the amount of 100*l.* for the given rate and time, is to the interest of 100*l.* for that time: So is the given sum or debt to the discount required.

EXAMPLES.

1. What is the discount of 635*l.* 17*s.* due two years hence, at $5\frac{1}{2}$ per cent. per annum?

```
Int. of 100l. } =5 10
per annum     }
                  2 years.
               ------
               11
       Add    100
              ---  £.   £.  s.
       As £111 : 11 :: 635  17
            20          20
          ----        -----
          2220        12717
                         11
                      -----
                      12717
                     12717
                    -------
          222|0)13988|7(63l. discount.
                1332
                ----
                 668
                 666
                 ---
                  27
                  20
                  --
           222|0)540(0
                  12
                 ---
          222|0)648|0(2
                444
                ---
                204
```

Or,

As £111 : £100 :: £635 17*s.* : £572 16*s.* $9\frac{1}{4}$*d.* present worth.

And £635 17*s.* − £572 16*s.* $9\frac{1}{4}$*d.* = £63 0*s.* $2\frac{3}{4}$*d.* discount.

Ans. £63 0*s.* $2\frac{3}{4}$*d.*

2. What

it before it is due, I give that benefit to another; therefore, we have only to inquire what discount ought to be allowed. And here, many suppose that, since by not paying till it becomes due they may employ it at interest, therefore, by paying it before due, they shall lose that interest, and for that reason all such interest ought to be discounted; but the supposition is false; for they cannot be said to lose that interest till the time arrives, when the debt becomes due; whereas we are to consider what would properly be lost, at present, by paying the debt before it becomes

2. What is the prefent worth of 350*l.* payable in half a year, difcounting at 6*l.* per cent. per annum?

Anf. £339 16*s.* 1¼*d.*

3. What is the prefent worth of 65*l.* due 15 months hence, at 6*l.* per cent. per annum? *Anf.* £60 9*s.* 3½*d.*

4. What is the difcount on 97*l.* 10*s.* due January 22d, this being September 7th, reckoning intereft at 5*l.* per cent.?

Anf. £1 15*s.* 11*d.*

5. What ready money will difcharge a debt of 475*l.* 10*s.* due 5 months and 20 days hence, at 6 per cent.?

Anf. £462 7*s.* 11½*d.*

6. Bought a quantity of goods for 250*l.* ready money, and fold them for 300*l.* payable 9 months hence: What was the gain, in ready money, fuppofing difcount to be made at 6*l.* per cent.?

Anf. £37 1*s.* 7½*d.*

7. What is the prefent worth of 275*l.* payable as follows; viz. ½ at 3 months, ⅓ at 6 months, and the reft at 9 months, fuppofing the difcount to be made at 6*l.* per cent.? *Anf.* £268 6*s.* 5¾*d.*

RULE II.

As any fum of money, at 6 per cent. per annum, will double, at fimple intereft, in 200 months; therefore,

To 200 add the number of months for which the difcount is required, by which fum divide the product of the money and time, (in months,) and the quotient will be the difcount.

EXAMPLES.

1. What is the difcount of 150*D.* 75*c.* for a year?

200

becomes due; this can, in point of equity, be no other than fuch a fum, which being put out at intereft till the debt fhall become due, would amount to the intereft of the debt for the fame time.—It is befides plain that the advantage arifing from difcharging a debt due fome time hence, by a prefent payment, according to the principles above mentioned, is exactly the fame as employing the whole fum at intereft till the time, when the debt becomes due, arrives: For, if the difcount allowed for prefent payment be put out at intereft for that time, its *amount* will be the fame as the intereft of the whole debt for the fame time; thus the difcount of 106*l.* due one year hence, reckoning intereft at 6*l.* per cent. will be 6*l.* and 6*l.* put out to intereft at 6*l.* per cent. for one year, will amount to 6*l.* 7*s.* 2¼*d.* which is exactly equal to the intereft of 106*l.* for one year at 6*l.* per cent.

The truth of the rule for working is evident from the nature of Simple Intereft; for fince the debt may be confidered as the *amount* of fome principal (called here the prefent worth) at a certain rate per cent. and for the given time, that amount muft be in the fame proportion either to its principal or intereft, as the amount of any other fum, at the fame rate, and for the fame time, to its principal or intereft.

```
 200    15075
  12       12
 ---  -------       D. c. m.
212)1809,00(8,533=8,53,3 discount, Ans.
     1696
     ----
      1130
      1060               150,75
      ----                 8 533
        700              ------
        636 Present worth 142,217
        ---
         640
         636
         ---
           4
```

2. What is the present worth of 426*D.* 55*c.* at 6 per cent. &c. to be paid 8 months hence? *Ans.* 410*D.* 14*c.* 5*m.*

3. What is the discount of 361*l.* 15*s.* 6*d.* to be paid 1 year and 8 months hence? *Ans.* £32 17*s.* 9¼*d.*

ABBREVIATIONS *in* DISCOUNT.

Any principal to be discounted for one year, at any of the following rates, (or for any rate and time, whose product is equal to any of the following rates) being (multiplied by the multiplier, if any, and) divided by the corresponding divisor, the quotient will be the discount.

Rates.	
At $1\frac{1}{4}$	$\div 81$ (or by 9 and 9)
2	$\div 51$
$2\frac{1}{2}$	$\div 41$
4	$\div 26$
5	$\div 21$ (or by 7 and 3)
6	$\div 53$ and $\times 3$
$7\frac{1}{2}$	$\div 43$ and $\times 3$
8	$\div 27$ and $\times 2$ (or $\times 2$ and $\div 9$ and 3)
$8\frac{1}{3}$	$\div 13$
10	$\div 11$
12	$\div 28$ and $\times 3$ (or $\times 3$, and $\div 7$ and 4)
$12\frac{1}{2}$	$\div 9$

1. How much must I abate of 5394*l.* 10*s.* due 3 years hence, at $2\frac{2}{3}$ per cent. per annum?

```
                                   £5394 10s.
                                           2
  2⅔                               ---------
 ×3                  27÷3=9)10789  0
 ---                        ----------
  8, therefore, ×2, and ÷27  3)1198 15 6½
                            ------------
                            £399 11 10 Ans.
```

2. What

2. What is the discount of 546*l.* 12*s.* 6*d.* for $8\frac{1}{3}$ years, at 1 per cent. per annum, (or for 1 year at $8\frac{1}{3}$ per cent. per annum ?)

	£.	s.	d.
13)	546	12	6
	£42	0	$11\frac{1}{2}$ *Ans.*

3. What is the discount of 125*l.* at $1\frac{1}{4}$ per cent. per annum, for four years, (or, at 4 per cent. per annum, for $1\frac{1}{4}$ year ?)

$1\frac{1}{4}$
$\times 4$

5, therefore, $\div$ 21)125

£5 19 $0\frac{1}{2}$ *Ans.*

DISCOUNT BY DECIMALS.*

The sum to be discounted, the time and the ratio given, to find the present worth.

Rule.—Multiply the ratio by the time, add unity to the product for a divisor; by which sum divide the sum to be discounted, and the quotient will be the present worth.

Subtract the present worth from the principal or sum to be discounted, and the remainder will be the discount.

Or, as the amount of 1*l.* for the given time, is to 1*l.* so is the interest of the debt for the said time, to the discount required.

Subtract the discount from the principal, and the remainder will be the present worth.

Examples.

First Method.

1. What is the present worth of 600*l.* due 3 years hence, at 6*l.* per cent. per annum ?

Ratio

* As in Simple Interest, let a=amount of any debt, p=present worth, t=time, and r=ratio ; then will the following Theorems exhibit all the cases in Discount at Simple Interest.

I. $\frac{a}{tr+1}=p$. II. $ptr+p=a$. III. $\frac{a-p}{tp}=r$. IV. $\frac{a-p}{rp}=t$.

Note. When the ratio is ,06 per cent. per annum, and the given time is expressed in months, whether less or more than a year, if the debt be divided by 1 plus half as many hundredths of an unit, as there are months in the given time, the quotient will be the present worth.—Thus, for 1 month $\frac{a}{1,005}$, 2 months $\frac{a}{1,01}$, 3 months $\frac{a}{1,015}$, 36 months $\frac{a}{1,18}$, 42 months $\frac{a}{1,21}$, &c. &c.

$$\begin{array}{rl} \text{Ratio} = & ,06 \\ \text{Multiply by the time} = & 3 \\ \hline \text{Product} = & ,18 \\ \text{Add} & 1 \end{array}$$

£.
Divisor=1,18)600(508,4745 pref. worth.

Or, $\frac{600}{,06 \times 3 + 1} = £508\ 9s.\ 5\frac{3}{4}d.$ *Anf.*

Prefent worth=508,4745=£508 9*s.* $5\frac{3}{4}$*d.* which fubtracted from the principal, will give the difcount =£91 10*s.* $6\frac{1}{4}$*d.*

Second Method.

What is the difcount of 600*l.* due 3 years hence, at 6*l.* per cent. per annum?

$$\begin{array}{rl} \text{Ratio} = & ,06 \\ \text{Multiply by} & 3 \\ \hline & ,18 \\ \text{Add} & 1 \end{array} \qquad \begin{array}{r} \text{As } 1,18 : 1 :: 108 \\ 1 \\ \hline 1,18)108,00(91,5254 \end{array}$$

Amount of 1*l.* for the given time $\}$ =1,18

And 600×,06×3=108=intereft of the debt for the given time. Difcount=91,5254=£91 10*s.* 6*d.* which taken from the principal will leave the prefent worth=£508 9*s.* 6*d.*

2. What is the prefent worth of 312*l.* 10*s.* due 2 years hence, at $4\frac{1}{2}$*l.* per cent. per annum? *Anf.* 286*l.* 13*s.* $11\frac{1}{4}$*d.*

3. What is the prefent worth of 1650*l.* 15*s.* 6*d.* at $6\frac{3}{4}$*l.* per cent. per annum, due 18 months hence? *Anf.* 1499 0*s.* $0\frac{1}{4}$*d.*

4. What ready money will difcharge a debt of 1354*l.* 8*s.* due 3 years, 3 months and 12 days hence, at $5\frac{7}{8}$*l.* per cent. per annum? *Anf.* £1135 7*s.* 9*d.*

BARTER

Is the exchanging of one commodity for another, and teaches traders to proportion their quantities without lofs.

CASE I.

When the quantity of one commodity is given, with its value, or that of its integer, that is, of 1lb, 1Cwt. 1yd. &c. as alfo the value of the integer of fome other commodity, to be given for it, to find the quantity of this; or, having the quantity thereof given, to find the rate of felling it.

Rule.—Find the value of the given quantity by the concifeft method, then find what quantity of the other, at the rate propofed, you may have for the fame money: *Or,* if the quantity be given, find from thence the rate of felling it.

Examples.

EXAMPLES.

1. How much tea, at 9*s*. 6*d*. per ℔, must be given in barter for 156 gallons of wine, at 12*s*. $3\frac{1}{2}$*d*. per gallon?

$$\begin{array}{l|l|r}
3d. & \frac{1}{4} & \text{Galls.}\ 156 \\
\frac{1}{2} & \frac{1}{6} & 12 \\
\hline
 & & 1872 \\
 & & 39 \\
 & & 6\ \ 6 \\
\hline
 & & 1917\ \ 6 \\
 & & 12 \\
\hline
 & & 23010
\end{array}$$

9*s*. 6*d*. = 114*d*.

$$\begin{array}{l}
\ \ d.\quad ℔\qquad d. \\
\text{As } 114 : 1 :: 23010 \\
\qquad\qquad\qquad 1 \\
\hline
114)23010(201 \\
\quad 228 \\
\hline
\quad\ \ 210 \\
\quad\ \ 114 \\
\hline
\qquad 96 \\
\qquad 16 \\
\hline
\quad\ 576 \\
\quad\ \ 96 \\
\hline
114)1536(13\frac{54}{114}\ \textit{Ans.}\ 201℔\ 13\frac{54}{114}oz. \\
\quad\ 114 \\
\hline
\quad\ 396 \\
\quad\ 342 \\
\hline
\qquad 54
\end{array}$$

2. How much cloth, at 15*s*. 8*d*. per yard, must be given for 5*Cwt*. 3*qrs*. 19℔ of steel, at 5 guineas per *Cwt*.

Ans. 52*yds*. 3*qrs*. 2*n*.

3. Suppose A has 350 yards of linen, at 1*s*. 4*d*. per yard, which he would truck with B for sugar, at 25*s*. 6*d*. per *Cwt*. How much sugar will the linen come to? *Ans*. 18*Cwt*. 1*qr*. $5\frac{2}{3}$℔.

4. A has broadcloths at 44*D*. per piece, and B has mace, at 1*D*. 42*c*. per ℔; How many pounds of mace must B give A for 35 pieces of cloth? *Ans*. $1084\frac{1}{2}$℔.

5. A

5. A has $7\frac{1}{2}$ *Cwt.* of ſugar at 12 cents per ℔, for which B gave him $12\frac{1}{2}$ *Cwt.* of flour; What was the flour rated at per ℔?

Anſ. 7*c.* 2*m.*

CASE II.

If the quantities of two commodities be given, and the rate of ſelling them, to find, in caſe of inequality, how much of ſome other commodity muſt be given.

Rule.—Find the ſeparate values of the two given commodities; ſubtract the leſs from the greater, and the difference will be the amount of the third commodity, whoſe quantity and rate may be eaſily found.

Examples.

1. Two merchants barter; A has 30 *Cwt.* of cheeſe, at 23*s.* 6*d.* per cwt. and B has 9 pieces of broadcloth, at 3*l.* 15*s.* per piece; Which muſt receive money, and how much?

Anſ. B muſt pay A £1 10*s.*

2. A and B would barter; A has 150 buſhels of wheat, at 5*s.* 9*d.* per buſhel, for which B gives 65 buſhels of barley, worth 2*s.* 10*d.* per buſhel, and the balance in oats at 2*s.* 1*d.* per buſhel; What quantity of oats muſt A receive from B? *Anſ.* $325\frac{15}{25}$ *buſhels.*

CASE III.

Sometimes, in bartering, one commodity is rated above the ready money price; then, to find the bartering price of the other, ſay,

As the ready money price of the one, is to its bartering price; ſo is *that* of the other, to *its* bartering price: Next, find the quantity required, according to either the bartering or ready money price.

Examples.

1. A has ribbands at 2*s.* per yard ready money; but in barter he will have 2*s.* 3*d.* B has broadcloth at 32*s.* 6*d.* per yard ready money:—At what rate muſt B value his cloth per yard, to be equivalent to A's bartering price, and how many yards of ribband, at 2*s.* 3*d.* per yard, muſt then be given by A for 488 yards of B's broadcloth?

Anſ. B's broadcloth, at £1 16*s.* $6\frac{1}{2}$*d.* per *yd.* 7930 *yds.* ribband.

2. A and B barter; A has 150 gallons of brandy, at 7*s.* 3*d.* per gallon, ready money, but in barter he will have 8*s.* per gallon: B has linen at 3*s.* 6*d.* per yard ready money; How muſt B ſell his linen per yard in proportion to A's bartering price, and how many yards are equal to A's brandy?

Anſ. barter price is 3*s.* $10\frac{1}{4}$*d.* and he muſt give A 310*yds.* 2*qrs.* 3*n.*

3. P

3. P and Q barter. P has Iriſh linen, at 3*s.* 7*d.* per yard; but in barter will have 3*s.* 10*d.* Q delivers him broadcloth at 1*l.* 16*s.* 6*d.* per yard, worth only 1*l.* 13*s.* per yard: Pray, which has the advantage in Barter, and how much linen does P give Q for 148 yards of broadcloth?

Yd. *Yds.* *s.*
As 1 : 36/6 :: 148 : 5402 price of the broadcloth. As 3/10 : 1*yd.* :: 5402*s.* : $1409\frac{5}{23}$ *yards* of linen. Q has the advantage; for as 3/7 : 3/10 :: 33*s.* : $35/3\frac{27}{43}$ his proportional price.

4. A has 200 yards of linen, at 1*s.* 6*d.* ready money per yard, which he barters with B, at 1*s.* 9*d.* per yard, taking buttons at $7\frac{1}{2}$*d.* per groſs, whioh are worth but 6*d.*: How many groſs of buttons will pay for the linen; who gets the beſt bargain, and by how much both in the whole and per cent.?

Yd. d. *Yds.* *d.* *d. Groſs. d.* *Groſs.* *Yd. d.* *Yds.* *£.*
As 1 : 21 :: 200 : 4200. As $7\frac{1}{2}$: 1 :: 4200 : 560. As 1 : 18 :: 200 : 15
gr. d. *gr.* *£.* [value of A's linen.
As 1 : 6 :: 560 : 14 value of B's goods. So that B gained 1*l.* of A.
£. *£.* *£.* *£.*
As 14 : 1 :: 100 : 7 2*s.* 10*d.* per cent.

5. A has linen cloth, at 30*c.* per yard, ready money, in barter 36*c.* B has 3610 yards of ribband, at 22*c.* per yard ready money, and would have of A 200*dolls.* in ready money, and the reſt in linen cloth; what rate does the ribband bear in barter per yard, and how much linen muſt A give B?

Anſ. The rate of ribband is 26*c.* 4*m.* per yard, and B muſt receive $1980\frac{2}{3}$ *yards* of linen, and 200*dolls.* in caſh.

LOSS AND GAIN

Is an excellent rule, by which merchants and traders diſcover their profit, or loſs per cent. or by the groſs: It alſo inſtructs them to raiſe or fall the price of their goods, ſo as to gain or loſe ſo much per cent. &c.

Queſtions in this rule are performed by the Rule of Three.

CASE I.

To know what is gained or loſt per cent.

RULE.

Firſt ſee what the gain or loſs is, by ſubtraction; then, as the price it coſt is to the gain or loſs: So is 100*l.* to the gain or loſs per cent.

EXAMPLES.

1. If I buy ſerge at 5*s.* per yard, and ſell it again, at 5*s.* 8*d.* per yard; What do I gain per cent. or in laying out 100*l.*?

Sold

	s.	d.		d.		d.		£.
Sold for	5	8	As	60	:	8	::	100
Cost	5	0						8

8 gain per yard.

```
60)800(13
   60
   ---
   200
   180
   ---
    20
    20
   ---
60)400(6
   360
   ---
    40
    12
   ---
60)480(8
   480   Ans. £13 6s. 8d.
```

N. B. The first questions, in the several Cases, serve to elucidate each other.

2. If I buy serge at 5/8 per yard, and sell it again at 5s. per yard; What do I lose per cent. or in laying out 100l.?

	s.	d.		d.		d.		£.
Cost	5	8	As	68	:	8	::	100
Sold for	5	0						8

Loss per yd. 8d.

```
68)800(11
   68
   ---
   120
    68
   ---
    52
    20
   ---
68)1040(15
   68
   ---
   360
   340
   ---
    20
    12
   ---
68)240(3
   204
   ---
    36
     4
   ---
68)144(2
   136
   ---
     8   Ans. £11 15 3 1/2.
```

3. If

3. If I buy a *Cwt.* of tobacco for 9*l.* 6*s.* 8*d.* and sell it again at 1*s.* 10*d.* per ℔, do I gain or lose, and what per cent.?

	£.	s.	d.	
Sold for	10	5	4	
Cost	9	6	8	
	0	18	8	gained in the gross.

℔
112

		£.	s.	
2*d.*	$\frac{1}{12}$	11	4	value at 2*s.* per ℔.
		0 18	8	value at 2*d.* per ℔.
		10 5	4	value at 1*s.* 10*d.* per ℔.

£. s. d. s. d. £. £.
As 9 6 8 : 18 8 :: 100 : 10 *Ans.* £10 per cent.

4. A draper bought 60 yards of cloth at 28*s.* per yard, and 38 yards of ditto at 14*s.* per yard, and sold them, one with another, at 26*s.* per yard: Did he gain or lose, and what per cent.?

yd. s. yds. £.
As 1 : 28 :: 60 : 84

yd. s. yds. £. s.
As 1 : 14 :: 38 : 26 12

£110 12

yd. s. yds. £. s.
As 1 : 26 :: 98 : 127 8

	£.	s.
Sold for	127	8
Prime cost	110	12
Gain in the gross	16	16

£. s. £. s. £. £. s. d.
As 110 12 : 16 16 :: 100 : 15 3 9½ per cent.

5. Bought sugar at 6½*d.* per ℔, and sold it at 2*l.* 3*s.* 9*d.* per *Cwt.* What was the gain or loss per cent?

℔ d. ℔ £. s. d.
As 1 : 6½ :: 112 : 3 0 8

Prime cost £3 0 8 per *Cwt.*
Sold at 2 3 9 per *Cwt.*

Lost £0 16 11 in the whole.

£. s. d. s. d. £. £. s. d.
As 3 0 8 : 16 11 :: 100 : 27 17 8¼ loss per cent. *Ans.*

6. At 2½*d.* in the shilling profit; How much per cent.?

s. d. £. £. s. d.
As 1 : 2½ :: 100 : 20 16 8 *Ans.*

7. At 4*s.* 6*d.* in the pound profit; How much per cent.?

£. s. d. £. £. s.
As 1 : 4 6 :: 100 : 22 10 *Ans.*

8. If I buy candles, at 1*s.* 6*d.* per ℔, and sell them again, at 2*s.* per ℔, and allow 3 months for payment; What do I gain per cent.?

d. d. £. £. s. d. Mo. £. Mo. £. s.
As 18 : 24 :: 100 : 133 6 8; then by Discount. As 12 : 6 :: 3 : 1 10

£. s. £. s. £. s. d. £. s. d.
Then, As 101 10 : 1 10 :: 133 6 8 : 1 19 4¾, which taken from 133*l.* 6*s.* 8*d.* leaves 131*l.* 7*s.* 3¼*d.* therefore, *Ans.* £31 7 3¼.

9. If

9. If I buy cloth at 6*s.* per yard, ready money, and ſell it again at 6*s.* 8*d.* per yard on 3 months credit; What is gained per cent.?

s. d. s. *s. d. £. £. s. d.* *Mo. £. Mo. £. s.*
6 8—6=8*d.* As 6 : 8 :: 100 : 11 2 $2\frac{1}{2}$ As 12 : 6 :: 3 : 1 10
£. s. £. £. s. d. £. s. d.
As 101 10 : 100 :: 111 2 $2\frac{1}{2}$: 109 9 $4\frac{1}{2}$; therefore
£. s. d. £. £. s. d.
109 9 $4\frac{1}{2}$—100=9 9 $4\frac{1}{2}$ *Anſ.*

10. If I buy cloth at 13*s.* per yard, on 8 months credit, and ſell it again at 12*s.* ready money, Do I gain or loſe, and what per cent.?

Mo. £. Mo. £. £. s. £. s. d.
As 12 : 6 :: 8 : 4 As 104 : 13 :: 100 : 12 6: So that 13*s.* on 8 months credit at 6*l.* per cent. is equal to 12*s.* 6*d.* ready money; then, *s. d. d. £. £.*
As 12 6 : 6 :: 100 : 4

s. d.
Prime coſt 12 6 ready money,
Sold at 12 0 ready money,

Loſt 0 6 in the yard. *Anſ.* loſt £4 per cent.

11. If I buy gloves at 1*D.* 25*c.* per pair; How long credit muſt I have, to gain 13*D.* per cent. when I ſell them at 1*D.* 36*c.* per pair?

D. c.
Sold at 1,36
Prime coſt 1,25

Gained ,11 per pair.

D. c. c. D. D. c.
As 1,25 : ,11 : 100 : 8,80 gain per cent. *D. D. c. D. c.* [ready money.
Then, 13—8,80=4,20 Now,
D. Mo. D. c. Mo. days.
As 6 : 12 :: 4,20 : 8 12 *Anſ.*

In caſting up the amount of goods bought, imported or exported; to the prime coſt of ſuch goods we muſt add all the charges upon them, in order to fix the price they ſtand us in.

12. Suppoſe I import from France, 12 bales of cloth, containing 10 pieces each, which, with the charges there, amounted to 360*D.*; I pay duty here 92*c.* per piece, for freight 12*D.* and portage 1*D.* 25*c.*; What does it ſtand me in per piece, and how muſt I ſell it per piece to gain 10*D.* per cent.?

12 bales.
10

120 pieces.

	D. c.
Firſt coſt	360
Duty	110,40
Freight	12
Porterage	1,25
Whole coſt,	483,65

Pieces. D. c. Piece. D. c.
As 120 : 483,65 :: 1 : 4,03 coſt per piece.

D. Pieces. D. c. c. m.
Again, As 100 : 10 :: 4,03 : 40,3 gain.
40,3

D. 4,43,3 the price, at which it muſt be ſold per piece.

CASE

CASE II.

To know how a commodity muſt be ſold, to gain or loſe ſo much per cent.

RULE.—As 100*l.* is to the price; ſo is 100*l.* with the profit added, or loſs ſubtracted, to the gaining or loſing price.

EXAMPLES.

1. If I buy a quantity of ſerge, at 5*s.* per yard; How muſt I ſell it per yard, to gain 13*l.* 6*s.* 8*d.* per cent.?

```
 £.   s.     £. s. d.
As 100 : 5 :: 113 6 8
     20        20
    ----      ----
    2000      2266
      12        12
    ----      ----
   24000     27200
                 5
             -----
24,000)136,000(5
       120
       ---
        16
        12
       ---
24,000)192,000(8
       192
       ---
       000
```

Or thus:

```
   £.  s. d.
   113  6 8
          5
   ---------
s. 5|66 13 4
      12
     ---
     792
Add    8 = 2/3 of 12 = 13s. 4d.
     ---
d. 8|00

        s. d.
   Anſ. 5 8 per yard.
```

2. If a barrel of powder coſt 4*l.* how muſt it be ſold to loſe 10*l.* per cent.?

```
 £.  £.  £.
As 100 : 4 :: 90
               4
              ---
       100)360(3
           300
           ---
            60
            20
           ---
      100)1200(12   Anſ. £3 12s.
          1200
```

Or thus:

```
     90
      4
    ---
 £3|60
    20
   ---
s. 12|00
```

3. Bought cloth, at 15*s.* per yard, which not proving ſo good as I expected, I am content to loſe $17\frac{1}{2}$ per cent. by it; How muſt I ſell it per yard?

£. s. £. s. d.
As 100 : 15 :: $82\frac{1}{2}$: 12 $4\frac{1}{2}$ *Anſ.* 12*s.* $4\frac{1}{2}$*d.*

Or thus:

$$
\begin{array}{r}
82\ 10 \\
3\times5=15 \\
\hline
247\ 10 \\
5 \\
\hline
s.\ 12|37\ 10 \quad s. \\
2+6=\tfrac{1}{2}\text{ of } 12\times10 \\
\hline
d.\ 4|50 \\
4 \\
\hline
qrs.\ 2|0
\end{array}
$$

4. If 120lb of ſteel coſt 7*l.* How muſt I ſell it per lb to gain $15\frac{1}{2}$*l.* per cent.?

lb £. lb s. d. £. s. d. £. s. d.
As 120 : 7 :: 1 : 1 2 As 100 : 1 2 :: $115\frac{1}{2}$: 1 4 per lb *Anſ.*

5. A gentleman bought 10 tons of iron for 200*l.* the freight and duties came to 25*l.* and his own charges to 8*l.* 6*s.* 8*d.*; How muſt he ſell it per lb to gain 20*l.* per cent. by it?

£. £ £. s. d. £. s. d. £. s. d. £. s. d. £.
As 100 : 20 :: 233 6 8 : 46 13 4 Then, 233 6 8+46 13 4=280

Tons. £. lb d.
As 10 : 280 :: 1 : 3 per lb *Anſ.*

6. If a bag of cotton, weighing 8*Cwt.* 0*qr.* 20lb coſt 45*D.* 55*c.* How muſt it be ſold per Cwt. to loſe 8*D.* per cent?

*Cwt.qrs.*lb *D. c. Cwt. D. c. m. D. c. m. D. c. m.*
As 8 0 20 : 45,55 :: 1 : 5,56,9 As 100 : 92 :: 5,56,9 : 5,12,3 *Anſ.*

7. Bought fiſh in Newburyport, at 10*s.* per quintal, and ſold it at Philadelphia, at 17*s.* 6*d.* per quintal; now, allowing the charges at an average, or one with another, to be 2*s.* 6*d.* per quintal, and conſidering I muſt loſe 20*l.* per cent. by remitting my money home; What do I gain per cent.?

	s.	d.	
Selling price	17	6	Philadelphia currency, per quintal.
Charges	2	6	ditto.
	15	0	ditto.

£. s. £. s.
As 100 : 15 :: 80 : 12 Newengland currency.

Sold at 12*s.* per quintal.
Bought at 10*s.* per quintal.

Gained 2*s.* per quintal.

s. s. £. £.
As 10 : 2 :: 100 : 20 per cent. gained, *Anſ.*

8. Bought

8. Bought 50 gallons of brandy, at 4*s*. per gallon, but, by accident, 10 gallons leaked out; At what rate must I sell the remainder per gallon, to gain upon the whole prime cost, at the rate of 10*l*. per cent.?

50 galls. at 4*s*. per gall. = £10
10 gallons leaked out.
—
40 gallons remain.

	Galls.	£.		*Gall.*	*s.*	
As	40 :	10 ::		1 :	5	
	£.	*s.*		£.	*s. d.*	
As	100 :	5 ::		110 :	5 6	*Ans.*

CASE III.

When there is gained or lost per cent. to know what the commodity cost.

Rule.—As 100*l*. with the gain per cent. added, or loss per cent. subtracted, is to the price: So is 100*l*. to the prime cost.

Examples.

1. If 1 yard of cloth be sold, at 5*s*. 8*d*. and there is gained 13*l*. 6*s*. 8*d*. per cent. What did the yard cost?

	£. *s. d.*	*s. d.*	£.
As	113 6 8 :	5 8 ::	100
	20	12	68
	2266	68	800
	12		600
	27200		272,00)68|00(0
			20

272,00)1360|00(5*s*. *Ans.* 5*s*. prime cost.
1360

2. If 12 yards of cloth are sold at 15*s*. per yard, and there is 7*l*. 10*s*. loss per cent. in the sale; What is the prime cost of the whole?

	Yd.	*s.*	*Yds.*	£.		£. *s.*	£.	£.	£. *s. d.*	
As	1 :	15 ::	12 :	9	As	92 10 :	9 ::	100 :	9 14 7	*Ans.*

3. If 40lb of chocolate be sold at 1*s*. 6*d*. per lb, and I gain 9*l*. per cent. What did the whole cost me?

	lb	*s. d.*	lb	£.		£.	£.	£.	£. *s. d.*	
As	1 :	1 6 ::	40 :	3	As	109 :	3 ::	100 :	2 15 0½	*Ans.*

4. If 19½ *Cwt.* sugar be sold at 14*D*. 50*c*. per Cwt. and I gain 15*D*. per cent. What did it cost per Cwt.?

	D.	*D.*	*D. c.*	*D. c. m.*	
As	114 :	100 ::	14,50 :	12,60,8	*Ans.*

CASE IV.

If by wares sold at such a rate, there is so much gained or lost per cent. to know what would be gained or lost per cent. if sold at another rate.

Rule.—As the first price is to 100*l*. with the profit per cent. added, or loss per cent. subtracted; so is the other price, to the gain or loss per cent, at the other rate.

N. B.

N. B. If your anſwer exceed 100*l.* the exceſs is your gain per cent. but if it be leſs than 100*l.* the deficiency is your loſs per cent.?

EXAMPLES.

1. If cloth, ſold at 5*s.* 8*d.* per yard, be 13*l.* 6*s.* 8*d.* profit per cent. What gain or loſs per cent. ſhall I have, if I ſell the ſame at 5*s.* per yard?

```
   s. d.   £.  s. d.    s.
As 5  8 : 113  6  8 :: 5
     12       20
     --     ----
     68     2266
              12
           -----
           27200
               5
           ----- 2,0
    68)136000(200,0  100—100=0  Anſ. I neither gain nor loſe.
       136       ----
       ---   £100
          ,000
```

2. If cloth, ſold at 4*s.* per yard, be 10*l.* per cent. profit; What ſhall I gain or loſe per cent. if ſold at 3*s.* 6*d.* per yard?

```
   s.   £.     s. d.
As 4 : 110 :: 3  6
   12          12                 £.    £.   £.
   --          --          Then, 100—96¼=3¾
   48          42
              110
              ----
        48)4620(96¼ Anſ. I loſt £3¾ per cent. by the laſt ſale.
           432
           ---
            300
            288
            ---
             12
```

3. If I ſell a gallon of wine for 1*D.* 50*c.* and thereby loſe 12 per cent. What ſhall I gain or loſe per cent. if I ſell 4 gallons of the ſame wine for 6*D.* 75*c.*?

```
     D.   D.    D. c.   D.
As 6 : 88 :: 6,75 : 99   And 100—99=1 per cent. loſs.
```

4. I ſold a watch for 50*l.* and by ſo doing, loſt 17*l.* per cent. whereas I ought in trading to have cleared 20*l.* per cent. How much was it ſold under its real value?

```
   £.   £.    £.   £. s. d.        £.   £. s. d.      £.   £. s. d.
As 83 : 50 :: 100 : 60 4 9¾   As 100 : 60 4 9¾ :: 120 : 72 5 9¼
        £. s. d.  £.   £. s. d.
Then, 72 5 9¼—50=22 5 9¼ Anſ.
```

EQUATION

EQUATION OF PAYMENTS

Is the finding of a time to pay, at once, ſeveral debts due at different times, ſo that no loſs ſhall be ſuſtained by either party.

RULE I.*

Multiply each payment by the time at which it is due; then divide the ſum of the products by the ſum of the payments, and the quotient will be the equated time, or that required.

EXAMPLES.

1. A owes B 380*l.* to be paid as follows, viz. 100*l.* in 6 months, 120*l.* in 7 months, and 160*l.* in 10 months; What is the equated time for the payment of the whole debt?

$$100 \times 6 = 600$$
$$120 \times 7 = 840$$
$$160 \times 10 = 1600$$

$$100+120+160=380)3040(8 \text{ months, } Anſ.$$
$$3040$$

2. A owes B 104*l.* 15*s.* to be paid in $4\frac{1}{2}$ months, 161*l.* to be paid in $3\frac{1}{2}$ months, and 152*l.* 5*s.* to be paid in 5 months; What is the equated time for the payment of the whole?

Anſ. 4 months and 8 days.

3. There is owing to a merchant 698*l.* to be paid 178*l.* ready money, 200*l.* at 3 months, and 320*l.* in 8 months; I demand the indifferent time for the payment of the whole?

Anſ. $4\frac{1}{2}$ months.

4. The ſum of 164*D.* 16*c.* 6*m.* is to be paid $\frac{1}{2}$ in 6 months, $\frac{1}{3}$ in 8 months, and $\frac{1}{6}$ in 12 months; What is the mean time for the payment of the whole? *Anſ.* $7\frac{2}{3}$ months.

RULE II.

See, by rule 1ſt, at what time the firſt man, mentioned, ought to pay in his whole money; then, as his money is to his time, ſo is the other's money, to his time, inverſely, which, when found, muſt be added to, or ſubtracted from, the time at which the ſecond ought to have paid in his money, as the caſe may require, and the ſum, or remainder, will be the true time of the ſecond's payment.

EXAMPLES.

* This rule is founded upon a ſuppoſition, that the ſum of the intereſts of the ſeveral debts, which are payable before the equated time, from their terms to that time, ought to be equal to the ſum of the intereſts of the debts payable after the equated time, from that time to their terms. Some, who defend this principle, have endeavoured to prove it to be right by this argument; that what is gained by keeping ſome of the debts after they are due, is loſt by paying others before they are due; but this cannot be the caſe; for though, by keeping a debt after it is due, there is gained the intereſt of it for that time; yet by paying a debt before it is due, the payer does not loſe the intereſt for that time, but the diſcount only, which is leſs than the intereſt, and therefore the rule is not accurately true; however, in moſt queſtions, which occur in buſineſs, the error is ſo trifling, that it will always be made uſe of as the moſt eligible method.

The true rule will be given in Equation of Payments by Decimals.

EXAMPLES.

1. A is indebted to B 150*l.* to be paid, 50*l.* at 4 months, and 100*l.* at 8 months: B owes A 250*l.* to be paid at 10 months: It is agreed between them, that A ſhall make preſent pay of his whole debt, and that B ſhall pay his ſo much the ſooner, as to balance that favour; I demand the time at which B muſt pay the 250*l.* reckoning ſimple intereſt.

$$50\times 4=200$$
$$100\times 8=800$$

$$50+100=15|0)100|0(6\tfrac{2}{3}\text{ months, A's equated time.}$$
$$\begin{array}{r}90\\ \hline 10\end{array}$$

As 150 £. : $6\frac{2}{3}$ mo. :: 250 £. : 4 mo. Then, 10 mo. − 4 mo. = 6 mo. time of B's payment.

2. A merchant has 120*l.* due to him, to be paid at 7 months; but the debtor agrees to pay $\frac{1}{2}$ ready money, and $\frac{1}{3}$ at 4 months; I demand the time he muſt have to pay in the reſt, at ſimple intereſt, ſo that neither party may have the advantage of the other?

Debt £120

$\frac{1}{2}=$ 60 muſt be paid down.
$\frac{1}{3}=$ 40 muſt be paid at 4 months.
$\frac{1}{6}=$ 20 unpaid.

Now, as he pays 60*l.* 7 months, and 40*l.* 3 months before they are reſpectively due: Say, As the intereſt of 20*l.* for 1 month, is to 1 month, ſo is the ſum of the intereſt of 60*l.* for 7 months, and of 40*l.* for 4 months, to a 4th number, which, added to the 7 months, will give the time for which the 20*l.* ought to be retained.

Anſ. 2 years and 10 months.

3. A merchant has 1200*l.* due to him, to be paid $\frac{1}{6}$ at 2 months, $\frac{1}{3}$ at 3 months, and the reſt at 6 months; but the debtor agrees to pay $\frac{1}{2}$ down; How long may the debtor detain the other half, ſo that neither party may ſuſtain loſs?

	mo.		mo.
$\frac{1}{6}\times$	2	=	$0\frac{1}{3}$
$\frac{1}{3}\times$	3	=	1
$\frac{1}{2}\times$	6	=	3
Equated time		=	$4\frac{1}{3}$

Now, as $\frac{1}{2}$ was paid $4\frac{1}{3}$ months before it was due, it is reaſonable that he ſhould detain the other $\frac{1}{2}$ $4\frac{1}{3}$ months after it became due, which, added, gives $8\frac{2}{3}$ months, the true time for the ſecond payment.

EQUATION

EQUATION OF PAYMENTS BY DECIMALS.

RULE.*

1. To the ſum of both payments add the continual product of the firſt payment, the ratio, and the time between the payments, and call this the firſt number.

2. Multiply twice the firſt payment by the ratio, and call this the ſecond number.

3. Divide the firſt number by the ſecond, and call the quotient the third number.

4. Call the ſquare of the third number the fourth number.

5. Divide the product of the ſecond payment and time between the payments by the product of the firſt payment and the ratio, and call the quotient the fifth number.

6. From the fourth number take the fifth, and call the ſquare root of the difference the ſixth number.

7. Then the difference of the third and ſixth numbers is the equated time, after the firſt payment.

EXAMPLE.

There are 100*l.* payable in 2 years, and 106*l.* at 6 years hence; What is the equated time, allowing ſimple intereſt, at 6 per cent. per annum?

1ſt payment=100	1ſt payment 100
Ratio=,06	Multiply by 2
6,00	200
Time between the payments=4 years.	Multiply by the ratio=,06
24	12,00=2d. number.
Add both payments= { 100, 106	
Div. by the 2d. numb.=12)230=1ſt number.	
19,166+=3d number.	
19,166+	
3d number ſquared=367,345556=4th number.	

2d payment

* Suppoſe a ſum of money be due immediately, and another ſum at the expiration of a certain given time forward, and it is propoſed to find a time, ſo that neither party ſhall ſuſtain loſs.

Now, it is plain that the equated time muſt fall between the two payments; and that what is gotten by keeping the firſt debt after it is due, ſhould be equal to what is loſt by paying the ſecond debt before it is due; but the gain ariſing from the keeping of a ſum of money after it is due, is evidently equal to the *intereſt* of the debt for that time: And the loſs, which is ſuſtained by the paying of a ſum of money before it is due, is evidently equal to the *diſcount* of the debt for that time: Therefore it is obvious that the debtor muſt retain the ſum immediately due, or the firſt payment, till its *intereſt* ſhall be equal to the *diſcount* of the ſecond ſum for the time it is paid before due; becauſe in that caſe the gain and loſs will be equal, and conſequently neither party can be a loſer.

2d payment $=106$
Multiplied by the time $= 4$

1st payment mult. by the ratio $=6)424=$ { product of the 2d product and time between the payments.

$70,666+=$ 5th number.

From the 4th number $=367,345556$
Take the 5th number $= 70,666666$

296,678890(17,224 square root $=$ 6th number.

From the 3d number $=19,166$
Take the 6th number $17,224$

1,942 $=$ equated time from the first payment; therefore 3,942 years $=3y.\ 11m.\ 14d.=$ whole equated time.

$$Or, \frac{100+106+\overline{100\times,06\times4}}{100\times2\times,06}-\left|\overline{\frac{100+106+\overline{100\times,06\times4}}{100\times2\times,06}}\right|^{2}-\frac{106\times4}{100\times,06}\Bigg|^{\frac{1}{2}}=1,942.$$

COMMISSION OR FACTORAGE*

Is an allowance of so much per cent. to a factor, or correspondent, abroad, for buying and selling goods, for his employer.

EXAMPLES.

1. What comes the commission on 539*l.* 12*s.* 9*d.* to, at $4\frac{1}{2}$ per cent. ?

	£.	s.	d.
	539	12	9
			$4\frac{1}{2}$
	2158	11	0
Product of $\frac{1}{2}=$	269	16	$4\frac{1}{2}$
	£24|28	7	$4\frac{1}{2}$
	20		
	s. 5|67		
	12		
	d. 8|08		

2. My factor receives 1008*l.* to lay out, after having deducted his commission of 5*l.* per cent. What does his commission amount to?

Here,

* The method of working questions in this rule, Brokage, and the first case of Insurance, is the same as in Simple Interest.

Here, as his commission is to be deducted from the given sum, it is evident that I ought not to pay him commission on his own money, (which, however, is often unjustly practised) therefore,

$$\begin{array}{ccccccc} £. & & £. & & £. & & £. \\ \text{As } 105 & : & 5 & :: & 1008 & : & 48 \text{ } \textit{Ans.} \end{array}$$

3. My correspondent writes me, that he has purchased goods to the value of 673*l.* 12*s.*; What does his commission on that sum amount to, at $3\frac{1}{2}$*l.* per cent.? *Ans.* £23 11*s.* 6*d.*

4. What must I allow my correspondent, at $2\frac{1}{4}$*l.* per cent. for disbursing on my account 395*l.* 15*s.* 5*d.*? *Ans.* £8 18*s.* 1*d.*

BROKERAGE

Is an allowance of so much per cent. to a person called a broker, for assisting merchants or factors in purchasing or selling goods.

EXAMPLES.

1. What is the brokerage upon 525*l.* 10*s.* at 5*s.* or $\frac{1}{4}$*l.* per cent.?

$$\begin{array}{l|l|l} & & £.\ \ s. \\ 5s. & \frac{1}{4} & 525\ \ 10 \\ \hline & & 1|31\ \ 7\ \ 6 \\ & & \ \ \ 20 \\ \hline & & 6|27 \\ & & \ \ \ 12 \\ \hline & & 3|30 \\ & & \ \ \ \ 4 \\ \hline & & 1|20 \end{array}$$

5 shillings being an aliquot part of 1*l.*; divide by that part, and the quotient is the answer.

Ans. £1 6*s.* $3\frac{1}{4}$*d.*

2. What is the brokerage upon 673*l.* 16*s.* at $\frac{5}{8}$*l.* per cent.? *Ans.* £4 4*s.* $2\frac{1}{4}$*d.*

3. If a broker sell goods to the amount of 2864*D.* 46*c.* What is his demand at $1\frac{1}{2}$ per cent.? *Ans.* 42*D.* 96*c.* $6\frac{9}{10}$*m.*

BUYING AND SELLING STOCKS.

Stock is a general name for the capitals of trading companies, banks, &c. and the buying and selling certain sums of money in those funds, is not unusual.

EXAMPLES.

1. What is the purchase of 275*l.* 15*s.* bank stock, at $71\frac{1}{2}$*l.* per cent.?

$74\frac{1}{2}l.$ want $25\frac{1}{2}l.$ of $100l.$ therefore, take parts for the deficiency, and ſubtract the ſum of thoſe parts from the given ſum.

£.		£. s. d.
20	$\frac{1}{5}$	275 15 0
5	$\frac{1}{4}$	55 3 0
$\frac{1}{2}$	$\frac{1}{10}$	13 15 9
		1 7 $6\frac{3}{4}$
	Subtract	70 6 $3\frac{3}{4}$
		£205 8 $8\frac{1}{4}$ *Anſ.*

Or, If I had taken parts for $74\frac{1}{2}l.$ the rate per cent. then the ſum of the ſeveral quotients would have been the anſwer as above.

When the price is above 100, take parts for the ſurplus of the price above 100, and add them to the given ſum for the anſwer.

2. What is the purchaſe of $1029l.$ $10s.$ $6d.$ bank ſtock, at $110\frac{1}{4}l.$ per cent.?

£.		£. s. d.		
10	$\frac{1}{10}$	1029 10 6	at	100 per cent.
$\frac{1}{4}$	$\frac{1}{40}$	102 19 $0\frac{1}{2}$	at	10 per cent.
		2 11 $5\frac{1}{2}$	at	$\frac{1}{4}$ per cent.
	Anſ.	£1135 1 0	at	£$110\frac{1}{4}$ per cent.

3. What is the purchaſe of $1058l.$ $12s.$ bank ſtock, at $115\frac{3}{4}l.$ per cent.? *Anſ.* £1225 $6s.$ $6\frac{1}{2}d.$

4. What does $1600l.$ capital ſtock, in the Maſſachuſetts bank, come to, at $120\frac{1}{8}l.$ per cent.? *Anſ.* £1922.

POLICIES OF INSURANCE.

Inſurance is a ſecurity, or aſſurance, by mean of a writ called a *Policy*, to indemnify the inſured of ſuch loſſes as ſhall be ſpecified in the policy ſubſcribed by the inſurer, or inſurers, by which the underwriters oblige themſelves to make good and effectual the property inſured, in conſideration of a certain premium at a ſtipulated rate per cent. (which varies according to the riſque) to be immediately paid down, or otherwiſe ſecured according to the tenor of the agreement.

The average loſs is 10 per cent.; that is, if the inſured ſuffer any damage or loſs, not exceeding 10 per cent. he bears it himſelf, and the inſurers are free.

A policy ſhould be taken out for a ſum ſufficient to cover the principal and premium, and the buſineſs of this rule is, in general, to calculate what that ſum ſhould be.

CASE I.

When the premium, at a certain rate per cent. for inſuring a ſum, is required, the operation is the ſame as in intereſt, or commiſſion.

1. What

1. What is the premium upon 537*l.* 15*s.* 9*d.* at $6\frac{1}{2}$ per cent.?

$$\begin{array}{rrrr}
 & £. & s. & d. \\
 & 537 & 15 & 9 \\
 & & & 6\frac{1}{2} \\
\hline
 & 3226 & 14 & 6 \\
\frac{1}{2}= & 268 & 17 & 10\frac{1}{2} \\
\hline
 & 34|95 & 12 & 4\frac{1}{2} \\
 & 20 & & \\
\hline
 & 19|12 & & \\
 & 12 & & \\
\hline
 & 1|48 & & \\
 & 4 & & \\
\hline
 & 1|94 & &
\end{array}$$

Anſ. £34 19*s.* $1\frac{1}{4}$*d.*

CASE II.

To find the ſum for which a policy ſhould be taken out to cover a given ſum.

RULE.—Take the premium from 100*l.* and ſay, As the remainder is to 100; ſo is the ſum adventured, to the policy.*

1. It is required to cover 759*l.* premium 8 per cent.; For what ſum muſt the policy be taken?

$$\begin{array}{l}
100 \\
\underline{\ \ 8} \\
92 : 100 :: 759 \\
\qquad\qquad\quad \underline{100} \\
\qquad\qquad\qquad\qquad £ \\
92)75900(825\ \textit{Anſ.} \\
\qquad \underline{736} \\
\qquad\ \ 230 \\
\qquad\ \ \underline{184} \\
\qquad\quad\ 460 \\
\qquad\quad\ 460
\end{array}$$

2. A

* Now it is plain, that if I want to recover 92*l.* I muſt, in this caſe, inſure upon 100*l.*; therefore, to recover 759*l.* I muſt inſure upon 825*l.*; for when 8 per cent. for premium is deducted, I ſhall have 759*l.* remaining nett.

For £825 = ſum inſured upon at 8 per cent.
66 = premium to be deducted.
759 = the firſt outſet.

In this and the following caſes, let $x=100$, $p=$ premium, $a=$ amount to be inſured upon, and $s=$ ſum to be covered; then $x-p : x :: s : a$, or $\frac{xs}{x-p}=a$.

2. A merchant ſent a veſſel and cargo to ſea, valued at 1525*l.*; What ſum muſt the policy be taken out for, to cover his property, premium $19\frac{1}{2}$ per cent.?

$$\begin{array}{r} 100 \\ 19{,}5 \\ \hline 80{,}5 \end{array} : 100 :: 1525 : £1894\ 8s.\ 2\tfrac{1}{4}d.\ \textit{Anſ.}$$

CASE III.

When a policy is taken out for a certain ſum in order to cover a given ſum:

To find the premium, ſay, As the policy is to the covered ſum; ſo is 100*l.* to a fourth number, which, being taken from 100, will leave the premium.*

If a policy be taken out for 1250*l.* to cover 500*l.* What is the premium per cent.?

$$\begin{array}{l} 1250 : 500 :: 100 \\ \qquad\quad\ 100 \\ \hline 1250)50000(40 \text{ and } £100-40=£60\ \textit{Anſ.} \end{array}$$

CASE IV.

When the policy for covering any ſum and the premium per cent. are given, to find the ſum to be covered.

RULE.—Deduct the premium per cent. from 100, and ſay, As 100 is to the remainder; ſo is the policy to the ſum required to be covered.

If a policy be taken out for 1250*l.* at 60 per cent.; What is the adventure, or ſum to be covered?†

$$\begin{array}{r} 100 \\ 60 \\ \hline 100 : 40 :: 1250 \\ 40 \\ \hline £ \\ 100)50000(500\ \textit{Anſ.} \end{array}$$

CASE V.

When a given ſum is adventured ſeveral voyages round from one place to another, either at the ſame, or different riſques, from place to place, and it is required to take out a policy for ſuch a ſum as will cover the adventure all round, ſuppoſing the riſque out and home to be equal and tantamount to the ſeveral given riſques.

RULE.

* $a : s :: x : \overline{x-p}$, or $x - \frac{sx}{a} = p$.

† $x : x-p :: a : s$, or, $\frac{a \times \overline{x-p}}{x} = s$.

RULE.

1. Raife 100*l.* to that power denoted by the number of rifques, and multiply the faid power by the fum adventured, (or to be covered) for a dividend.

2. Subtract the feveral premiums, each, from 100*l.* and multiply the feveral remainders continually together for a divifor, and the quotient, arifing from this divifion, will give the policy to cover the adventure the voyage round.*

1. A merchant adventured 480*l.* 10*s.* from Newburyport to Southcarolina, from thence to Jamaica, and from thence, home, and the premium was 5 per cent. from port to port; What fum muft he take out a policy for, to cover his adventure the voyage round, fuppofing the rifque to be equal out and home, and tantamount to the feveral given rifques?

$100\times100\times100\times480{,}5=480500000=$ Dividend.

100	100	100
—5	—5	—5
95 ×	95 ×	95 = 857375 = Divifor.

857375)480500000(560*l.* 8*s.* $7\frac{1}{2}$*d.* *Anf.*

2. A merchant adventured 500*l.* from Bofton to Philadelphia, at 3 per cent. from thence to Guadaloupe, at 4, from thence to Nantz, at 5, and from thence home, at 6 per cent.; For what fum muft he take out a policy to cover his adventure the voyage round, fuppofing the rifque to be equal out and home, and tantamount to the feveral given rifques?

$$\frac{100\times100\times100\times100\times500}{\overline{100-3}\times\overline{100-4}\times\overline{100-5}\times\overline{100-6}}=£601{,}278,\ Anf.$$

CASE

* For the firft Voyage.

$\overline{x-p} : x :: s : a.$

$\frac{xs}{x-p}=a.$

$\frac{a\times\overline{x-p}}{x}=s.$

$x-\frac{sx}{a}=p.$

Second Voyage.

$\overline{x-p} : x :: \frac{xs}{x-p} : a.$

$\frac{xxs}{\overline{x-p}^2}=a.$

$\frac{a\times\overline{x-p}^2}{x^2}=s.$

$x-\sqrt{\frac{xxs}{a}}=p.$

Third Voyage.

$\overline{x-p} : x :: \frac{xxs}{\overline{x-p}^2} : a.$

$\frac{xxxs}{\overline{x-p}^3}=a.$

$\frac{a\times\overline{x-p}^3}{x^3}=s.$

$x-\frac{\sqrt[3]{x^3s}}{a}=p.$ and fo on for as many voyages as may be required. Hence, making $m=$exponent of any given power, $\frac{x^m s}{\overline{x-p}\times\overline{x-p}\times\overline{x-p}, \&c.}=$ fum to be infured upon, all round:——And $x-\sqrt[m]{\frac{x^m\times s}{a}}=$ the premium all round, tantamount to the feveral given premiums; s, in this Theorem, being equal to the firft adventure, and $a=$ amount which will cover that adventure the voyage round.

CASE VI.

When a given ſum is adventured ſeveral voyages round, as in the laſt caſe, either at the ſame, or different riſques, from port to port, and the premium for the voyage round is required, tantamount to the ſeveral given rates per cent.

RULE.

1. Find the ſum for which the policy muſt be taken, by the laſt caſe.

2. Multiply the ſum adventured by 100, and divide that product by the policy.

3. Take the quotient from 100*l.* and the remainder will be the premium per cent. on the policy, tantamount to the ſeveral premiums given in the queſtion.

1. A merchant adventured 480*l.* 10*s.* from Newburyport to Southcarolina, from thence to Jamaica, and from thence, home, and the premium was 5 per cent. from port to port: What will be the premium tantamount to the ſeveral given premiums, (allowing the riſques out and home to be equal) on the policy which will cover the firſt adventure of 480*l.* 10*s.*

In queſtion 1, Caſe 5, we found the policy to be £560 8*s.* $7\frac{1}{2}$*d.*=£560,43125

$480,5 \times 100 = 48050$ and $560,43125)48050,00000(85,7375$ and $100 - 85,7375 = £14,2625$ the premium required: Or thus,

$$100 - \frac{480,5 \times 100}{560,43125} = 14,2625.$$

2. A merchant adventured 500*l.* from Boſton to Philadelphia, at 3 per cent. from thence to Guadaloupe, at 4; from thence to Nantz, at 5; and from thence, home, at 6 per cent.; What will be the premium, tantamount to thoſe given in the queſtion, on a policy for covering the firſt adventure, the voyage, ſuppoſing the riſques out and home equal?

In queſtion 2d, Caſe 5, we found the policy, which would cover the adventure the voyage round, to be £601,278.

Then, $100 - \frac{500 \times 100}{601,278} = £16,844 =$ the premium per cent. on the policy the voyage round, and tantamount to the ſeveral given premiums.

CASE VII.

If a policy be taken out for a given ſum, to cover a certain adventure from one port to another, on to ſeveral ports, at equal premiums from one place to the other, to find what that equal premium is.

RULE.

1. Involve 100 to that power denoted by the number of riſques, and multiply this power by the ſum adventured, (or covered.)

2. Divide the laſt product by the policy.

3. Extract that root of the quotient denoted by the number of riſques.

4. Take

4. Take this root from 100*l.* and the remainder will be the equal premium from one port to the other.

1. A merchant adventured 480*l.* 10*s.* from Newburyport to Southcarolina, from thence to Jamaica, and from thence home; to cover which all round he took out a policy for 560*l.* 8*s.* $7\frac{1}{2}$*d.* and the premium was equal from one place to the other; What was the premium per cent.?

$$100-\sqrt[3]{\frac{100\times 100\times 100\times 480,5}{560,43125}}=5 \text{ per cent.}$$

CASE VIII.

When an adventure is insured out and home at one risque, at a given rate per cent. and the voyage terminates short of what was at first intended: To find what the underwriter must receive per cent.

Rule 1.—If just half the voyage is performed, it must be considered as *two* equal risques: If one third, then, as *three* equal risques; if but one fourth, then, as *four* risques, and so on; and by Case 2d must be found the amount which will cover the adventure the voyage round.

2. Involve 100 to that power denoted by the number of risques, and multiply this power by the sum adventured.

3. Divide this product by the aforesaid amount.

4. Extract that root of the quotient denoted by the number of risques.

5. Take this root from 100*l.* and the remainder will be the sum per cent. which the underwriter must receive.

1. A merchant covers 200*l.* at 6 per cent. from Newburyport to the Westindies and home again; but the voyage terminating in the Westindies, what must the insurer receive per cent.?

100
6
―――
94 : 100 :: 200 : 212,765957=amount to cover £200 voyage round.

$100\times 100\times 200=2000000$ and $\frac{2000000}{212,765957}=9400$

and $100-\sqrt{9400}=3,0465$ to be paid the insurer per cent. upon the above amount.

2. A merchant insures 350*l.* to the Westindies, from thence, to France, and from thence home, at 10 per cent. the voyage round; but there is but one third part of the voyage performed; What must the insurer receive per cent.?

There being but $\frac{1}{3}$ of the voyage performed, we must suppose three equal risques.

100
10
―――
90 : 100 :: 350 : 388,8889, & $100\times 100\times 100\times 350=350000000$

$\frac{350000000}{388,8889}=899999,9999$ & $100-\sqrt[3]{899999,9999}=£3,4515$ *Ans.*

COMPOUND

COMPOUND INTEREST

Is that which arises from the interest being added to the principal, and (continuing in the hands of the borrower) becomes a part of the principal, at the end of each stated time of payment.

METHOD I.

RULE.*

Find the *amount* of the given principal, for the time of the first payment, by Simple Interest; next, find the interest of that sum, or principal, and add it as before, and thus proceed for any number of years, still accounting the last *amount* as the principal for the next payment. The given principal being subtracted from the last *amount*, the remainder will be the compound interest.

EXAMPLES.

1. What will 480*l.* amount to in 5 years, at 6 per cent. per annum?

	£.
Principal	480
Rate of interest	6
	28|80
	20
	16|00

	£.	s.	d.
Principal for the 1st year	480	0	
Interest of ditto	28	16	
Principal for the 2d year	508	16	
		6	
	30|52	16	
	20		
	10|56		
	12		
	6|72		
	4		
	2|88		

	£.	s.	d.
Prin. for the 2d year	508	16	0
Interest for ditto	30	10	6½
Prin. for the 3d year	539	6	6½
			6
	32|35	19	3
	20		
	7|19		
	12		
	2|31		
	4		
	1|24		

	£.	s.	d.
Principal for the 3d year	£539	6	6½
Interest for ditto	32	7	2¼
Principal for the 4th year	571	13	8¾

Principal

* As all the computations relating to Simple Interest, are founded upon Arithmetical Progression, the simple interest of one pound being a series of terms in Arithmetical Progression increasing; whose first term and common difference is the interest of one pound for one year, and the number of years shewing the number of all the terms; therefore, the last term will always be equal to the product of the time and rate, equal to the interest of one pound for any given time: So those relating to Compound Interest are founded upon a series of terms increasing in Geometrical Progression, wherein the number of years assigns the the index of the last and highest term: Therefore, as one pound is to the amount of one pound, for any given time; so is any proposed principal, or sum, to its amount for the same time.

	£.	s.	d.
Prin. for the 4th year	571	13	$8\frac{1}{4}$
			6
	34|30	2	$4\frac{1}{2}$
	20		
	6|02		
	12		
	0|28		
	4		
	1|14		

	£.	s.	d.
Prin. for the 4th yr.	571	13	$8\frac{1}{4}$
Intereſt for ditto.	34	6	$0\frac{1}{4}$
Prin. for the 5th year	605	19	9
			6
	36|35	18	6
	20		
	7|18		
	12		
	2|22		

	£.	s.	d.
Prin. for the 5th year	605	19	9
Intereſt for ditto	36	7	2
Amount for 5 years	642	6	11
Subtract the firſt prin.	480	0	0
Comp. int. for 5 years	162	6	11

2. What is the compound intereſt of 740*l.* for 6 years, at 4*l.* per cent. per annum? *Anſ.* £195 6*s.* $8\frac{1}{2}$*d.*

3. What will 400*l.* amount to in 5 years, at 4*l.* per cent. per annum? *Anſ.* £486 13*s.* $2\frac{1}{2}$*d.*

4. What will 150*l.* amount to in a year, at 2*l.* per cent. per month? *Anſ.* £190 4*s.* 5*d.*

METHOD II.

When the rate is at 5 per cent. per annum.

1. Divide the principal by 20, and this quotient, added to the principal, will be the amount for the firſt year, and the principal for the ſecond.

2. In like manner find the amount for every ſucceeding year.

When the rate is at 6 per cent. per annum.

1. Divide the principal by 20, and that quotient by 5: Theſe quotients, added to the principal, will be the amount for the firſt year, and the principal for the ſecond.

2. In like manner obtain the amount for every ſucceeding year.

EXAMPLES.

What is the amount of 480*l.* at 6 per cent. per annum, for 5 years?

20)	480			
5)	24			
	4	16		
20)	508	16		amount of 1st year.
5)	25	8	$9\frac{1}{2}$	
	5	1	9	
20)	539	6	$6\frac{1}{2}$	ditto of 2d.
5)	26	19	$3\frac{3}{4}$	
	5	7	$10\frac{1}{4}$	
20)	571	13	$8\frac{1}{2}$	ditto of 3d.
5)	28	11	8	
	5	14	4	
20)	605	19	$8\frac{1}{2}$	ditto of 4th.
5)	30	5	$11\frac{3}{4}$	
	6	1	$2\frac{1}{4}$	
	£642	6	$10\frac{1}{2}$	do. of 5th, *Ans.*

Of the same sum at 5 per cent. per annum, for 5 years.

	£.			
20)	480			
	24			
20)	504			amount of 1st year.
	25	4		
20)	529	4		ditto of 2d.
	26	9	$2\frac{1}{4}$	
20)	555	13	$2\frac{1}{4}$	ditto of 3d.
	27	15	$7\frac{3}{4}$	
20)	583	8	10	ditto of 4th.
	29	3	$5\frac{1}{4}$	
	£612	12	$3\frac{1}{4}$	do. of 5th, *Ans.*

COMPOUND INTEREST BY DECIMALS.

A TABLE *of the Amount of* 1*l.* *at* ½ *per Cent. per Month, as practised at the Massachusetts Bank.*

Months.	£. Dec. pts.	Months.	£. Dec. pts.	Months.	£. Dec. pts.
1	1,005	5	1,025	9	1,045
2	1,01	6	1,03	10	1,05
3	1,015	7	1,035	11	1,055
4	1,02	8	1,04	12	1,06

A TABLE *of the Amount of* 1*l.* *from* 1 *Day to* 31 *Days, at* 6 *per Cent. per Annum.*

Days.	£. Dec. parts.	Days.	£. Dec. parts.	Days.	£. Dec. parts.
1	1,00016	12	1,00197	22	1,00361
2	1,00032	13	1,00213	23	1,00378
3	1,00049	14	1,00230	24	1,00394
4	1,00065	15	1,00246	25	1,00410
5	1,00082	16	1,00263	26	1,00427
6	1,00098	17	1,00279	27	1,00443
7	1,00115	18	1,00295	28	1,00460
8	1,00131	19	1,00312	29	1,00476
9	1,00147	20	1,00328	30	1,00493
10	1,00164	21	1,00345	31	1,00509
11	1,00180				

CASE I.*

When the principal, the rate of intereſt, and time, are given, to find either the amount or intereſt.

RULE 1.—Find the amount of 1*l.* for 1 year, at the given rate per cent.

2. Involve the amount, thus found, to ſuch power, as is denoted by the number of years; or, in Table I, at the end of Annuities,

* Let $r=$ amount of 1*l.* for 1 year, and $p=$ principal, or given ſum; then, ſince r is the amount of 1*l.* for 1 year, r^2 will be its amount for 2 years, r^3 for 3 years, and ſo on; therefore, it will be as $1 : r :: r : r^2 =$ amount for the ſecond year, or principal for the third: Again, As $1 : r :: r^2 : r^3 =$ amount for the third year, or principal for the fourth, &c. to any number of years.—And if the time, or number of years, be denoted by t, the amount of 1*l.* for t years, will be r^t; from hence it will appear that the amount of any other principal ſum p, for t years, is pr^t; for, as $1 : r^t :: p : pr^t$, the ſame as in the rule.

If the rate of intereſt be determined to any other time than a year, as $\frac{1}{4}$, $\frac{1}{2}$, &c. the rule is the ſame, and then t will repreſent that ſtated time.

Let $\begin{cases} r = \text{amount of 1}l.\text{ for 1 year, at the given rate per cent,} \\ p = \text{principal, or ſum put out at intereſt.} \\ i = \text{intereſt.} \\ t = \text{time.} \\ m = \text{amount for the time } t. \end{cases}$

Then the following Theorems will exhibit the ſolutions of all the caſes in Compound Intereſt.

I. $pr^t = m$. II. $prt - p = i$. III. $\frac{m}{r^t} = p$. IV. $\left.\overline{\frac{m}{p}}\right|^{t} = r$.

The moſt convenient way of giving the Theorems, eſpecially that for the time, will be by Logarithms, as follows:

I. $t \times Log.\ r + Log.\ p = Log.\ m$. II. $Log.\ \overline{m - t} \times L.\ r = L.\ p$.

III. $\frac{L.m - L.p}{L.r} = t$. IV. $\frac{L.m - L.p}{t} = L.r$.

If the compound intereſt, or amount of any ſum, be required for the parts of a year, it may be determined as follows:

I. *When the time is an aliquot part of a year.*

RULE 1.—Find the amount of 1*l.* for 1 year, as before, and that root of it, which is denoted by the aliquot part, will be the amount of 1*l.* for the time ſought.

2. Multiply the amount, thus found, by the principal, and it will be the amount of the given ſum required.

II. *When the time is not an aliquot part of a year.*

RULE 1.—Reduce the time into days, and the 365th root of the amount of 1*l.* for 1 year is the amount for 1 day.

2. Raiſe this amount to that power, whoſe index is equal to the number of days, and it will be the amount of 1*l.* for the given time.

3. Multiply this amount by the principal, and it will be the amount of the given ſum required.

To avoid extracting very high roots, the ſame may be done by logarithms, thus: Divide the logarithm of the rate, or amount of 1*l.* for 1 year, by the denominator of the given aliquot part, and the quotient will be the logarithm of the root ſought.

nuities, under the rate, and against the given number of years, you will find the power.*

3. Multiply this power by the principal, or given sum, and the product will be the amount required, from which, if you subtract the principal, the remainder will be the interest.

EXAMPLES.

1. What is the compound interest of 600*l.* for 4 years, at 6 per cent. per annum?

$$\begin{array}{rl} 1,06 = & \left\{\begin{array}{l}\text{amount of } 1l. \text{ for } 1 \text{ year, at 6 per} \\ \quad\text{cent. per annum.}\end{array}\right. \\ \text{Multiply by } \underline{1,06} & \\ 1,1236 = & \text{2d power.} \\ \text{Multiply by } \underline{1,1236} & \\ 1,26247696 = & \text{4th power.} \\ \text{Multiply by } \underline{600} = & \text{principal.} \\ 757,48617600 = & \text{amount.} \\ \text{Subtract } \underline{600} & \\ 157,486176 = & \pounds 157\ 9s.\ 8\tfrac{1}{2}d. = \text{interest required.} \end{array}$$

By TABLE I.

Tabular amount of 1*l.* for 4 years, at 6 per cent. per ann. = 1,2624769
Multiply by the principal = 600

Amount = 757,4861400

2. What is the amount of 570*l.* 10*s.* for 12 years, at $3\frac{1}{2}$ per cent. per annum?

Ans. £862 1*s.* $3\frac{1}{4}$*d.*

Another method of working Compound Interest for *years*, *months* and *days*, which is much more concise than the preceding method.

RULE.

To the logarithm of the principal, found in any Table of logarithms, add the several logarithms, answering to the number of *years*,

* The amounts of 1*l.* in this Table, are so many powers of the amount of 1*l.* for 1 year, whose indices are denoted by the number of years.

Note. When the given time consists of years and months, or years, months and days; first seek the amount of 1*l.* in the Table for years, then in the Table of months, &c. multiply these several amounts and the principal continually together, and the last product will be the amount required.

Thus, if the amount of 480*l.* in $5\frac{1}{2}$ years, at 6 per cent. per annum, were required; the amount of 1*l.* for 5 years = £1,33822, ditto for 6 months = £1,02956, Now, 1,33822 × 1,02956 × 480 = £661,2341 *Answer.*

years, months and *days*, found in the following tables, and their ſum will be the logarithm of the amount for the given time, which being found in any table of logarithms, the natural number correſponding thereto will be the anſwer.*

Logarithmic TABLES, *at* 6 *per Cent. per Annum, for Years, Months and Days.*

Years.	*dec. pts.*	*Y.*	*dec. pts.*	*Y.*	*dec. pts.*	*Y.*	*dec. pts.*	*Months*	*dec. pts.*
1	,025306	11	,278366	21	,531426	31	,784586	1	,002166
2	,050612	12	,303672	22	,556732	32	,809792	2	.004321
3	,075918	13	,328978	23	,582038	33	,835098	3	,006466
4	,101224	14	,354284	24	,607344	34	,860404	4	,0086
5	,12653	15	,37969	25	,63265	35	,88571	5	,010724
6	,151836	16	,404896	26	,657956	36	,911016	6	,012837
7	,177142	17	,430202	27	,683262	37	,936322	7	,01494
8	,202448	18	,455508	28	,708568	38	,961628	8	,017033
9	,227754	19	,480814	29	,733974	39	,986934	9	,019116
10	,25306	20	,50612	30	,75938	40	1,01224	10	,021189
								11	,023252
Days.		*D.*		*D.*		*D.*		*D.*	
1	,000071	8	,000571	14	,000999	20	,001426	26	,001852
2	,000143	9	,000642	15	,00107	21	,001497	27	,001923
3	,000215	10	,000713	16	,001142	22	,001568	28	,001994
4	,000287	11	,000785	17	,001213	23	,001639	29	,002065
5	,000358	12	,000857	18	,001284	24	,00171	30	,002136
6	,000429	13	,000928	19	,001355	25	,001781	31	,002207
7	,0005								

6. What is the amount of 132*l.* 10*s.* at 6 per cent. per annum, for 9 years, 8 months, and 15 days?

To the log. of £132,5 = 2,122216

Add { Log. for 9 years = ,227754
ditto for 8 months = ,017033
ditto for 15 days = ,00107 }

2,368073

Becauſe 8 months are paſt, deduct 4 per cent. upon the logarithm of 15 days } = ,0000428

Remains 2,3680302, the neareſt to which, in the Table of logarithms, is 2,368101, and the natural number anſwering thereto is 233,4=£233 8*s*. *Anſ.*

CASE

* Although there is a ſmall error in the logarithms for days, yet they are exact enough for common uſe.—And if after the firſt month you deduct ½ per cent. for each month paſt (that is, ½ per cent. after 1 month, 1½ per cent. after 3 months, &c.) from the logarithm of the number of days, it will give the true anſwer.

Note, That, after 1 month, ½ per cent, on the logarithm of 1 day, is ,00000355, on 2 days, is ,00000715: After 2 months, 1 per cent. on the logarithm of 1 day, is, ,00000071, on 2 days, ,00000143: After 10 months, 5 per cent. on the logarithm for 1 day, is, ,00000355, on 6 days, is, ,00002145, &c.

CASE II.

When the amount, rate and time, are given, to find the principal.

RULE.

Divide the amount by the amount of 1*l.* for the given time, and the quotient will be the principal.

Or, If you multiply the prefent value of 1*l.* for the given number of years, at the given rate per cent. by the amount, the product will be the principal, or prefent worth.*

EXAMPLES.

1. What is the prefent worth of 757*l.* 9*s.* 8½*d.* due 4 years hence, difcounting at the rate of 6*l.* per cent. per annum?

By TABLE I.

Divide by the tabular amount of 1*l.* for 4 years, } = 1,2624769)757,4861400(£600 *Anf.*

By TABLE II.

Mult. by the prefent worth of 1*l.* for 4 years, at 6 per cent. per ann. } Amount = 757,48614
= ,79 20936

Anf. 599,999923582704+ = £600.

2. What principal muft be put to intereft 6 years, at 5½ per cent. per annum, to amount to 965*l.* 3*s.* 9½*d*,3616? *Anf.* £700.

CASE III.

When the principal, rate and amount, are given, to find the time.

RULE.

Divide the amount by the principal; then divide this quotient by the amount of 1*l.* for 1 year, this quotient by the fame till nothing remain, and the number of the divifions will fhew the time.

Or, Divide the amount by the principal, and the quotient will be the amount of 1*l.* for the given time, which feek under the given rate in Table 1, and, in a line with it, you will fee the time.

EXAMPLES.

1. In what time will 600*l.* amount to 757*l.* 9*s.* 8½*d.* at 6 per cent. per annum, compound intereft?

Divide the amount by the principal } = 600)757,486176(1,26247696 = quotient to be divided by 1,06 till it can be had no more; or you may find it in Table 1, under 6 per cent. and againft 4 years.

Divide

* See Table II. fhewing the prefent value of 1*l.* difcounting at the rates of 4, 4½, &c. per cent. the conftruction of which is thus:

Amount. Pref. worth. Amount. Pref. worth.

As 1,06 : 1 :: 1 : ,9433962, and fo on, for any other rate per cent. and time.

Divide by 1,06)1,26247696

1,06)1,191016

Four divisions shew the time to be 4 years. 1,06)1,1236

1,06)1,06

1

CASE IV.

When the principal, amount and time, are given, to find the rate per cent.

RULE.—Divide the amount by the principal, and the quotient will be the amount of 1*l.* for the given time, then, extract such root as the time denotes, and that root will be the amount of 1*l.* for 1 year, from which subtract unity, and the remainder will be the ratio.

Or, Having found the amount of 1*l.* for the time, as above directed, look for it in Table 1, even with the given time, and directly over the amount you will find the ratio.

EXAMPLE.

At what rate per cent. per annum will 600*l.* amount to 757*l.* 9*s.* 8½*d.* in 4 years?

600)757,48614(1,2624769. Now, the time being 4 years, the 4th root of this quotient minus 1 will be the ratio.

1,26247690(1,123599+ and 1,123599(1,05999+ and 1,05999 — 1 = ,06 *Answer.*

DISCOUNT BY COMPOUND INTEREST.

CASE I.*

The sum, or debt to be discounted, the time and rate, given, to find the present worth.

RULE.

* Let m = sum or debt to be discounted, and the other letters as before: Then the following Theorems will shew all the cases in Discount by Compound Interest.

I. $\frac{m}{r^t} = p$. II. $pr^t = m$. III. $\frac{m}{p} = r^t$ which being continually divided by r, till nothing remain, the number of these divisions will be equal to t.

IV. $\frac{m}{p} = r^t$ which being extracted, (the time, given in the question, shewing the power) will be equal to r.

Note. Case 2d may be wrought by Table 1, thus: Find that power of 1*l.* for 1 year, denoted by the time; multiply the present worth by it, and the product will be the answer.

Or, by Table 2d, thus: Find the present worth of 1*l.* for the given time, by which divide the present worth, and the quotient will be the debt or principal.

Case 3d, thus: Divide the debt by its present worth, and seek the quotient in Table 1, under the given rate, and in a line with it you will see the time.

Case 4th is wrought in the same manner, only, seek the quotient in a line with the time, it will shew the rate atop.

RULE.—Divide the debt by that power of the amount of 1*l.* for 1 year, denoted by the time, and the quotient will be the present worth, which, subtracted from the debt, will leave the discount.

EXAMPLES.

1. What is the present worth, and discount, of 600*l.* due 3 years hence, at 6*l.* per cent. per annum, compound interest?

Divide by $\overline{1,06}|^3 = 1,19101$)600,00000(503,7741 = £503 15*s.* $5\frac{3}{4}$*d.* present worth, and £600—£ 503 15 $5\frac{3}{4}$ = £ 96 4*s.* $6\frac{1}{4}$*d.* = Discount.

$$Or, \frac{600}{1,19101} = £503,7741, \ \& \ 600 - \frac{600}{1,19101} = £96,2259$$

By TABLE II.

In this Table, corresponding to the time and rate, we have ,839619 = present worth of 1*l.* for the time and rate.
Multiply by 600 = debt, or principal.

503,771400 = present worth of the debt.

2. What is the present worth of 312*l.* 10*s.* due 2 years hence, at $4\frac{1}{2}$ per cent. per annum, compound interest?

Ans. £286 3*s.* 3*d.* 2,97*qrs.*

3. What ready money will discharge a debt of 1000*D.* due 4 years hence, at 5*D.* per cent. per annum, compound interest?

Ans. 822*D.* 70*c.* 2*m.*

ANNUITIES OR PENSIONS IN ARREARS, AT COMPOUND INTEREST.

CASE I.

When the annuity, or pension, the time it continues, and the rate per cent. are given, to find the amount.

RULE* 1.—Make 1 the first term of a Geometrical Progression, and the amount of 1*l.* for 1 year at the given rate per cent. the ratio.

2. Carry

* It is plain that upon the first year's annuity there will be due so many year's compound interest, as the given number of years less 1, and gradually one year less, upon every succeeding year, to that preceding the last, which has but one year's interest, and the last bears none.

Let r = rate, or amount of 1*l.* for 1 year, then the series of amounts of 1*l.* annuity for several years, from the first to the last, is 1, r, r^2, r^3, &c. to r^{t-1}; and the sum of this, according to the rule in Geometrical Progression, will be $\frac{r^t-1}{r-1}$ = amount of 1*l.* annuity for t years. And all annuities are proportional to their amounts; therefore, $1 : \frac{r^t-1}{r-1} :: n : \frac{r^t-1}{r-1} \times n$ = amount of any given annuity n.

Let r = rate, or amount of 1*l.* for 1 year, and the other letters as before, then, $\frac{r^t-1}{r-1} \times n = a$, and $\frac{ar-a}{r^t-1} = n$.

And from these equations, all the cases relating to annuities or pensions in arrears, may be conveniently exhibited in logarithmic terms, thus,

I. L.

2. Carry the ſeries to ſo many terms as the number of years, and find its ſum.

3. Multiply the ſum, thus found, by the given annuity, and the product will be the amount ſought.

Or, Multiply the amount of 1*l.* for one year into itſelf ſo many times as there are years leſs by 1; then multiply this product by the annuity; and ſubtract the annuity therefrom: *Laſtly*, Divide the remainder by the ratio leſs 1, and the quotient will be the amount.

EXAMPLES.

1. What will an annuity of 60*l.* per annum, payable yearly, amount to in 4 years, at 6*l.* per cent.?

Firſt Method.

$1+1,06+\overline{1,06}|^2+\overline{1,06}|^3 = 4,374616 =$ ſum.
Multiply by 60 = annuity.

262,476960
20

9,53920
12

6,4704
4

1,8816 *Anſ.* £262 9*s.* 6¼*d.*

Or, $1+1,06+\overline{1,06}|^2+\overline{1,06}|^3\times 60 =$ £262 9*s.* 6¼*d.*

Second

I. $L.n+L.r^t-1+L.\overline{r-1}=L.a.$

II. $L.a-L.r^t-1+L.\overline{r-1}=L.n$

III. $\dfrac{L.\overline{ar-a+n}-L.n}{L.r}=t.$ IV. $r^t-\dfrac{ar}{n}-1=\epsilon.$

Or thus, I. $\dfrac{nr^t-n}{r-1}=a.$ II. $\dfrac{ar-a}{r^t-1}=n.$ III. $\dfrac{ar+n-a}{n}=r^t$

which continually divided by r till nothing remain, the number of thoſe diviſions will be equal to t; Or, being extracted, (the given time ſhewing the power) will be equal to r.

Second Method.

$1,06 \times 1,06 \times 1,06 \times 1,06 = 1,26247$
Multiply by 60 annuity.

75,74820
Subtract 60

Divide by 1,06—1=,06)15,7482(262,47 = £262 9*s.* $4\frac{3}{4}$*d.* *Ans.*
12

37
36

14
12

28
24

42
42

$$Or,\ \frac{\overline{1,06 \times 1,06 \times 1,06 \times 1,06} \times 60 - 60}{1,06 - 1} = £262,47$$

Or, By TABLE III.*

Multiply the tabular number under the rate, and opposite to the time, by the annuity, and the product will be the amount.

2. What will an annuity of 60*l.* per annum amount to in 20 years, allowing 6*l.* per cent. compound interest?

Under 6*l.* per cent. and opposite 20, in Table 3d, you will find,
Tabular number = 36,78559
Multiply by 60 = annuity.

2207,13540 = £2207 2*s.* $8\frac{1}{4}$*d.* *Ans.*

3. What will a pension of 75*l.* per annum, payable yearly, amount to in 9 years, at 5*l.* per cent. compound interest.

Ans. £826 19*s.* 10*d.*

4. If a salary of 100*l.* per annum, to be paid yearly, be forborne 5 years, at 6*l.* per cent. What is the amount?

Ans. £563 14*s.* 2*d.*

CASE

* Table 3d is calculated thus: Take the first year's amount, which is 1*l.* multiply it by 1,06+1 = 2,06 = second year's amount, which also multiply by 1,06+1 = 3,1836 = third year's amount, &c. and in this manner proceed in calculating Tables at any other rates.

CASE II.

When the amount, rate per cent. and time are given, to find the annuity, penſion, &c.

Rule.—Multiply the whole amount by the amount of 1*l.* for 1 year, from which ſubtract the whole amount; divide the remainder by that power of the amount of 1*l.* for 1 year, ſignified by the number of years, made leſs by unity, and the quotient will be the anſwer.

Examples.

1. What annuity, being forborne 4 years, will amount to £262,47696, at 6*l.* per cent. compound intereſt?

262,47696 = amount.
Multiply by 1,06 = amount of 1*l.* for 1 year.

157486176	1,06
262476960	1,06
278,2255776	636
Subtract 262,47696	1060
,26247696)15,7486176(£60 *Anſ.*	1,1236
15 7486176	1,06
•	67416
	112360
	1,191016
	1,06
	7146096
	11910160
	1,26247696
	Subtract 1
	Diviſor = ,26247696

Or, $\frac{262,47696 \times 1,06 - 262,47696}{1,06 \times 1,06 \times 1,06 \times 1,06 - 1} = 60.$

Or, By Table III.

Divide the amount by the tabular number under the rate, and oppoſite to the time, and the quotient will be the annuity.

2. What annuity, being forborne 20 years, will amount to £2207,1354, at 6*l.* per cent. compound intereſt?

Tabular amount = 36,78559)2207,13540(£60 *Anſ.*
2207,1354

CASE

CASE III.

When the annuity, amount and ratio are given, to find the time.

Rule.—Multiply the amount by the ratio, to this product add the annuity, and from the sum subtract the amount; this remainder being divided by the annuity, the *quotient* will be that power of the ratio signified by the time, which being divided by the amount of 1*l.* for 1 year, and this *quotient* by the same, till nothing remain, the number of those divisions will be equal to the time. Or, look for this number under the given rate in Table 1. and, in a line with it, you will see the time.

Examples.

1. In what time will 60*l.* per annum, payable yearly, amount to £262,47696, allowing 6*l.* per cent. compound interest, for the forbearance of payment?

```
              262,47696 = amount.
Multiply by        1,06 = ratio.
              ---------
              157486176
             262476960
             ----------
             278,2255776
Add           60         = annuity.
             ----------
             338,2255776
Subtract     262,47696
             ----------
Divide by 60)75,7486176
             ----------
Divide by 1,06)1,26247696
             ----------
Divide by 1,06)1,191016
             ----------
Divide by 1,06)1,1236
             ----------
Divide by 1,06)1,06
             ----------
                  1
```

The number of divisions by 1,06, being 4, gives the number of years = 4 the answer.

Or, In Table 1. under the given rate, you will find 1,262476, and in a line, under years, you will find 4.

2. In what time will an annuity of 60*l.* payable yearly, amount to £2207,1354, allowing 6*l.* per cent. for the forbearance of payment? *Ans.* 20 years.

PRESENT

PRESENT WORTH OF ANNUITIES, &c. AT COMPOUND INTEREST.

CASE I.

When the annuity, &c. rate and time are given, to find the present worth.

RULE* 1.—Divide the annuity by the ratio, or the amount of 1*l.* for 1 year, and the quotient will be the present worth of 1 year's annuity.

2. Divide the annuity by the square of the ratio, and the quotient will be the present worth for two years.

3. In like manner, find the present worth of each year by itself, and the sum of all these will be the present value of the annuity, sought.

Or, Divide the annuity, &c. by that power of the ratio signified by the number of years, and subtract the quotient from the annuity: This remainder being divided by the ratio less 1, the quotient will be the present worth.

EXAMPLES.

* The reason of this rule is evident from the nature of the question, and what was said upon the same subject in the purchasing of annuities by simple interest.

Let p = present worth of the annuity, and the other letters as before: Then,

$n\times\frac{r^t-1}{r^{t+1}-r^t}=p$. And $p\times\frac{r^{t+1}-r^t}{r^t-1}=n$.

And from these Theorems, all the cases, where the purchase of annuities is concerned, may be exhibited in logarithmic terms, as follows:

I. $L.n+L.\overline{1-\frac{1}{r^t}}-L.r-1=L.p$.

II. $L.p+L.\overline{r-1}-L.1-\frac{1}{r^t}=n$.

III. $\frac{L.n-L.\overline{n+p-pr}}{L.r}=t$.

Or, thus, I. $\frac{n-\frac{n}{r^t}}{r-1}=p$, II. $\frac{pr^t\times r-pr^t}{r^t-1}=n$.

III. $\frac{n}{p+n-pr}=r^t$ which being continually divided by r till nothing remain, the number of those divisions will be equal to t.

Let t express the number of half years, or quarters, n the half years, or quarter's payment, and r the sum of 1*l.* and $\frac{1}{2}$ or $\frac{1}{4}$ year's interest, then all the preceding rules will be applicable to half yearly, and quarterly payments, the same as to whole years.

The *amount* of an annuity may also be found for years and parts of a year, thus:

1. Find the amount for the whole years, as before.
2. Find the interest of that amount for the given parts of a year.
3. Add this interest to the former account, and it will give the whole amount required.

The *present worth* of an annuity for years and parts of a year may be found thus:

1. Find the present worth for the whole years, as before.
2. Find the present worth of this present worth, discounting for the given parts of a year, and it will be the whole present worth required.

EXAMPLES.

1.* What ready money will purchase an annuity of 60*l.* to continue 4 years, at 6*l.* per cent. compound interest.

First Method.

$\text{Ratio} = 1{,}06)60{,}00000(56{,}603 =$ present worth for 1 year.
$\overline{\text{Ratio}}|^2 = 1{,}1236)60{,}00000(53{,}399 =$ do. for 2 years.
$\overline{\text{Ratio}}|^3 = 1{,}191016)60{,}00000(50{,}377 =$ do. for 3 years.
$\overline{\text{Ratio}}|^4 = 1{,}26247696)60{,}00000(47{,}525 =$ do. for 4 years.
$207{,}904 = £\,207\ 18s.\ 0\tfrac{3}{4}d.$ *Ans.*

Second Method.

$\left.\begin{array}{l}\text{4th power of}\\ \text{the ratio}\end{array}\right\} = 1{,}26247696)60{,}0000000(47{,}525$

From 60
Subtract 47,525

Divis. 1,06 — 1 = ,06)12,475

207,916 = £207 18*s.* 3¾*d.* *Ans.*

Or, $\dfrac{60}{\overline{,06}|^4} = 47{,}525$ $60 - 47{,}525 = 12{,}475$

And $\dfrac{12{,}475}{,06} = 207{,}916.$

By TABLE III.

Under 6*l.* per cent. and opposite 4, we find
4,37461 = amount of 1*l.* annuity for 4 years.
Multiply by 60 = annuity.

262,47660 = amount of 60*l.* for 4 years.

Then, opposite 4 years, and under 6*l.* per cent. in Table 2d,
We have ,792093
Multiply by 262,7466

4752558
4752558
3168372
5544651
1584186
4752558
1584186

208,1157426338 = £208 2*s.* 4½*d.*

Or, Opposite 4 years, and under 6*l.* per cent. in Table 1st, we have 1,26247 = the amount of 1*l.* for 4 years:

Then, 262,7466 ÷ 1,26247 = 208,1209 = £208 2*s.* 5*d.* *Ans.*

By

* Questions in this Case may also be answered by first finding the amount of the given annuity by Case 1. of Annuities in Arrears, page 304, and then the present worth, or principal, by Case 2. of Compound Interest, page 302.

By TABLE IV.*

Multiply the tabular number, under the rate, and oppoſite the time, into the annuity, and the product will be the preſent worth.

Thus, in Example 1ſt, What ready money will purchaſe 60*l.* annuity, to continue 4 years, at 6*l.* per cent. compound intereſt?

Under 6*l.* per cent. and even with 4 years,

We have 3,4651 = preſent worth 1*l.* for 4 years.
Multiply by 60 = annuity.

Anſ. = 207,9060 = £207 18*s.* $1\frac{1}{4}$*d.*

2. What is the preſent worth of an annuity of 60*l.* per annum, to continue 20 years, at 6*l.* per cent. compound intereſt?

Anſ. £688 3*s.* $10\frac{3}{4}$*d.*

CASE II.

When the preſent worth, time and rate are given, to find the annuity, rent, &c.

RULE 1.—From that power of the ratio, denoted by the number of years plus 1, ſubtract that power of it, denoted by the number of years.

2. Divide the remainder by that power of the ratio, ſignified by the time made leſs by unity.

3. Multiply the preſent worth into this quotient, and the product will be the annuity, penſion, rent, &c.

Or, 1. Multiply that power of the ratio, denoted by the number of years plus 1, by the preſent worth.

2. Multiply that power of the ratio, denoted by the time, by the preſent worth, and ſubtract this product from the former.

3. Divide the remainder by that power of the ratio, denoted by the time made leſs by unity, and the quotient will be the annuity.

EXAMPLES.

1. What annuity, to continue 4 years, will £207,904 purchaſe, compound intereſt, at 6*l.* per cent.?

Firſt Method.

From $1,06 \times 1,06 \times 1,06 \times 1,06 \times 1,06 = 1,3382255776$
Subt. $1,06 \times 1,06 \times 1,06 \times 1,06 = 1,26247696$

Div. by $\overline{1,06}|^4 - 1 =$,26247696),07574861 76(,2885898
,2885898
Multiply by 207,9 preſent worth.

25973082
20201286
5771796 0

Anſ. 59,99781942 = £60.

Second

* Table 4th is thus made: Divide 1*l.* by 1,06 = ,94339 the preſent worth of the firſt year, which, divided by 1,06, is equal to ,88999, which, added to the firſt year's preſent worth, is = 1,83339, the ſecond year's preſent worth, then ,88999 divided by 1,06, and the quotient added to 1,83339, gives 2,6701 for the third year's preſent worth, &c.

Second Method.

From $1,06 \times 1,06 \times 1,06 \times 1,06 \times 1,06 \times 207,9 = 278,217097573$
Take $1,06 \times 1,06 \times 1,06 \times 1,06 \times 207,9 \quad = 262,468959984$

Divide by $\overline{1,06}|^{4} - 1 = ,26247696)15,748137589(59,998 = £60.$

By TABLE V.*

Multiply the tabular number, correſponding with the rate and time, by the purchaſe money, and the product will be the annuity.

Under 6*l.* per cent. and oppoſite 4 years, you will find
,28859 = annuity which 1*l.* will purchaſe in 4 years.
Mult. by 207,9

```
  259731
 202013
577180
59,997861 = £60.
```

2. What ſalary, to continue 20 years, will 688*l.* 3*s.* 10¼*d.* purchaſe, at 6*l.* per cent. compound intereſt? *Anſ.* £60.

CASE III.

When the annuity, preſent worth, and ratio are given, to find the time.

RULE.

Divide the annuity by the product of the preſent worth and ratio ſubtracted from the ſum of the preſent worth and annuity, and the quotient will be that power of the ratio, denoted by the number of years, which being divided by the ratio, and this quotient by the ſame, till nothing remain, the number of diviſions will ſhew the time: *Or*, the above quotient being ſought in Table 1ſt, under the given rate, in a line with it, you will ſee the time.

EXAMPLES.

1. For how long may an annuity of 60*l.* per annum be purchaſed for £207,9063367 62, at 6*l.* per cent. compound intereſt?

Multiply

* Table 5th is made in this manner: Divide 1*l.* by the preſent worth of 1*l.* for 1 year, and the quotient will be the annuity, which 1*l.* will purchaſe for 1 year; divide 1*l.* by the preſent worth of 1*l.* for 2 years, and the quotient will be the annuity, which 1*l.* will purchaſe for 2 years, &c.

Multiply 207,906336762		To 207,906336762	= present worth.
by	1,06	Add 60	= annuity.
	124743802057 2	From 267,906336762	
	2079063367620	Subt. 220,380716967	
	220,38071696772	47,525619795	= divisor.

47,525619795)60,000000000(1,26247696

Divide by 1,06)1,26247696

1,06)1,191016

1,06)1,1236

1,06)1,06

$1 \left\{ \begin{array}{l} \text{The number of divisions} \\ = \text{time} = 4 \text{ years.} \end{array} \right.$

Or, $\dfrac{60}{207{,}906336762+60-207{,}906336762\times 1{,}06}=1{,}26247696,$

which being sought in Table 1. under the given rate, in a line with it, is 4 = 4 years.

2. How long may a lease of 75*l.* yearly rent be had for 533*l.* 1*s.* 8½*d.* allowing 5*l.* per cent. compound interest, to the purchaser? *Ans.* 9 years.

ANNUITIES, LEASES, &c. TAKEN IN REVERSION AT COMPOUND INTEREST.

CASE I.

When the annuity, time and ratio are given, to find the present worth of the annuity in reversion.

RULE* 1.—Divide the annuity by that power of the ratio denoted by the time of its continuance.

2. Subtract this quotient from the annuity; divide the remainder by the ratio less 1, and the quotient will be the present worth, to commence immediately.

3. Divide this quotient by that power of the ratio denoted by the time of reversion, (or, time to come, before the annuity commences)

* Let v denote the time in reversion, and the other letters as before. Then, the two cases under this rule will be expressed by the following Theorems.

I. $n-\dfrac{n}{r^t}=p$. Then change p into m, and $\dfrac{m}{r^v}=p$.

II. $pr^v=m$. Change m into p, and $\dfrac{pr^t\times r-pr^t}{r^t-1}=n$.

Or, I. $\dfrac{r^t-1\times n}{r-1\times r^t\times r^v}=p$. II. $\dfrac{r-1\times r^t\times r\times vp}{r^t-1}=n$.

mences) and the quotient will be the present worth of the annuity in reversion.

Or, 1. Multiply the annuity by that power of the ratio denoted by the time of its continuance, minus unity, for a dividend.

2. Multiply that power of the ratio denoted by the time of its continuance, that power of it denoted by the time of reversion, and the ratio less 1, continually together for a divisor, and the quotient arising from the division of these two numbers will be the present worth of the annuity in reversion.

EXAMPLES.

1. What is the present worth of 60*l.* payable yearly, for 4 years; but not to commence till 2 years hence at 6*l.* per cent. ?

First Method.

Ratio = 1,06
1,06
636
1060
2d power = 1,1236
1,1236
67416
33708
22472
11236
11236

Or, In Table 4th, find the present value of 1*l.* at the given rate, both for the time in being and the time in reversion added together, and subtract the present worth of the time in being from the other, multiply the remainder by the annuity, and the product will be the answer.

Pres. worth of the time in being & reversion = 4,91732
Pres. worth of the time in being = 1,8333
3,08402
60
£185,04120

Div. by 4th pow. = 1,26247696)60,0000000(47,525621378467
Subtract the quotient = 47,525621378467
Divide by 1,06—1 = ,06)12,474378621533

Divide by 1,06×1,06 = 1,1236)207,9063103588(185,035876 = £185 0*s.* 8½*d.* the present worth of the annuity in reversion,

Or, $\frac{60}{1,26247696} = 47,5256$ $\qquad \frac{60-47,5256}{1,06-1} = 207,906.$

And $\frac{207,906,}{1,1236} = 185,035876$

Second Method.

,26247696 = 4th power—1
Multiply by 60 = annuity.
15,74861760 = dividend.
1,26247696 = 4th power.
1,1236 = 2d power.
757486176
378743088
252495392
126247696
126247696
1,418519112256
,06 = ratio—1
,08511114673536 = divisor.

,08511115)15,74861760(185,036 *Ans.*

Or, $\frac{\overline{1,06}|^4 - 1 \times 60}{\overline{1,06}|^4 \times \overline{1,06}|^2 \times 1,06-1} = 185,036$

2. What

2. What is the present worth of a reversion of a lease of 60*l.* per annum, to continue 20 years, but not to commence till the end of 8 years, allowing 6*l.* per cent. to the purchaser?

Ans. £431 15*s.* 7*d.* 2,78199*qrs.*

An annuity, several times in reversion, and rate being given, to find the several present values.

Find the present value of 1*l.* per Table 4. at the given rate, and for the several given times, which being severally multiplied by the annuity, the products will be the several present values of that annuity, for the several times given; subtract the several present values, the one from the other, and the several remainders will answer the question.

3. A has a term of 6 years in an estate of 60*l.* per annum. B has a term of 14 years in the same estate, in reversion, after the 6 years are expired; and C hath a further term of 16 years, after the expiration of 20 years. I demand the present values of the several terms, at 6 per cent.?

		£.	s.	d.	
Pres. val. of 1*l.* for 36 years	= 14,61722 × 60 =	877	0	$7\frac{3}{4}$	
Ditto of ditto for 20	= 11,46992 × 60 =	688	3	$10\frac{3}{4}$	
Ditto of ditto for 6	= 4,91732 × 60 =	295	0	$9\frac{1}{4}$	= A's term.

Therefore, 877 0 $7\frac{3}{4}$ — 688 3 $10\frac{3}{4}$ = £188 16 9 C's term, and 688 3 $10\frac{3}{4}$ — 295 0 $9\frac{1}{4}$ = £393 3 $1\frac{1}{2}$ = B's term.

4. For a lease of certain profits for 7 years, A offers to pay 300*l.* gratuity, and 300*l.* per annum, B offers 800*l.* gratuity and 250*l.* per annum, C bids 1300*l.* gratuity and 200*l.* per annum, and D bids 2500*l.* for the whole purchase, without any yearly rent; which is the best offer, computing at 6 per cent.?

	£.
By Table 4. the present worth of 300*l.* per annum, for 7 years, at 6 per cent. is	1674,714
To which add	300
Value of A's offer =	1974,714
Present worth of 250*l.* per annum, for 7 years =	1395,595
To which add	800
Value of B's offer =	2195,595
Present worth of 200*l.* per annum for 7 years =	1116,476
To which add	1300
Value of C's offer =	2416,476
D's offer =	2500

Hence it appears that D's offer is the best.

The

The above questions may be answered by the 4th and 2d Tables. Take question 1st for Example.

1. Multiply the tabular number in Table 4, corresponding to the rate and the time of continuance, into the annuity, and the product will be the present worth, to commence immediately.

2. Multiply this present worth by the tabular number in Table 2. corresponding to the rate and the time of reversion, and the product will be the present worth of the annuity in reversion.

In Table 4th we have 3,4651
Multiply by 60 = annuity.

207,9060
In Table 2d we have ,889996

1247436
1871154
1871154
1871154
1663248
1663248

185,035508376 = pres. worth of the revers.

CASE II.

When the present worth of the reversion, rate and time are given, to find the annuity.

RULE 1.—Multiply that power of the ratio signified by the time of reversion, by the present worth, and the product will be the amount of the present worth for the time before the annuity commences.

2. Multiply that power of the ratio signified by the time of continuance plus 1 by the last product.

3. Multiply that power of the ratio, signified by the time, by the aforesaid product, and this last product, divided by that power of the ratio denoted by the time, minus unity, will give the annuity.

Or, Divide the continual product of the present worth, that power of the ratio denoted by the time of continuance, that power of it denoted by the time of reversion, and the ratio minus 1, by that power of the ratio denoted by the time of continuance minus 1, and the quotient will be the annuity.

EXAMPLES.

1. What annuity, to be entered upon 2 years hence, and then to continue 4 years, may be purchased for £185,035876, at 6*l.* per cent.?

First

First Method.

1,06×1,06 = 1,1236 = 2d power of the ratio.
Multiply by 185,036 = present worth.

```
      67416
     33708
    561800
   89888
  11236
  207,9064496 amount for the time of reversion.
```

Multiply by 1,33822 = 5th power of the ratio.

```
   415812
   415812
  1663248
  623718
 623718
207906
```

From 278,22396732
Take 262,47508782

4th power of the ratio = 1,26247
Multiply by 207,906

```
     757482
  11362230
   883729
2524940
262,47508782
```

Divide by $\overline{1,06}|^4 - 1$ = ,26247)15,74887950(60 the annuity required.

Or, 185,036×1,1236 = 207,906

Then, $\frac{207,906\times1,33822 - 207,906\times1,26247}{1,26247-1}$ = £60 *Ans.*

Second Method.

185,036 = present worth of the reversion.
1,26247 = 4th power of the ratio.

```
   1295252
   740144
  370072
 1110216
 370072
185036
 233,6024
   1,1236 = 2d power of the ratio.
```

Or by Table 4th, divide the present worth of the reversion by the difference between the present worth of 1*l.* for the time both in being and reversion, and the time in being and the quotient will be the annuity.

```
   14016144
   7008072
  4672048
 2336024
2336024
262,47565664
        ,06 = ratio—1.
```

4,91732
1,8333

3,08402)185,0412(60 *Ans.*

$\overline{1,06}|^4 - 1$ = ,26247)15,7485393984(60.

Or, $\frac{185,036\times1,26247\times1,1236\times\overline{1,06-1}}{1,26247-1}$ = 60.

2. The present worth of a lease of an house is 431*l.* 15*s.* 7*d.* 2,7819 *qrs.* taken in reversion for 20 years; but not to commence till the end of 8 years, allowing 6*l.* per cent. to the purchaser; What is the yearly rent? *Ans.* £60.

PURCHASING

PURCHASING ANNUITIES FOREVER, *or* FREEHOLD ESTATES, *at* COMPOUND INTEREST.

CASE I.

When the annuity, or yearly rent, and the rate are given, to find the present worth, or price.

RULE.*—As the rate per cent. is to 100*l.*; so is the yearly rent, to the value required.

Or, Divide the yearly rent by the ratio less 1, and the quotient will be the value required.

EXAMPLES.

1. What is the worth of a freehold estate of 60*l.* per annum, allowing 6*l.* per cent. to the purchaser?

```
£.   £.     £.
6 : 100 :: 60                    Or, 1,06—1=,06)60,00
       60                                      ———
     ————                                      1000
    6)6000
     ————
   £1000 Ans.
```

2. An estate brings in yearly, 75*l.*; What would it sell for, allowing the purchaser 5*l.* per cent. compound interest?

Ans. £1500.

CASE II.

When the price, or present worth, and rate are given, to find the annuity, or yearly rent.

RULE.—As 100*l.* is to the rate, so is the present worth to its rent.

Or, Multiply the present worth by the ratio less 1, and the product will be the yearly rent.

EXAMPLES.

1. If a freehold estate be bought for 1000*l.* allowing 6*l.* per cent. to the purchaser; What is the yearly rent?

```
 £.   £.    £.
100 : 6 :: 1000
              6
           ————                  Or, 1000×,06=£60.
     100)6000(£60 Ans.
         600
         ———
           0
```

2. If

* The reason of this rule is obvious; for since a year's interest of the price, which is given for it, is the annuity, there can neither more nor less be made of that price, than of the annuity, whether it be employed at simple or compound interest.

The following Theorems shew all the varieties of this rule.

I. $\frac{n}{r-1}=p$. II. $\overline{r-1}\times p=n$. III. $\frac{n}{p}+1=r$, or $\frac{n}{p}=r-1$; or $\frac{p+n}{p}=r$.

2. If an eſtate be ſold for 1500*l.* and 5 per cent. allowed to the buyer; What is the yearly rent? *Anſ.* £75.

CASE III.

When the preſent worth, or price, and yearly rent are given, to find the rate.

RULE.—As the preſent worth is to the rent; ſo is 100*l.* to the rate.

Or, Divide the rent by the preſent worth; add 1 to the quotient, and the ſum will be the ratio of the rate per cent.

Or, Divide the ſum of the preſent worth and rent by the preſent worth, and the quotient will be the ratio.

EXAMPLES.

1. If an eſtate of 60*l.* per annum be bought for 1000*l.*; What rate of intereſt was allowed the purchaſer for his money.

£. £. £.
1000 : 60 :: 100
100

1000)6000(£6 *Anſ.*
6000

Or, 1000)60,00(,06+1=1,06
60 00

Or, to 1000 = preſent worth,
Add 60 = rent.

1000)1060(1,06
1000

6000
6000

2. An eſtate of 75*l.* per annum was purchaſed for 1500*l.*; What rate of intereſt had the buyer for his money? *Anſ.* £5.

To find at how many year's purchaſe an eſtate may be bought.

CASE I.

When the rate of intereſt is given, to find the number of years.

RULE.—Divide 100*l.* by the rate, and the *quotient* will be the years.

EXAMPLES.

1. How many years' purchaſe ſhould a gentleman offer for the purchaſe of an eſtate, to have 6*l.* per cent. for his money?

6)100

16,666+ = $16\frac{2}{3}$ years.

2. How many years' purchaſe is an eſtate worth, allowing 5*l.* per cent. to the purchaſer? *Anſ.* 20 years.

CASE II.

When the number of year's purchaſe, at which an eſtate is bought, or ſold, is given, to find the rate of intereſt.

RULE.—Divide 100*l.* by the number of years, and the quotient will be the rate.

EXAMPLES,

EXAMPLES.

1. A gentleman gives $16\frac{2}{3}$ years' purchaſe for a farm; What intereſt is he allowed?

$16\frac{2}{3} = 16,666+)100,000($£$6$ *Anſ.*

2. A gentleman gives 20 years' purchaſe for an eſtate; What intereſt has he? *Anſ.* £5.

PURCHASING FREEHOLD ESTATES IN REVERSION.

CASE I.

The rate and rent of a freehold eſtate being given, to find the preſent worth of reverſion.

RULE 1.*—Find the preſent worth of the annuity or rent, (by Caſe 1. of purchaſing Freehold Eſtates, page 318,) as though it were to be entered on immediately.

2. Divide the laſt preſent worth by that power of the ratio denoted by the time of reverſion (by Caſe 1. of Diſcount by Compound Intereſt) and the quotient will be the anſwer required.

Or, 1. Having found the preſent value of the eſtate, ſuppoſing it to be immediate: Multiply the annuity, or rent, by the preſent worth of 1*l.* correſponding with the time of reverſion and rate in Table 4th, and the product will be the preſent worth of the annuity or rent, for the time of reverſion; or the value of the preſent poſſeſſion.

2. Subtract the value of the poſſeſſion from the value of the eſtate, and the remainder will be the value of reverſion.

EXAMPLES.

1. Suppoſe a freehold eſtate of 60*l.* per annum to commence 2 years hence, be put up to ſale; What is its value, allowing the purchaſer 6*l.* per cent.?

Firſt Method.

$1,06-1 = ,06)60,00$ = rent per annum.

1000 = preſent worth, if entered on immediately.

$1,06|^2 = 1,1236)1000,000(889,996 =$ £889 19*s.* 11*d.* = preſent worth of 1000*l.* for 2 years, or the whole preſent worth required.

Second

* The following Theorems expreſs all the Caſes under this rule.

I. $\frac{n}{r-1} = p$; then change p into m, and $\frac{m}{r^v} = p$.

II. $pr^v = m$; then change m into p, and $\frac{prr-pr}{r} = n$.

Second Method.

1,06—1 = ,06)60,00

1000 = present worth, for immediate possession.
In Table 4th we have, 1,83339 = value of 1*l.* for 2 years.
Multiply by 60 = rent.

110,00340 = value of possession.
From 1000,0000
Subtract 110,0034

889,9966 = value required.

2. Suppose an estate of 75*l.* per annum, to commence 10 years hence, were to be sold, allowing the purchaser 5*l.* per cent.; What is its worth? *Ans.* £920 17*s.* 5*d.*

CASE II.

The Value of a Reversion, the Time prior to its Commencement, and Rate of Interest given, to find the Annuity or Rent.

Rule 1.—Multiply the price of the reversion by that power of the amount of 1*l.* for 1 year, denoted by the time of reversion, and the product will be its amount (by Case 1, of Compound Interest.)

2. Find the interest of the amount (by Case 1st, Simple Interest) and it will be the annuity, or yearly rent.

Examples.

1. A freehold estate is bought for £889,9966 which does not commence till the end of 2 years; the buyer being allowed 6*l.* per cent. for his money; I desire to know the yearly income?

889,9966 = price of the reversion.
Multiply by $\overline{1,06}|^2$ = 1,1236 denoted by the time of reversion.

53399796
26699898
17799932
8899966
8899966

1000,00017976 = amount of the reversion.
,06

Ans. £60,00

2. If a freehold estate, to commence 10 years hence, be sold for £920 17*s.* 5*d.* allowing the purchaser 5*l.* per cent.; What is the yearly income? *Ans.* £75.

 Table

TABLE I. *Shewing the amount of £1 from 1 year to 50.*

ys.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.
1	1,0300000	1,0350000	1,0400000	1,0450000	1,0500000	1,0550000	1.0600000
2	1,0609000	1,0712250	1,1816000	1,0920250	1,1025000	1,1130250	1,1236000
3	1,0927270	1,1087178	1,1248640	1,1411661	1,1576250	1,1742413	1,1910160
4	1,1255088	1,1475230	1,1698585	1,1925186	1,2155062	1,2388245	1,2624796
5	1,1592740	1,1876863	1,2166529	1,2461819	1,2762815	1.3069598	1,3382256
6	1,1940523	1,2292553	1,2653190	1.3022601	1,3400956	1,3788426	1,4185191
7	1,2298738	1,2722792	1,3159317	1,3608618	1,4071004	1,4546789	1,5036302
8	1,2667700	1,3168090	1,3685690	1,4221006	1,4774554	1,5346862	1,5938480
9	1,3047731	1,3628973	1,4233118	1,4860951	1,5513-82	1,6190939	1,6894789
10	1,3439163	1,4105987	1,4802842	1,5529694	1,6288946	1,7081440	1,7908476
11	1,3842338	1,4599697	1,5394540	1,6228530	1,7103393	1,8020919	1,898298[illegible]
12	1,4257608	1,5110686	1,6010322	1,6958814	1,7958563	1,9012069	2,0121964
13	1,4685337	1,5639560	1,6650735	1,7721961	1,8856491	2,0057732	2,1329282
14	1,5125897	1,6186945	1,7316764	1,8519449	1.9799316	2,1160907	2 2609039
15	1,5579674	1,6753488	1,8009435	1,9352824	2 0789281	2,2324756	2.39655[illegible]
16	1,6047064	1,733986	1,8729812	2,0223701	2,1828745	2,3552617	2,5472716
17	1,6528476	1,7946755	1,9479005	2,1133768	2,2920183	2 4848011	2,6927727
18	1,7024330	1,8574892	2,0258161	2,2084787	2 4066192	2,6214652	2.8543391
19	1,7535060	1.9225013	2,1068491	2,3078603	2,5269502	2.7656458	3.0255995
20	1,8061112	1,9897888	2,1911231	2,4117140	2 6532977	2.9177563	3 2071355
21	1,8602945	2,0594314	2,2787680	2,5202411	2,7859625	3.0782329	3,3995636
22	1,9161034	2,1315115	2.3699187	2.6336520	2,9252607	3.2475357	3,6035374
23	1,9735865	2,2061144	2,4647155	2,7521663	3,0715237	3.4261502	3,8197496
24	2,0327941	2,2833284	2,5633041	2,8760138	3,2250999	3,6145885	4,0489346
25	2,0937779	2 3632449	2,6658363	3,0054344	3,3863549	3,8133910	4 2918707
26	2,1565912	2,4459585	2,7724697	3,1406790	3,5556726	4,0231279	4 5493829
27	2,2212890	2,5315671	2,8833685	3,2820095	3,7334563	4,2443999	4,822345[illegible]
28	2,2879276	2,6201719	2,9987033	3,4296999	3,9201291	4,4778419	5,1116867
29	2,3565655	2,7118779	3,1186514	3,5840364	4,1161356	4.7241232	5,4183879
30	2,4270624	2,8067937	3,2433975	3 7453181	4,3219423	4.9839499	5,7434912
31	2,5000803	2,9050314	3,3731334	3,9138574	4,5380394	5.2580671	6,0881007
32	2,5750827	3,0067075	3,5080587	4,0899810	4,7649414	5,5472608	6.4533867
33	2,6522352	3,1119423	3.6483811	4,2740301	5.0031885	5,8523600	6.8405899
34	2,7319053	3,2208603	3,7943163	4.4663615	5.2533479	6,1742398	7,2510253
35	2,8138624	3.3335904	3.9460889	4 6673478	5 5160152	6.5138230	7,6860868
36	2,8982783	3,4502661	4,1039325	4,8773784	5.7918101	6,8720832	8,147252
37	2,9852266	3,5710254	4,2680898	5,0968604	6,0814069	7.2500478	8,6360871
38	3,0747834	3,6960113	4,4388134	5,3262192	6,3854772	7,6488004	9.1542523
39	3,1670269	3,8253717	4,6163659	5,5658990	6,7047511	8,0694844	9,7035074
40	3,2620377	3,9592597	4,8010206	5.8164645	7,0399887	8,5133060	10,2857178
41	3,3598988	4,0978337	4,9930614	6,0782054	7.3919881	8,9815378	10,9028608
42	3,4606958	4,2412579	5,1927838	6,3517246	7,7615875	9,4755224	11,557032[illegible]
43	3,5645167	4,3897020	5,4004952	6,6375522	8,149666[illegible]	9,9966761	12,2504547
44	3,6714522	4,5433415	5,616515	6,9362421	8,5571502	10 5464933	12.9854817
45	3,7815957	4.7023585	5.8411756	7.[illegible]18373	8.9850077	11 1265504	13.7626109
46	3,8950436	4,8669411	6,0748236	7,5745497	9.4342581	11,7385217	14,5883673
47	4,0118949	5,0372840	6,3168166	7,9154045	9,9059710	12.3841404	15 4636693
48	4,1322518	5,2135889	6,5694892	8,2715977	10,4012696	13.0652681	16,3914804
49	4.2562193	5,3960645	6,8322688	8,6438196	10 9213331	13,7838579	17 3749788
50	4,3830059	5.5849268	7.1055596	9,0327915	11.4673697	14,5410000	18,2174775

TABLE

TABLE II. *Shewing the present value of £1, due at the end of any number of years, from 1 to 40.*

yrs.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.	yrs.
1	,961538	,956938	,952381	,947867	,943396	1
2	,924556	,91573	,90703	,898513	,889996	2
3	,888996	,876297	,863838	,851728	,839619	3
4	,854804	,838561	,822702	,807397	,792093	4
5	,821927	,802451	,783526	,765392	,747258	5
6	,790314	,767896	,746215	,725587	,70496	6
7	,759918	,734828	,710681	,687869	,665057	7
8	,730690	,703185	,676839	,652125	,627412	8
9	,702587	,672904	,644609	,618253	,591898	9
10	,675564	,643928	,613913	,586153	,558394	10
11	,649581	,616199	,584679	,573733	,562787	11
12	,624597	,589664	,556837	,526903	,496969	12
13	,600574	,564271	,530321	,49958	,468839	13
14	,577475	,539973	,505068	,473684	,442301	14
15	,555264	,516720	,481017	,449141	,417265	15
16	,533908	,494409	,458311	,425979	,393647	16
17	,513373	,473176	,436297	,40383	,371364	17
18	,493628	,4528	,415521	,382932	,350343	18
19	,474642	,433302	,395734	,363123	,330513	19
20	,456387	,414643	,3[illegible]6889	,344346	,311804	20
21	,438833	,396787	,358042	,326568	,294155	21
22	,421955	,379701	,34185	,[illegible]09677	,277505	22
23	,405726	,36335	,325571	,293684	,261797	23
24	,390121	,347703	,310068	,278523	,246978	24
25	,375117	,332731	,305303	,26915	,232998	25
26	,360689	,318402	,281241	,250525	,21981	26
27	,340816	,304691	,267848	,237608	,207368	27
28	,333477	,291571	,255094	,225362	,19563	28
29	,320651	,279015	,242946	,213715	,184556	29
30	,308309	,267	,231377	,202743	,17411	30
31	,290460	,255504	,220359	,192307	,164255	31
32	,285058	,2445	,209866	,182411	,154957	32
33	,274094	,233971	,199872	,173029	,146185	33
34	,263552	,223896	,190355	,164133	,137912	34
35	,254415	,214251	,18129	,155692	,130105	35
30	,243609	,205028	,172057	,147399	,122741	36
37	,234207	,196299	,164436	,140114	,115793	37
38	,225285	,18775	,156605	,132893	,109182	38
39	,216671	,179665	,149148	,126075	,103002	39
40	,208289	,171929	,142046	,119608	,09717	40

TABLE

TABLE III. *Shewing the amount of £1 annuity for any number of years, from 1 to 40.*

yrs.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.	yrs.
1	1,	1,	1,	1,	1,	1
2	2,04	2,045	2,05	2,055	2,06	2
3	3,1216	3,137025	3,1525	3,16802	3,1836	3
4	4,246464	4,278191	4,310125	4,34226	4,374616	4
5	5,416322	5,47071	5.525631	5,58109	5,637093	5
6	6,632975	6,716892	6,801913	6,888051	6.975318	6
7	7,898294	8,019152	8,142008	8,266894	8,393837	7
8	9,214266	9,380014	9,549109	9,721573	9,897467	8
9	10,582795	10,802114	11,026564	11,256259	11,491315	9
10	12,006107	12,2882	12,577892	12.875354	13,180794	10
11	13.486351	13,841179	14,206787	14,583498	14,971642	11
12	15,025805	15,464032	15,917126	16,38559	16,86994	12
13	16,626838	17,159913	17,712983	18,286798	18,882132	13
14	18,291911	18,932109	19,598632	20.292572	21,015064	14
15	20.023588	20,784054	21,578563	22,408663	23,275968	15
16	21,824531	22,719337	23,657492	24,64114	25,672527	16
17	23,697512	24,741707	25,840366	26,996402	28,212879	17
18	25,645413	26,855084	28,132385	29,481205	30,905652	18
19	27,671229	29,063562	30,529004	32,102671	33,759991	19
20	29,778078	31,371423	33,065954	34,868318	36,78559	20
21	31,969202	33,783137	35,719252	37,786075	39,992725	21
22	34,24797	36.303378	38,505214	40,864309	43,392289	22
23	36,617888	38,93703	41,430475	44.111846	46,995826	23
24	39,082604	41,689196	44,501999	47,537998	50,815576	24
25	41,645908	44,56521	47,727099	51,152588	54,86451	25
26	44.311745	47,570645	51,113454	54,96598	59,156381	26
27	47,084214	50,711324	54,669126	58,989109	63,705763	27
28	49,967583	53,993333	58,402583	63,23351	68,528109	28
29	52,966286	57,423033	62,322712	67,711353	73,639796	29
30	56,084938	61.007067	66,438847	72,435478	79,058183	30
31	59,328335	64,752388	70,76079	77,419429	84,801674	31
32	62,701469	68,666245	75,298829	82,677498	90,889775	32
33	66,209527	72,756226	80,063771	88,22476	97,343161	33
34	69,857908	77,030256	85,066959	94,077122	104,183751	34
35	73,652225	81,496618	90.320307	100.251363	111,434776	35
36	77,598314	86,103966	95,836323	106,765188	119,120863	36
37	81,702246	91,041344	101,628139	113,637274	127,268114	37
38	85.970336	96,138205	107,709546	120,887324	135,904201	38
39	90,40915	101,464424	114,095025	128,536127	145,058453	39
40	95.025516	107.030323	120.799774	135,605614	154.761961	40

TABLE IV. *Shewing the present worth of £1, annuity, for any number of years, from 1 to 40.*

yrs.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.
1	0,96154	0,95694	0,95238	0,94786	0,94339
2	1,88609	1,87267	1,85941	1,8463	1,83339
3	2,77509	2,74896	2,72325	2,6979	2,67301
4	3,62989	3,58752	3,54595	3,49862	3,4651
5	4,45182	4,38997	4,32948	4,25759	4,21236
6	5,24214	5,15787	5,07569	4,97699	4,91732
7	6,00205	5,8927	5,78637	5,65888	5,58238
8	6,73274	6,59589	6,46321	6,30522	6,20979
9	7,43533	7,26879	7,10782	6,91786	6,80169
10	8,11089	7,91272	7,72173	7,49856	7,36008
11	8,76048	8,52892	8,30641	8,04898	7,88687
12	9,38507	9,11858	8,86325	8,5707	8,38384
13	9,98565	9,68285	9,39357	9,06522	8,85268
14	10,56312	10,22282	9,89864	9,53395	9,29498
15	11,11839	10,73954	10,37966	9,97824	9,71225
16	11,65229	11,23401	10,83777	10,39936	10,10589
17	12,16567	11,70719	11,27407	10,79852	10,47726
18	12,65929	12,15099	11,68958	11,17687	10,8276
19	13,13394	12,59329	12,08532	11,53549	11,15811
20	13,59032	13,00793	12,46221	11,87541	11,46992
21	14,02916	13,40472	12,82115	12,1976	11,76407
22	14,45111	13,79442	13,163	12,50299	12,04158
23	14,85684	14,14777	13,48807	12,79245	12,30338
24	15,24696	14,49548	13,79864	13,06682	12,55035
25	15,62208	14,82821	14,09391	13,3688	12,78335
26	15,98277	15,14661	14,37518	13,57338	13,00316
27	16,32959	15,4513	14,64303	13,80702	13,21053
28	16,66306	15,74287	14,89813	14,02848	13,40616
29	16,98371	16,02189	15,14107	14,23838	13,59072
30	17,20202	16,28889	15,37245	14,43733	13,76483
31	17,58849	16,54439	15,59281	14,6259	13,92908
32	17,87355	16,78889	15,80268	14,80463	14,08398
33	18,14764	17,02286	16,00255	14,97404	14,22917
34	18,4112	17,24676	16,1929	15,13461	14,36613
35	18,66461	17,46101	16,37419	15,2868	14,49533
36	18,90828	17,66604	16,54685	15,43105	14,61722
37	19,14258	17,86224	16,71129	15,56779	14,73211
38	19,36787	18,04999	16,86789	15,6974	14,84048
39	19,58448	18,22965	17,01704	15,82024	14,9427
40	19,79277	18,40158	17,15909	15,93667	15,03913

TABLE V. *The annuity which £1 will purchase for any number of years to come, from 1 to 40.*

yrs.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.
1	1,04	1,045	1,05	1,055	1,06
2	,5302	,534	,5378	,54162	,54544
3	,36035	,36377	,36721	,37065	,37411
4	,27549	,27874	,28201	,28582	,28859
5	,22463	,22779	,23097	,23487	,23739
6	,19076	,19388	,19702	,20092	,20336
7	,16661	,1697	,17282	,17671	,17913
8	,14853	,15161	,15473	,15859	,16103
9	,13449	,13757	,14069	,14455	,14702
10	,12329	,12638	,1295	,13334	,13587
11	,11415	,11725	,12039	,12424	,12679
12	,10655	,10967	,11282	,11667	,11927
13	,10014	,10327	,10645	,11031	,11296
14	,09467	,09782	,10102	,10489	,10758
15	,08994	,09311	,09624	,10022	,10296
16	,08582	,08901	,09227	,0962	,09895
17	,0822	,08542	,0887	,0926	,09544
18	,07899	,08224	,08555	,08947	,09235
19	,07614	,07941	,08274	,08699	,08962
20	,07359	,07688	,08024	,08427	,08718
21	,07128	,0746	,078	,08198	,085
22	,0692	,07254	,07597	,07998	,08303
23	,06731	,07068	,07414	,07825	,08128
24	,06559	,06899	,07247	,07653	,07968
25	,06401	,06744	,07095	,07503	,07823
26	,06257	,06602	,06956	,07367	,0769
27	,06124	,06472	,06829	,07242	,0757
28	,06001	,06352	,06712	,07128	,07459
29	,05888	,06241	,06604	,07023	,07358
30	,05783	,06139	,06505	,06926	,07272
31	,05685	,06044	,06413	,06837	,07179
32	,05595	,05956	,06328	,06754	,071
33	,0551	,05874	,06249	,06678	,07027
34	,05431	,05798	,06175	,06607	,06959
35	,05358	,05727	,06107	,06541	,06899
36	,05289	,0566	,06043	,0648	,06839
37	,05224	,05224	,05984	,06423	,06785
38	,05163	,0554	,05928	,0637	,06735
39	,05106	,05485	,05876	,06321	,06689
40	,05052	,05434	,05828	,06274	,06646

CIRCULATING DECIMALS

Are produced from Vulgar Fractions, whose denominators do not measure their numerators, and are distinguished by the continual repetition of the same figures.

1. The circulating figures are called *repetends;* and, if one figure only repeats, it is called a *single repetend:* As ,1111 &c. ,6666 &c.

2. A *compound repetend* has the same figures circulating alternately: As ,010101 &c. ,379379379 &c.

3. If other figures arise before those which circulate, the decimal is called a *mixed repetend;* thus, ,375555 &c. is a *mixed single repetend.* and ,378123123 &c. a *mixed compound repetend.*

4. A single repetend is expressed by writing only the circulating figure with a point over it; thus, ,1111 &c. is denoted by $,\dot{1}$, and ,6666 &c. by $,\dot{6}$.

5. Compound repetends are distinguished by putting a point over the first and last repeating figures; thus, ,010101 &c. is written $,\dot{0}\dot{1}$ and .379379379 &c. thus $,\dot{3}7\dot{9}$.

6. *Similar circulating decimals* are such as consist of the same number of figures, and begin at the same place, either before or after the decimal point; thus, $,\dot{3}$ and $,\dot{5}$ are similar circulates; as are also $\dot{3},\dot{5}4$ and $\dot{7},\dot{3}6$, &c.

7. *Dissimilar repetends* consist of an unequal number of figures, and begin at different places.

8. *Similar and conterminous circulates* are such as begin and end at the same place; as $47,34\dot{5}7\dot{6}$, $9,73\dot{5}2\dot{8}$ and $,05\dot{4}6\dot{3}$, &c.

REDUCTION *of* CIRCULATING DECIMALS.

CASE I.

To reduce a simple Repetend to its equivalent Vulgar Fraction.

RULE* 1.—Make the given decimal the numerator, and let the denominator be a number, consisting of so many nines as there are recurring places in the repetend. 2. If

* If unity, with cyphers annexed, be divided by 9 *ad infinitum*, the quotient will be 1 continually; that is, if $\frac{1}{9}$ be reduced to a decimal, it will produce the circulate $,\dot{1}$, and since $,\dot{1}$ is the decimal equivalent to $\frac{1}{9}$, $,\dot{2}$ will $=\frac{2}{9}$, $\dot{3}=\frac{3}{9}$, and so on till $\dot{9}=\frac{9}{9}=1$. Therefore every single repetend is equal to a vulgar fraction, whose numerator is the repeating figure, and denominator 9.

Again, $\frac{1}{99}$ or $\frac{1}{999}$, being reduced to decimals, make ,010101 &c. and ,001001001 &c. *ad infinitum* $=,\dot{0}\dot{1}$ and $,\dot{0}0\dot{1}$; that is, $\frac{1}{99}=,\dot{0}\dot{1}$, and $\frac{1}{999}=,\dot{0}0\dot{1}$, consequently $\frac{2}{99}=,\dot{0}\dot{2}$, $\frac{3}{99}=,\dot{0}\dot{3}$, &c. and $\frac{2}{999}=,\dot{0}0\dot{2}$, $\frac{3}{999}=,\dot{0}0\dot{3}$, &c, and the same will hold universally.

2. If there be integral figures in the circulate, ſo many cyphers muſt be annexed to the numerator as the higheſt place of the repetend is diſtant from the decimal point.

E X A M P L E S.

1. Required the leaſt vulgar fractions equal to $,\dot{3}$ and $,\dot{3}2\dot{4}$.

$$,\dot{3} = \tfrac{3}{9} = \tfrac{1}{3}\text{; and } ,\dot{3}2\dot{4} = \tfrac{324}{999} = \tfrac{12}{37}\ \textit{Anſ.}\ \tfrac{1}{3}\text{ and }\tfrac{12}{37}.$$

2. Reduce $,\dot{7}$ to its equivalent vulgar fraction. *Anſ.* $\frac{7}{9}$.

3. Reduce $\dot{2},3\dot{7}$ to its equivalent vulgar fraction. *Anſ.* $\frac{2370}{999}$.

4. Required the leaſt vulgar fraction equal to $,\dot{3}8461\dot{5}$. *Anſ.* $\frac{5}{13}$.

C A S E II.

To reduce a mixed Repetend to its equivalent Vulgar Fraction.

RULE* 1.—To ſo many nines as there are figures in the repetend, annex ſo many cyphers as there are finite places, (that is, as there are decimal places before the repetend) for a denominator.

2. Multiply the nines in the ſaid denominator by the finite part, and add the repeating decimals to the product for the numerator.

3. If the repetend begins in ſome integral place, the finite value of the circulating part muſt be added to the finite part.

E X A M P L E S.

1. What is the vulgar fraction equivalent to $,15\dot{3}$?

There being 1 figure in the repetend, and 2 finite places, I annex 2 cyphers to 9 for a denominator, viz. 900; then I multiply the 9 in the denominator by the two figures in the finite part, and add the repeating figure for a numerator; thus, $9 \times 15 + 3 = 138$ numerator.

Therefore, $,15\dot{3} = \frac{138}{900} = \frac{23}{150}$ the *Anſ.*

2. What is the leaſt vulgar fraction equal to $,41\dot{2}\dot{3}$? *Anſ.* $\frac{4079}{9990}$.

C A S E III.

To make any number of diſſimilar repetends ſimilar and conterminous; that is, of an equal number of places.

RULE

* In like manner for a mixed circulate; conſider it as diviſible into its finite and circulating parts, and the ſame principle will be ſeen to run through them alſo; thus the mixed circulate $1\dot{3}$ is diviſible into the finite decimal $,1$, and the repetend $,0\dot{3}$; but $,1 = \frac{1}{10}$, and $,\dot{0}3$ would be equal to $\frac{3}{9}$ provided the circulation began immediately after the place of units; but as it begins after the place of tenths, it is $\frac{3}{9}$ of $\frac{1}{10} = \frac{3}{90}$, and ſo the vulgar fraction $= ,1\dot{3}$ is $\frac{1}{10} + \frac{3}{90} = \frac{9}{90} + \frac{3}{90} = \frac{12}{90}$, and is the ſame as by the rule.

RULE.*

Change them into other repetends, which ſhall each conſiſt of ſo many figures, as the leaſt common multiple of the ſums of the ſeveral numbers of places, found in all the repetends, contains units.

EXAMPLES.

1. Make $6,\dot{3}1\dot{7}$; $3,4\dot{5}$; $52,3$; $191,\dot{0}\dot{3}$; $,\dot{0}5\dot{7}$; $5,\dot{3}$ and $1,35\dot{9}$ ſimilar and conterminous.

Here, in the firſt repetend, there are three places, in the ſecond, one, in the third, none, in the fourth, two, in the fifth, three, in the ſixth, one, and in the ſeventh, one.

Now find the leaſt common multiple of theſe ſeveral ſums, thus:

$$3\,\big)\,\underline{3,\ 1,\ 2,\ 3,\ 1,\ 1}$$
$$\quad 1,\ 1,\ 2,\ 1,\ 1,\ 1$$

and $2\times 3 = 6$ units; therefore, the ſimilar and conterminous repetends muſt contain 6 places.†

Diſſimilar made ſimilar and conterminous.

$$6,\dot{3}1\dot{7} = 6,31\dot{7}3173\dot{1}$$
$$3,4\dot{5} = 3,45\dot{5}5555\dot{5}$$
$$52,3 = 52,3\dot{0}0000\dot{0}$$
$$191,\dot{0}\dot{3} = 191,03\dot{0}3030\dot{3}$$
$$,\dot{0}5\dot{7} = ,05\dot{7}0570\dot{5}$$
$$5,\dot{3} = 5,33\dot{3}3333\dot{3}$$
$$1,35\dot{9} = 1,35\dot{9}9999\dot{9}$$

2. Make $,\dot{5}3\dot{1}$, $,73\dot{4}\dot{8}$, $,\dot{0}\dot{7}$ and $,05\dot{0}\dot{3}$ ſimilar and conterminous.

CASE IV.

To find whether the decimal fraction, equal to a given vulgar one, be finite or infinite, and how many places the repetend will conſiſt of.

RULE

* Any given repetend whatever, whether ſingle, compound, pure, or mixed, may be transformed into another repetend, which ſhall conſiſt of an equal or greater number of figures at pleaſure; thus $,\dot{3}$ may be transformed into $,3\dot{3}$, or $,\dot{3}\dot{3}3$, &c. alſo $,\dot{7}\dot{9} = ,\dot{7}97\dot{9} = ,7\dot{9}\dot{7}$, and ſo on.

† The learner may obſerve that the ſimilar and conterminous repetends begin juſt ſo far from unity, as is the fartheſt among the diſſimilar repetends; and is ſo in all caſes.

RULE* 1.—Reduce the given fraction to its least terms, and divide the denominator by 2, 5 or 10, as often as possible.

2. Divide 9999, &c. by the former result, till nothing remain, and the number of 9s used will shew the number of places in the repetend; which will begin after so many places of figures as there were 10s, 2s, or 5s, divided by.

If the whole denominator vanish in dividing by 2, 5 or 10, the decimal will be finite, and will consist of so many places as you perform divisions.

EXAMPLES.

1. Required to find whether the decimal equal to $\frac{475}{2800}$ be finite or infinite, and if infinite, how many places that repetend will consist of.

First $25)\frac{475}{2800} = \frac{19}{112}$ $\quad 2)112 = \overset{(2)}{56} = \overset{(2)}{28} = \overset{(2)}{14} = 7.$

Then, $\frac{7)999999}{142857}$; therefore, because the denominator 112 did not vanish in dividing by 2, the decimal is infinite, and, as six 9s were used, the circulate consists of 6 places, beginning at the fifth place, because four 2s were used in dividing.

2. Let $\frac{2}{11}$ be the fraction proposed.

3. Let $\frac{3}{5}$ be the fraction proposed.

ADDITION OF CIRCULATING DECIMALS.

RULE 1.—Make the repetends *similar and conterminous*, and find their sum as in common addition.

2. Divide this sum (of the repetends only) by so many nines as there are places in the repetend, and the remainder is the repetend of their sum; which must be set under the figures added, with cyphers on the left hand, when it has not so many places as the repetends.

3. Carry the quotient of this division to the next column, and proceed with the rest, as infinite decimals. EXAMPLES.

* In dividing 1,000, &c. by any prime number whatever, except 2 or 5, the figures in the quotient will begin to repeat over again as soon as the remainder is 1: And since 999, &c. is less than 1000, &c. by 1, therefore 999, &c. divided by any number whatever, will, when the repeating figures are at their period, leave 0 for a remainder.

Now, whatever number of repeating figures we have, when the dividend is 1, there will be exactly the same number, when the dividend is any other number whatever.

Thus, let ,390539053905 &c. be a circulate, whose repeating part is 3905. Now every repetend (3905,) being equally multiplied, must give the same product: For although these products will consist of more places, yet the overplus in each, being alike, will be carried to the next, by which mean, each product will be equally increased, and consequently every four places will continue alike. And the same will hold for any other number.

Now from hence it appears that the dividend may be altered at pleasure, and the number of places in the repetend will still be the same; thus $\frac{1}{11} = \dot{9}\dot{0}$ and $\frac{4}{11}$; or $\frac{1}{11} \times 4 = \dot{3}\dot{6}$, whence the number of places in each are alike.

EXAMPLES.

1. Let $5,\dot{3}+59,4\dot{3}5\dot{6}+397,\dot{6}+519+,\dot{3}\dot{9}+217,\dot{5}$ be added together.

$$
\begin{array}{rcr}
5,\dot{3} & = & 5,3\dot{3}3333\dot{3} \\
59,4\dot{3}5\dot{6} & = & 59,4\dot{3}5635\dot{6} \\
397,\dot{6} & = & 397,6\dot{6}6666\dot{6} \\
519 & = & 519,0\dot{0}0000\dot{0} \\
,\dot{3}\dot{9} & = & ,3\dot{9}3939\dot{3} \\
217,\dot{5} & = & \underline{217,5\dot{5}5555\dot{5}}
\end{array}
$$

1199,3851303 the sum.

In this question, the sum of the repetends is 2851303, which, divided by 999999, gives 2 to carry to the next column 5,3,0 &c. and the remainder is 851305.

2. Let $3275,3\dot{1}9+36,4\dot{5}+123,1\dot{9}+5,3173+112,35\dot{1}3+11,13\dot{1}$ $+,\dot{1}25+29,\dot{1}0053$ be added together. *Ans.* $3593,0004\dot{2}$.

SUBTRACTION OF CIRCULATING DECIMALS.

RULE.—Make the repetends *similar* and *conterminous*, and subtract as usual, observing, that if the repetend of the number to be subtracted be greater than the repetend of the number it is to be taken from, then the right hand of the remainder must be less by unity than it would be if the expressions were finite.

EXAMPLES.

1. From $57,\dot{0}\dot{3}$ take $29,73\dot{5}8\dot{7}$

$$
\begin{array}{rcr}
57,\dot{0}\dot{3} & = & 57,0\dot{3}03\dot{0} \\
29,73\dot{5}8\dot{7} & = & \underline{29,73\dot{5}8\dot{7}}
\end{array}
$$

$27,29\dot{4}4\dot{2}$ the difference.

2. From $325,1\dot{7}$ take $137,\dot{5}819$— *Ans.* $187,595\dot{7}$.

MULTIPLICATION

MULTIPLICATION OF CIRCULATING DECIMALS.

RULE 1.—Turn both the terms into their equivalent vulgar fractions, and find the product of thofe fractions as ufual.

2. Turn the vulgar fraction, expreffing the product, into an equivalent decimal one, and it will be the product required.

EXAMPLES.

1. Multiply $\dot{5}\dot{4}$ by $,1\dot{5}$. $\dot{5}\dot{4} = \frac{54}{99} = \frac{6}{11}$ and $,1\dot{5} = \frac{14}{90} = \frac{7}{45}$

$\frac{6}{11} \times \frac{7}{45} = \frac{42}{495} = ,0\dot{8}\dot{4}$ the product.

2. Multiply $378,\dot{5}$ by $23,\dot{6}$— *Anf.* $8959,\dot{1}48$.

DIVISION OF CIRCULATING DECIMALS.

RULE 1.—Change both the divifor and dividend into their equivalent vulgar fractions, and find their quotient as ufual.

2. Turn the vulgar fraction, expreffing the quotient, into its equivalent decimal, and it will be the quotient required.

EXAMPLES.

1. Divide $,\dot{5}\dot{4}$ by $,1\dot{5}$.

$,\dot{5}\dot{4} = \frac{54}{99} = \frac{6}{11}$ and $,1\dot{5} = \frac{14}{90} = \frac{7}{45}$

$\frac{6}{11} \div \frac{7}{45} = \frac{6}{11} \times \frac{45}{7} = \frac{270}{77} = 3\frac{39}{77} = 3,\dot{5}0649\dot{3}$ the quotient.

2. Divide $345,\dot{8}$ by $\dot{6}$. *Anf.* $518,8\dot{3}$.

ALLIGATION

Is the method of mixing two or more fimples of different qualities, fo that the compofition may be of a mean or middle quality: It confifts of two kinds, viz. Alligation Medial and Alligation Alternate.

ALLIGATION MEDIAL

Is, when the quantities and prices of feveral things are given, to find the mean price of the mixture compounded of thofe things.

RULE.

As the fum of the quantities, or the whole compofition, is to their total value; fo is any part of the compofition to its mean price or value.

EXAMPLES.

EXAMPLES.

1. A tobacconiſt would mix 60lb of tobacco, at 6*d.* per lb, with 50lb at 1*s.* 40lb at 1*s.* 6*d.* and 30lb at 2*s.* per lb : What is 1lb of this mixture worth ?

lb		s.	d.		£.	s.
60	at	0	6	is	1	10
50	—	1	0	—	2	10
40	—	1	6	—	3	0
30	—	2	0	—	3	0
Sum of the ſimples, 180	Total val.				10	0

lb £. lb
As 180 : 10 :: 1
1
—
10
20
—
180)200(1*s.*
180
—
20
12
180)240(1$\frac{1}{3}$*d.*
180
—
60

s. *d.*
Anſ. 1 1$\frac{1}{3}$ per lb.

2. A farmer would mix 20 buſhels of wheat at 6*s.* per buſhel, 16 buſhels of rye at 4*s.* per buſhel, 12 buſhels of barley at 3*s.* per buſhel, and 8 buſhels of oats at 2*s.* per buſhel ; What is the value of one buſhel of this mixture ? *Anſ.* 4*s.* 2$\frac{1}{2}$*d.*

3. A wine merchant mixes 12 gallons of wine, at 75*c.* per gallon, with 24 gallons, at 90*c.* and 16 gallons at 1*D.* 10*c.* ; What is a gallon of this compoſition worth ? *Anſ.* 92*c.* 6*m.*

4. A goldſmith melted together 8*oz.* of gold of 22 carats fine, 1lb 8*oz.* of 21 carats fine, and 10*oz.* of 18 carats fine : Pray what is the quality, or fineneſs of the compoſition ?

$$\frac{8\times 22+20\times 21+10\times 18}{8+20+10} = 20\tfrac{8}{19} \text{ carats fine, } Anſ.$$

5. A refiner melts 5lb of gold of 20 carats fine with 8lb of 18 carats fine ; How much alloy muſt he put to it, to make it 22 carats fine ?

$$22-\overline{\overline{5\times 20}+\overline{8\times 18}}\div\overline{5+8} = 3\tfrac{3}{13}$$

Anſwer, It is not fine enough by 3$\frac{3}{13}$ carats, ſo that no alloy muſt be added, but more gold.

ALLIGATION ALTERNATE*

Is the method of finding what quantity of each of the ingredients, whoſe rates are given, will compoſe a mixture of a given rate : So that it is the reverſe of Alligation Medial, and may be proved by it.

CASE

* *Demon.* By connecting the leſs rate with the greater, and placing the difference between them and the mean rate alternately, or one after the other in turn, the quantities reſulting are ſuch that there is preciſely as much gained by one quantity as is loſt by the other, and therefore the gain and loſs, upon the whole, are equal, and are exactly the propoſed rate.

In

CASE I.

The whole work of this caſe conſiſts in linking the extremes truly together and taking the differences between them and the mean price, which differences are the quantities ſought.

RULE 1.—Place the ſeveral prices of the ſimples, being reduced to one denomination, in a column under each other, the leaſt uppermoſt, and ſo gradually downward, as they increaſe, with a line of connection at the left hand, and the mean price at the left hand of all.

2. Connect, with a continued line, the price of each ſimple, or ingredient, which is leſs than that of the compound, with one or any number of thoſe, which are greater than the compound, and each greater rate or price with one, or any number of the leſs.

3. Place the difference, between the mean price (or mixture rate) and that of each of the ſimples, oppoſite to the rates with which they are connected.

4. Then, if only one difference ſtand againſt any rate, it will be the quantity belonging to that rate; but if there be ſeveral, their ſum will be the quantity.

EXAMPLES.

1. A merchant has ſpices, ſome at 1*s.* 6*d.* per ℔, ſome at 2*s.* ſome at 4*s.* and ſome at 5*s.* per ℔; How much of each ſort muſt he mix, that he may ſell the mixture at 3*s.* 4*d.* per ℔?

	d.	℔		*s.* *d.*	
Mean rate 40*d.*	18	20	at	1 6	per ℔.
	24	8	—	2 0	
	48	16	—	4 0	
	60	22	—	5 0	

	d.	℔		*s.* *d.*	
40*d.*	18	8	at	1 6	per ℔.
	24	20	—	2 0	
	48	22	—	4 0	
	60	16	—	5 0	

	d.		℔		*s.* *d.*	
d. 40	18	20+ 8	28	at	1 6	per ℔.
	24	8	8	—	2 0	
	48	16+22	38	—	4 0	
	60	22	22	—	5 0	

	d.		℔		*s.* *d.*	
d. 40	18	20	20	at	1 6	per ℔. 40*d.*
	24	8+20	28	—	2 0	
	48	16	16	—	4 0	
	60	22+16	38	—	5 0	

In like manner, let the number of ſimples be what it may, and with how many ſoever each one is linked, ſince it is always a leſs with a greater than the mean price, there will be an equal balance of loſs and gain between every two, and conſequently an equal balance on the whole.

It is obvious from the rule, that queſtions of this ſort admit of a great variety of anſwers; for having found one anſwer, we may find as many more as we pleaſe by only multiplying or dividing each of the quantities found by 2, 3, 4, &c. the reaſon of which is evident; for if two quantities of two ſimples make a balance of loſs and gain with reſpect to the mean price, ſo muſt alſo the double or triple, the half or third part, or any other ratio of theſe quantities, and ſo on *ad infinitum.*

If any one of the ſimples be of little or no value with reſpect to the reſt, its rate is ſuppoſed to be nothing, as water mixed with wine, and alloy with gold and ſilver.

40*d.*	18	20+ 8	28 at 1 6	per ℔.	
	24	8+20	28 — 2 0		
	48	16+22	38 — 4 0		
	60	22+16	38 — 5 0		

Note. These five answers arise from as many various ways of linking the rates of the ingredients together.

2.* A merchant has Canary wine, at 3*s.* per gallon, Sherry, at 2*s.* 1*d.* and claret at 1*s.* 5*d.* per gallon ; How much of each sort must he take, to sell it at 2*s.* 4*d.* per gallon ?

Mean rate 28*d.*	36	3+11	14 at 3 0	per gallon.
	25	8	8 — 2 1	
	17	8	8 — 1 5	

3. How much barley at 2*s.* 4*d.* rye at 3*s.* 9*d.* and wheat at 5*s.* per bushel, must be mixed together, that the compound may be worth 4*s.* 4*d.* per bushel ?

Ans. 8 bushels of barley, 8 of rye, and 31 of wheat.

4. A goldsmith would mix gold of 19 carats fine, with some of 16, 18, 23 and 24 carats fine, so that the compound may be 21 carats fine ; What quantity of each must he take ?

21	16
	18
	19
	23
	24

Ans. 5*oz.* of 16 carats fine, 5*oz.* of 18, 5*oz.* of 19, 10*oz.* of 23, and 10*oz.* of 24 carats fine.

5. It is required to mix several sorts of wine, at 60*c.* 90*c.* and 1*D.* 15*c.* per gallon, with water, that the mixture may be worth 75*c.* per gallon ; Of how much of each sort must the composition consist ?

75	0
	60
	90
	115

Ans. 40*galls.* of water, 15*galls.* of wine, at 60*c.* 15*galls.* do. at 90*c.* and 75*galls.* do at 1*D.* 15*c.*

CASE II.

When the rates *of* all *the* ingredients, *the* quantity *of but* one *of them, and the* mean rate *of the whole mixtures are given, to find the several quantities of the rest, in proportion to the quantity given.*

Rule.—Take the differences between each price, and the mean rate, and place them alternately, as in Case 1. Then, As the difference standing against that simple, whose quantity is given, is to that quantity ; so is each of the other differences, severally, to the several quantities required.

Examples.

1. A merchant has 40℔ of tea, at 6*s.* per ℔, which he would mix with some at 5*s.* 8*d.* some at 5*s.* 2*d.* and some at 4*s.* 6*d.* How much

* *Note,* the 2d and 3d questions admit but of one way of linking, and so but of one answer ; yet all numbers in the same proportion between themselves, as the numbers, which compose the answer, will likewise satisfy the condition of the question.

much of each sort must he take, to mix with the 40lb, that he may sell the mixture at 5*s.* 5*d.* per lb?

$$65\left\{\begin{array}{l}54\\62\\68\\72\end{array}\right.\quad\begin{array}{r}7+3\\3+7\\3+11\\11+3\end{array}\left|\begin{array}{l}\text{lb}\\10\\10\\14\\14\end{array}\right.$$
stands against the given quantity.

$$\text{As } 14 : 40 :: \left\{\begin{array}{lllll}\text{lb} & & \text{lb} & & s.\ d.\\ 10 & : & 28\frac{8}{14} & \text{at} & 4\ \ 6\\ 10 & : & 28\frac{8}{14} & — & 5\ \ 2\\ 14 & : & 40 & — & 5\ \ 8\end{array}\right\}\text{per lb.}$$

2. A farmer being determined to mix 20 bushels of oats, at 2*s.* per bushel, with barley, at 2*s.* 6*d.* rye at 4*s.* and wheat, at 5*s.* 6*d.* per bushel; I demand the quantity of each, which must be mixed with the 20 bushels of oats, that the whole quantity may be worth 4*s.* 6*d.* per bushel? *Ans.* 20 of barley, 20 of rye, and 100 of wheat.

3. How much gold of 16, 20 and 24 carats fine, and how much alloy, must be mixed with 10*oz.* of 18 carats fine, that the composition may be 22 carats fine?

Ans. 10*oz.* of 16 carats fine, 10 of 20, 170 of 24, and 10 of alloy.

ALTERNATION TOTAL.*

CASE III.

When the rates *of the several ingredients, the* quantity *to be compounded, and the* mean rate *of the whole mixture are given, to find how much of each sort will make up the quantity.*

RULE.—Place the differences between the mean rate, and the several prices alternately, as in Case 1;—then, As the sum of the quantities, or differences thus determined, is to the given quantity,

* To this Case belongs that curious question concerning king Hiero's crown.

Hiero, king of Syracuse, gave orders for a crown to be made, entirely of pure gold; but suspecting the workmen had debased it by mixing with it silver or copper, he recommended the discovery of the fraud to the famous Archimides, and desired to know the exact quantity of alloy in the crown.

Archimides, in order to detect the imposition, procured two other masses, the one of pure gold, and the other of silver, or copper, and each of the same weight with the former; and, by putting each separately into a vessel *full* of water, the quantity of water expelled by them, determined their specifick bulks; from which and their given weights it is easier to determine the quantities of gold and alloy in the crown by this case of Alligation, than by an Algebraic process.

Suppose the weight of each mass to have been 5lb, the weight of the water expelled by the alloy, 23*oz.* by the gold 13*oz.* and by the crown 16*oz.* that is, that their specifick bulks were as 23, 13, and 16, then, What were the quantities of gold and alloy respectively in the crown?

Here, the rates of the simples are 23 and 13, and of the compound 16; whence,

$$16\left\{\begin{array}{l}13\\23\end{array}\right.\begin{array}{l}7\text{ of gold}\\3\text{ of alloy}\end{array}\Big\}$$
And the sum of these is $7+3=10$, which should have been but 5, whence, by the rule,

$$10 : 5 :: \left\{\begin{array}{l}7 : 3\frac{1}{2}\text{lb of gold}\\3 : 1\frac{1}{2}\text{lb of alloy}\end{array}\right\}\text{ the } Ans.$$

tity, or whole compoſition ; ſo is the difference of each rate, to the required quantity of each rate.

EXAMPLES.

1. Suppoſe I have 4 ſorts of currants, at 8*d.* 12*d.* 18*d.* and 22*d.* per ℔ ; the worſt will not ſell, and the beſt are too dear ; I therefore conclude to mix 120 ℔, and ſo much of each ſort, as to ſell them at 16*d.* per ℔ ; How much of each ſort muſt I take ?

$$\begin{array}{rcc} & d. & ℔ \\ 16d. & \left\{\begin{array}{c}8\\12\\18\\22\end{array}\right. & \begin{array}{c}6\\2\\4\\8\end{array} \\ & & \overline{} \\ & \text{Sum} = & 20 \end{array} \quad \begin{array}{c} ℔ \quad ℔ \\ \text{As } 20 : 120 :: \end{array} \left\{\begin{array}{l} ℔ \quad ℔ \\ 6 : 36 \text{ at } 8d. \\ 2 : 12 \text{ — } 12d. \\ 4 : 24 \text{ — } 18d. \\ 8 : 48 \text{ — } 22d. \\ \quad\;\, \overline{120} \end{array}\right\} \text{per ℔.}$$

2. A goldſmith has ſeveral ſorts of gold ; viz. of 15, 17, 20 and 22 carats fine, and would melt together, of all theſe ſorts, ſo much as may make a maſs of 40*oz.* 18 carats fine ; How much of each ſort is required ?

Anſ. 16*oz.* 15 carats fine, 8*oz.* 17, 4*oz.* 20, and 12*oz.* of 22 carats fine.

3. A merchant would mix 4 ſorts of wine, of ſeveral prices, viz. at 4*s.* 6*s.* 8*s.* and 9*s.* per gallon ; of theſe he would have a mixture of 60 gallons, worth 7*s.* per gallon ; What quantity of each ſort muſt he have ?

Anſ. $17\frac{1}{7}$*gal.* at 4*s.* $8\frac{4}{7}$ at 6*s.* $8\frac{4}{7}$ at 8*s*, and $25\frac{5}{7}$ at 9*s.*

4. How many gallons of water, of no value, muſt be mixed with wine at 4*s.* per gallon, ſo as to fill a veſſel of 80 gallons, that may be afforded at 2*s.* 9*d.* per gallon ?

$$\begin{array}{rcc} & & Gal. \\ 33 & \left\{\begin{array}{c}0\\48\end{array}\right. & \begin{array}{c}15\\33\end{array} \\ & & \overline{} \\ & \text{Sum} & 48 \end{array} \quad \begin{array}{c} Gal.\;Gal. \\ \text{As } 48 : 80 :: \end{array} \left\{\begin{array}{l} Gal. \\ 15 : 25 \text{ gallons of water} \\ 33 : 55 \text{ gallons of wine} \end{array}\right\} Anſ.$$

CASE IV.*

When more than one of the ſimples are limited.

RULE.—Find, by Alligation Medial, what will be the *rate* of a mixture made of the *given quantities* of the *limited ſimples* only ; then, conſider this as the *rate* of a *limited ſimple*, whoſe *quantity* is the ſum of the firſt given *limited ſimples*, from *which* and the *rates* of the *unlimited ſimples*, by Caſe 2d, calculate the quantity.

EXAMPLES.

1. How much wine, at 4*s.* 6*d.* and at 5*s.* per gallon, muſt be mixed with 6 gallons at 4*s.* and 6 gallons at 3*s.* per gallon, that the mixture may be worth 4*s.* 4*d.* per gallon ?

Limited

* The three laſt Caſes need no demonſtration, as the 2d and 3d evidently reſult from the firſt, and the laſt, from Alligation Medial and the ſecond Caſe in Alternate.

$$\text{Limited ſimples}\left\{\begin{array}{l} 6 \text{ gallons at } 4s. = 24 \\ 6 \text{ gallons at } 3s. = 18 \end{array}\right\} \quad \begin{array}{l} \textit{Gal. s. Gal.} \\ As\ 12 : 42 :: 1 : 3/6 \textit{ per gal.} \end{array}$$

$$\begin{array}{cc} \overline{12} & \overline{42} \end{array}$$

Now, having found the rate of the limited ſimples, the queſtion may ſtand thus: How much wine at 4*s.* 6*d.* and 5*s.* per gallon, muſt be mixed with 12 gallons, at 3*s.* 6*d.* per gallon, that the mixture may be worth 4*s.* 4*d.* per gallon?

$$52\left\{\begin{array}{l} 42 \\ 54 \\ 60 \end{array}\right. \quad \begin{array}{c} 2+8 \\ 10 \\ 10 \end{array} \left| \begin{array}{c} 10 \\ 10 \\ 10 \end{array}\right. \quad \text{As } 10 : \left\{\begin{array}{c} 10 \\ 10 \end{array}\right\} :: 12 : \left\{\begin{array}{l} 12 \text{ gal. at } 4/6 \\ 12 \text{ gal. at } 5s. \end{array}\right\} \begin{array}{r} \text{per gal.} \\ \textit{Anſ.} \end{array}$$

2. How much gold, of 14 and 16 carats fine, muſt be mixed with 6*oz.* of 19, and 12 of 22 carats fine, that the compoſition may be 20 carats fine?

Anſ. $1\frac{8}{10}$*oz.* of each ſort.

POSITION.

Poſition is a rule, which, by falſe or ſuppoſed numbers, taken at pleaſure, diſcovers the true ones required. It is divided into two parts; *ſingle* and *double.*

SINGLE POSITION.

Single Poſition teaches to reſolve thoſe queſtions, whoſe reſults are proportional to their ſuppoſitions: Such are thoſe which require the multiplication or diviſion of the number ſought by any propoſed number; or when it is to be increaſed or diminiſhed by itſelf a certain propoſed number of times.

RULE* 1. Take any number, and perform the ſame operations with it, as are deſcribed to be performed in the queſtion.

2. Then ſay, As the ſum of the errors is to the given ſum; ſo is the ſuppoſed number, to the true one required.

Proof. Add the ſeveral parts of the ſum together, and if it agrees with the ſum, it is right.

EXAMPLES.

1. A ſchoolmaſter, being aſked how many ſcholars he had, ſaid, If I had as many more as I now have, three quarters as many, half

* The reaſon of this rule is obvious, it being evident that the reſults are proportional to the ſuppoſitions.

$$\text{Thus,}\left\{\begin{array}{l} nx : x :: na : a \\ \dfrac{x}{n} : x :: \dfrac{a}{n} : a \\ \dfrac{x}{n} \pm x \text{ \&c.} : x :: \dfrac{a}{n} \pm \dfrac{a}{m} \text{ \&c.} : a, \text{ and ſo on.} \end{array}\right.$$

half as many, one fourth and one eighth as many, I ſhould then have 435; Of what number did his ſchool conſiſt?

Suppoſe he had 80	As 290 : 435 :: 80	
As many = 80	80	
$\frac{3}{4}$ as many = 60	——	120
$\frac{1}{2}$ as many = 40	29|0)3480|0(120 *Anſwer.*	120
$\frac{1}{4}$ as many = 20	29	90
$\frac{1}{8}$ as many = 10	——	60
——	58	30
290	58	15
	——	——
	0	435 *Proof.*

2. A perſon lent his friend a ſum of money unknown, to receive intereſt for the ſame at 6*l.* per cent. per annum, ſimple intereſt, and, at the end of 12 years, received for principal and intereſt 860*l.* What was the ſum lent? *Anſ.* £500.

3. A, B and C joined their ſtocks, and gained 350*l.* of which A took up a certain ſum, B took up four times ſo much as A, and C, eight times ſo much as B; What ſhare of the gain had each?

	£.	s.	d.	qrs.	
Anſ.	9	9	2	$1\frac{3}{37}$	A's ſhare.
	37	16	9	$0\frac{12}{37}$	B's ditto.
	302	14	0	$2\frac{22}{37}$	C's ditto.

4. A, B, C, and D ſpent 35*s.* at a reckoning, and, being a little dipped, they agreed that A ſhould pay $\frac{2}{3}$, B $\frac{1}{2}$, C $\frac{1}{3}$, and D $\frac{1}{4}$; What did each pay in the above proportion?

		s.	d.
Anſ.	A,	13	4
	B,	10	0
	C,	6	8
	D,	5	0

5. A certain ſum of money is to be divided between 5 men, in ſuch a manner as that A ſhall have $\frac{1}{4}$, B $\frac{1}{5}$, C $\frac{1}{10}$, D $\frac{1}{20}$, and E the remainder, which is 40*l.*; What is the ſum?

Suppoſe 200*l.* then $\frac{1}{4}+\frac{1}{5}+\frac{1}{10}+\frac{1}{20}=120$.

200—120 = 80. As 80 : 40 :: 200 : 100 *Anſ.*

6. A perſon, after ſpending $\frac{1}{2}$ and $\frac{1}{3}$ of his money, had $26\frac{2}{3}$*l.* left; What had he at firſt? *Anſ.* £160.

7. A and B talking of their ages, B ſaid his age was once and an half the age of A; C ſaid his was twice and one tenth the age of both, and that the ſum of their ages was 93; What was the age of each? *Anſ.* A's 12, B's 18, and C's 63 years.

8. A veſſel has 3 cocks, A, B and C; A can fill it in $\frac{1}{2}$ an hour, B in $\frac{1}{4}$ of an hour, and C in $\frac{1}{3}$ of an hour; In what time will they all fill it, together? *Anſ.* $\frac{1}{9}$ hour.

9. A perſon having about him a certain number of dollars, ſaid that

that $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of them would make 57; Pray, how many had he? *Anſ.* 60.

10. A gentleman bought a chaiſe, horſe and harneſs for 100*l.* the horſe coſt $\frac{1}{4}$ more than the harneſs, and the chaiſe $\frac{1}{3}$ more than the horſe; What was the price of each?

Anſ. Harneſs £$25\frac{25}{47}$. Horſe £$31\frac{43}{47}$. Chaiſe £$42\frac{26}{47}$.

11. A and B, having found a purſe of money, diſputed who ſhould have it: A ſaid that $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{1}{20}$ of it amounted to 35*l.* and if B could tell him how much was in it, he ſhould have the whole, otherwiſe he ſhould have nothing; How much did the purſe contain? *Anſ.* £100.

12. A gentleman divided his fortune among his ſons; to A he gave 9*l.* as often as to B 5*l.* and to C, but 3*l.* as often as to B 7*l.* yet C's portion came to $1050\frac{4}{5}$*l.*; What was the whole eſtate? *Anſ.* £$7916\frac{2}{5}$.

13. Seven eighths of a certain number exceeds four fifths by 6; What is that number? *Anſ.* 80.

14. What number is that, which, being increaſed by $\frac{2}{5}$, $\frac{3}{8}$ and $\frac{5}{6}$ of itſelf, the ſum will be $234\frac{1}{4}$? *Anſ.* 90.

DOUBLE POSITION.

Double Poſition teaches to reſolve queſtions by making two ſuppoſitions of falſe numbers.

Thoſe queſtions, in which the reſults are not proportional to their poſitions, belong to this rule: Such are thoſe, in which the number ſought is increaſed or diminiſhed by ſome given number, which is no known part of the number required.

RULE* 1.—Take any two convenient numbers, and proceed with each according to the conditions of the queſtion.

2. Place the reſult or errors againſt their poſitions or ſuppoſed numbers, thus,

$$\begin{array}{ccc} \textit{Poſ.} & & \textit{Err.} \\ 30 & & 12 \\ & \times & \\ 20 & & 6 \end{array}$$

and if the error be too great, mark it with $+$; and if too ſmall with $-$.

3. Multiply

* The rule is founded on this ſuppoſition, that the firſt error is to the ſecond, as the difference between the true and firſt ſuppoſed number is to the difference between the true and ſecond ſuppoſed number: When that is not the caſe, the exact anſwer to the queſtion cannot be found by this rule.

That the rule is true, according to the ſuppoſition, may be thus demonſtrated.

Let A and B be any two numbers produced from a and b by ſimilar operations, it is required to find the number from which N is produced by a like operation.

Put $x =$ number required, and let $N-A = r$, and $N-B = s$. Then, according to the ſuppoſition on which the rule is founded, $r : s :: x-a : x-b$, whence, by multiplying means and extremes, $rx-rb = sx-sa$; and by tranſpoſition, $rx-sx = rb-sa$, and by diviſion, $x = \frac{rb-sa}{r-s} =$ number ſought; and if r and s be both negative, the Theorem is the ſame, and if r or s be negative, x will be equal to $\frac{rb+sa}{r+s}$ which is the rule.

3. Multiply them crosswise; that is, the first position by the last error, and the last position by the first error.

4. If the errors be alike, that is, both too small, or both too great, divide the difference of the products by the difference of the errors, and the quotient will be the answer.

5. If the errors be unlike; that is, one too small, and the other too great, divide the sum of the products by the sum of the errors, and the quotient will be the answer.

Note, When the errors are the same in quantity, and unlike in quality, half the sum of the suppositions is the number sought.

EXAMPLES.

1. A lady bought damask for a gown, at 8*s*. per yard, and, lining for it at 3*s*. per yard; the gown and lining contained 15 yards, and the price of the whole was 3*l*. 10*s*.; How many yards were there of each?

Suppose 6 yards damask, value 48*s*.
Then she must have 9 yards of lining, value 27*s*.
Sum of their values = 75*s*.
So that the first error is 5 too much, or +5
Again, suppose she had 4 yards of damask, value 32*s*.
Then she must have 11 yards of lining, value 33*s*.
Sum of their values = 65*s*.
So that the second error is 5 too little, or —5*s*.

```
       6     5+                        £. s. d.
Then      X          5 yds. at 8s. =  2  0  0
       4     5—     10 yds. at 3s. =  1 10  0
      --    --                        -------
      20    30                        3 10  0 proof.
            20
            --
Sum of err.=5+5=10)50
            --
```

Ans. 5 *yds*. damask, and 15—5 = 10 *yds*. lining.
Or, 6+4÷2 = 5 as before.

2. A and B have the same income; A saves $\frac{1}{8}$ of his; but B, by spending 30*l*. per annum more than A, at the end of 8 years finds himself 40*l*. in debt; What is their income, and what does each spend per annum?

```
         ⎧ 80     120+
Suppose  ⎨     X
         ⎩ 160     40+
```

Ans. Their income is 200*l*. per ann. Also, A spends 175*l*. and B 205*l*. per annum. Then 80—10 = 70 A's expense per annum, and 70+30 = 100, B's expense per annum. Then $\overline{100\times8}-\overline{80\times8}$ = 160, which should have been 40; therefore, 160—40 = 120 more than it should be, for the first error. In like manner proceed for the second error.

3. A

3. A and B laid out equal ſums of money in trade : A gained a ſum equal to $\frac{1}{4}$ of his ſtock, and B loſt 225*l.* then A's money was double that of B ; What did each lay out?

Suppoſe $\left\{\begin{array}{l} 300 \\ 900 \end{array}\right. \times \begin{array}{l} 225+ \\ 225- \end{array}$ *Anſ.* £600.

4. A labourer was hired for 60 days, upon this condition, that, for every day he wrought, he ſhould receive 3*s.* 4*d.* ; and for every day he was idle, ſhould forfeit 1*s.* 8*d.* ; at the expiration of the time he received 3*l.* 15*s.* How many days did he work, and how many was he idle?

Suppoſe he worked $\left\{\begin{array}{l} 20 \\ 40 \end{array}\right. \times \begin{array}{l} 900- \\ 300+ \end{array}$

Anſ. He was employed 35 days, and was idle 25.

5. A gentleman has two horſes of conſiderable value, and a carriage worth 100*l.* ; now, if the firſt horſe be harneſſed in it, he and the carriage together will be triple the value of the ſecond ; but if the ſecond be put in, they will be 7 times the value of the firſt ; What is the value of each horſe?

Suppoſe $\begin{array}{l} 32 \\ 44 \end{array} \times \begin{array}{l} 80- \\ 160- \end{array}$

Anſ. One £20, and the other £40.

6. There is a fiſh, whoſe head is 10 feet long ; his tail is as long as his head and half the length of his body ; and his body as long as the head and tail ; What is the whole length of the fiſh?

Firſt, ſuppoſe the body $20 \times 10-$ | Head = 10
2d ſuppoſe it $30 \times 5-$ | Tail = 30, Body = 40

Head	= 10
Tail	= 30
Body	= 40

Anſ. 80 feet.

7. What number is that, which, being increaſed by its $\frac{1}{2}$, its $\frac{1}{4}$, and 5 more, will be doubled?

Suppoſe $\begin{array}{l} 8 \\ 16 \end{array} \times \begin{array}{l} 3+ \\ 1+ \end{array}$ *Anſ.* 20.

8. A farmer having driven his cattle to market, received for them all 80*l.* being paid at the rate of 6*l.* per ox, 4*l.* per cow, and 1*l.* 10*s.* per calf ; there were as many oxen as cows, and 4 times as many calves as cows ; How many were there of each ſort?

Suppoſe cows $\begin{array}{l} 6 \\ 12 \end{array} \times \begin{array}{l} 16+ \\ 112+ \end{array}$

Anſ. 5 oxen, 5 cows, and 20 calves.

9. A, B and C built a ſhip, which coſt them 1000*l.* of which A paid a certain ſum, B paid 100*l.* more than A, and C 100*l.* more

more than both; having finiſhed her, they fixed her for ſea with a cargo worth twice the value of the ſhip: The outfits and charges of the voyage amounted to $\frac{1}{8}$ of the ſhip; upon the return of which, they found their clear gain to be $\frac{2}{3}$ of $\frac{3}{5}$ of the veſſel, cargo and expenſes; Pleaſe to inform me what the ſhip coſt them, ſeverally; what ſhare each had in her, and what, upon the final adjuſtment of their accompts, they had ſeverally gained?

$$\text{Suppoſe it coſt A}\ \begin{matrix} 100 & & 300- \\ & \times & \\ 200 & & 100+ \end{matrix}$$

A owned $\frac{7}{40}$ of the ſhip, which coſt him 175*l.* and his ſhare of the gain was 218*l.* 15*s.* B owned $\frac{11}{40}$, which coſt 275*l.* and his gain was 343*l.* 15*s.* C owned $\frac{11}{20}$, which coſt 550*l.* and his gain was 687*l.* 10*s.*

PERMUTATIONS *and* COMBINATIONS.

The permutation of quantities is, the ſhewing how many different ways any given number of things may be changed.

This is alſo called *variation*, *alternation*, or *changes*; and the only thing to be regarded here is the order they ſtand in; for no two parcels are to have all their quantities placed in the ſame ſituation.

The Combination of Quantities is the ſhewing how often a leſs number of things can be taken out of a greater, and combined together, without conſidering their places, or the order they ſtand in.

This is, ſometimes, called *election*, or *choice*; and here every parcel muſt be different from all the reſt, and no two are to have preciſely the ſame quantities, or things.

The Compoſition of Quantities is the taking of a given number of quantities out of as many equal rows of different quantities, one out of every row, and combining them together.

Here no regard is had to their places; and it differs from Combination only as that admits but of one row of things.

Combinations of the ſame form are thoſe in which there are the ſame number of quantities, and the ſame repetitions; thus, *abcc*, *bbad*, *deef*, &c. are of the ſame form; but *abbc*, *abbb*, *aacc* are of different forms.

PROBLEM I.

To find the number of permutations, or changes, that can be made of any given number of things, all different from each other.

RULE.*—Multiply all the terms of the natural ſeries of numbers, from 1 up to the given number, continually together, and the laſt product will be the anſwer required.

EXAMPLES.

* The reaſon of this rule may be ſhewn thus, any one thing *a* is capable of one poſition only, as *a*.

Any

EXAMPLES.

1. Chriſt's church, in Boſton, has 8 bells; How many changes may be rung on them?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40320$ *Anſ.*

2. Nine gentlemen met at an inn, and were ſo pleaſed with their hoſt, and with each other, that in a frolic they agreed to tarry ſo long as they, together with their hoſt, could ſit every day in a different poſition at dinner; Pray how long, had they kept their agreement, would their frolic have laſted?

Anſ. $9941\frac{335}{365}$ years.

3. How many changes, or variations, will the alphabet admit of? *Anſ.* 620448401733239439360000.

PROBLEM II.

Any number of different things being given, to find how many changes can be made out of them, by taking any given number of quantities at a time.

RULE.*—Take a ſeries of numbers, beginning at the number of things given, and decreaſing by 1, to the number of quantities to be taken at a time: The product of all the terms will be the anſwer required.

EXAMPLES.

1. How many changes may be rung with 4 bells out of 8?

$$\begin{array}{r} 8 \\ 7 \\ \hline 56 \\ 6 \\ \hline 336 \\ 5 \\ \hline 1680 \end{array}$$

Or, $8 \times 7 \times 6 \times 5$ (= 4 terms) = 1680 *Anſ.*

2. How many words can be made with 6 letters of the alphabet, admitting a number of conſonants may make a word?

Anſ. 96909120.

PROBLEM

Any two things a and b are capable of two variations only; as ab, ba; whoſe number is expreſſed by 1×2.

If there be three things a, b and c; then any two of them, leaving out the third, will have 1×2 variations; and conſequently when the third is taken in, there will be $1 \times 2 \times 3$ variations; and ſo on, as far as you pleaſe.

* This Rule, expreſſed in terms, is as follows; $m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}$ &c. to n terms; whence m = number of things given, and n = quantities to be taken at a time.

PROBLEM III.

Any number of things being given, whereof there are ſeveral things of one ſort, ſeveral of another, &c. to find how many changes may be made out of them all.

RULE* 1.—Take the ſeries $1\times2\times3\times4$, &c. up to the number of things given, and find the product of all the terms.

2. Take the ſeries $1\times2\times3\times4$, &c. up to the number of the given things of the firſt ſort, and the ſeries, $1\times2\times3\times4$, &c. up to the number of the given things of the ſecond ſort, &c.

3. Divide the product of all the terms of the firſt ſeries by the *joint* product of all the terms of the remaining ones, and the quotient will be the anſwer required.

EXAMPLES.

1. How many variations may be made of the letters in the word *Zaphnathpaaneah*?

$1\times2\times3\times4\times5\times6\times7\times8\times9\times10\times11\times12\times13\times14\times15$ (= number of letters in the word) $= 1307674368000$.

$1\times2\times3\times4\times5$ (= number of as) $= 120$
1×2 (= number of ps) $= 2$
1 (= number of ts) $= 1$
$1\times2\times3$ (= number of hs) $= 6$
1×2 (= number of ns) $= 2$

$2\times6\times1\times2\times120 = 2880)1307674368000(454053600$ *Anſ.*

2. How many different numbers can be made of the following figures, 1223334444? *Anſ.* 12600.

PROBLEM IV.

To find the number of combinations of any given number of things, all different from one another, taken any given number at a time.

RULE

* This Rule is expreſſed in terms thus; $\dfrac{1\times2\times3\times4\times5, \text{\&c. to } m.}{1\times2\times3, \text{\&c to } p.\times1\times2\times3, \text{\&c. to } q \text{ \&c.}}$ whence $m =$ number of things given, $p =$ number of things of the firſt ſort, $q =$ number of things of the ſecond ſort, &c.

Any 2 quantities, a, b, both different, admit of 2 changes; but if the quantities are the ſame, or ab become aa, there will be only one alteration, which may be expreſſed by $\dfrac{1\times2}{1\times2} = 1$.

Any 3 quantities, a, b, c, all different from each other, admit of 6 variations; but if the quantities are all alike, or, $a\,b\,c$ become aaa, then the 6 variations will be reduced to 1, which may be expreſſed by $\dfrac{1\times2\times3}{1\times2\times3} = 1$. Again, if two quantities out of three are alike, or abc become aac; then the 6 variations will be reduced to theſe 3, aac, caa, aca, which may be expreſſed by $\dfrac{1\times2\times3}{1\times2} = 3$, and ſo of any greater number.

RULE* 1.—Take the ſeries 1, 2, 3, 4, &c. up to the number to be taken at a time, and find the product of all the terms.

2. Take a ſeries of as many terms, decreaſing by 1, from the given number, out of which the election is to be made, and find the product of all the terms.

3. Divide the laſt product by the former, and the quotient will be the number ſought.

EXAMPLES.

1. How many combinations may be made of 7 letters out of 12?

$1\times2\times3\times4\times5\times6\times7$ (=the number to be taken at a time)=5040.
$12\times11\times10\times9\times8\times7\times6$ (=ſame number from 12) = 3991680.
5040)3991680(792 *Anſ.*

2. How many combinations can be made of 6 letters out of the 24 letters of the alphabet? *Anſ.* 134596.

3. A general was aſked by his king, what reward he ſhould confer on him for his ſervices; the general only required a penny for every file, of 10 men in a file, which he could make out of a company of 90 men; what did it amount to?

Anſ. £23836022841 7*s.* $11\frac{65}{1134}$*d.*

4. A farmer bargained with a gentleman for a dozen ſheep, (at 2 dollars per head) which were to be picked out of 2 dozen; but, being long in chooſing them, the gentleman told him that if he would give him a penny for every different dozen which might be choſen out of the two dozen, he ſhould have the whole, to which the farmer readily agreed; Pray what did they coſt him? *Anſ.* £11267 6*s.* 4*d.*

5. How many locks, whoſe wards differ, may be unlocked with a key of 6 ſeveral wards? *Anſ.* 63: 6 of which may have one ſingle ward, 15 double wards, 20 triple wards, 15 four wards, 6 five wards, and 1 lock, 6 wards.

Wards.		*Locks.*	*Wards.*		*Locks.*

$$\left\{\begin{matrix}1\\2\\3\\4\\5\\6\end{matrix}\right\}\text{in }6=\left\{\begin{matrix}6\\15\\20\\15\\6\\1\end{matrix}\right\}\qquad\left\{\begin{matrix}1\\2\\3\\4\\5\end{matrix}\right\}\text{in }5=\left\{\begin{matrix}5\\10\\10\\5\\1\end{matrix}\right\}$$

PROBLEM

* This Rule, expreſſed algebraically, is $\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}$, &c. to n terms; where m is the number of given quantities, and n, thoſe to be taken at a time.

Note, In any given number of quantities, the number of Combinations increaſes gradually till you come about the mean numbers, and then gradually decreaſe. If the number of quantities be *even*, half the number of places will ſhew the greateſt number of Combinations, that can be made of thoſe quantities; but if odd, then thoſe 2 numbers which are the middle, and whoſe ſum is equal to the given number of quantities, will ſhew the greateſt number of Combinations.

PROBLEM V.

To find the number of combinations of any given number of things, by taking any given number at a time; in which there are ſeveral things of one ſort, ſeveral of another, &c.

RULE.—Find the number of different *forms*, which the things, to be taken at a time, will admit of, in the following manner:

1. Place the things ſo that the greateſt indices may be firſt, and the reſt in order.

2. Begin with the firſt letter, and join it to the ſecond, third, fourth, &c. to the laſt.

3. Join the ſecond letter to the third, fourth, &c. to the laſt; and ſo on till they are all done, always rejecting ſuch combinations as have occurred before; and this will give the combinations of all the twos.

4. Join the firſt letter to every one of the twos; then join the ſecond, third, &c. as before; and it will give the combinations of all the threes.

5. Proceed in the ſame manner to get the combinations of all the fours, fives, &c. and you will at laſt get all the ſeveral *forms* of combination, and the number in each form.

6. Having found the number of combinations in each *form*, add them all together, and the ſum will be the number required.

EXAMPLE.

1. Let the things propoſed be *aaabbc*; It is required to find the number of combinations of every 2, of every 3, and of every 4 of theſe quantities.

Combinations at large.	Forms.	Comb. in each form.
aa,aa,ab,ab,ac	a^2,b^2	2
aa,ab,ab,ac	*ab,ac,bc*	3
ab,ab,ac		—
bb,bc		5 = ſum of the twos.
bc		
	a^3	1
aaa,aab,aab,aac	a^2b,a^2c,b^2a,b^2c	4
aab,aab,aac	*abc*	1
abb,abc		—
bbc		6 = ſum of the threes.
aaab,aaab,aaac	a^3b,a^3c	2
aabb,aabc	a^2b^2	1
abbc	a^2bc,b^2ac	2
		—
		5 = ſum of the fours.

Anſ. 5 Combinations of every 2; 6, of every 3, and 5 of every 4 quantities.

PROBLEM

PROBLEM VI.

To find the changes of any given number of things, taken a given number at a time; in which there are several given things of one sort, several of another, &c.

RULE 1.—Find all the different forms of combination of all the given things, taken, as many at a time, as in the queſtion, by Problem 5.

2. Find the number of changes in any form, (by Problem 3,) and multiply it by the number of combinations in that form.

3. Do the ſame for every diſtinct form, and the ſum of all the products will give the whole number of changes required.

EXAMPLE.

How many changes can be made of every 4 letters out of theſe 6; *aaabbc*?

No. of forms.	*Comb.*	*Changes.*
a^3b, a^3c	2	$\dfrac{1\times2\times3\times4}{1\times2\times3} = \dfrac{24}{6} = 4$
a^2b^2	1	$\dfrac{1\times2\times3\times4}{1\times2\times1\times2} = \dfrac{24}{4} = 6$
a^2bc, b^2ac	2	$\dfrac{1\times2\times3\times4}{1\times2} = \dfrac{24}{2} = 12$

$$\text{Therefore,}\left\{\begin{array}{l} 2\times4 = 8 \\ 1\times6 = 6 \\ 2\times12 = 24 \end{array}\right.$$

38 = number of changes required.

MISCELLANEOUS MATTERS.

A short method of reducing a Vulgar Fraction into its equivalent Decimal, by Multiplication.

RULE.

DIVIDE unity or 1 by the denominator, till the remainder is a single figure, 10, 100, &c. if convenient, then multiply the whole quotient, including the remainder after division, by the remainder (which is now the numerator, and the divisor, the denominator) and annex the product to the quotient, then multiply the quotient, thus increased, by the last numerator, and annex the product to the increased quotient; and thus it may be reduced to what exactness you please. But if the numerator of the given fraction exceed 1, you must finally multiply the last product by the said numerator.

EXAMPLES.

1. Reduce $\frac{1}{26}$ to its equivalent decimal.

26)1,00(,03846$\frac{4}{26}$
78
——
220
208
——
120
104
——
160
156
——
4

This multiplied by 4 (the numerator) is ,15384$\frac{16}{26}=\frac{8}{13}$
Which annexed to the quotient ,03846 is ,0384615384$\frac{8}{13}$
And ,0384615384$\frac{8}{13}\times 8$ and annexed to the last product
$=$,03846153843076923076$\frac{12}{13}$ &c.

2. Reduce $\frac{5}{246}$.

246)1,000000(,004065$\frac{10}{246}$ and ,0040650$\frac{10}{246}\times 10=$,0040650$\frac{100}{246}$ and this annexed to the quotient is ,00406540650$\frac{100}{246}$, and this multiplied by the given numerator 5, is ,0203270325$2\frac{8}{246}$.

For any number of pounds, avoirdupois, under 28, multiply the decimal ,00892857 by the given number of pounds, which generally gives the decimal true to the sixth place.

A short method of finding the duplicate, triplicate, &c. Ratio of any two numbers, whose difference is small, compared with the two numbers.

For the Duplicate Ratio.

RULE.—Assume two numbers, whose difference is small; subtract *half* their difference from the least, and add it to the greatest, and the two numbers, thus found, will be in the same proportion nearly as the squares of the assumed numbers.

EXAMPLE.

Let the assumed numbers be 10 and 11: Then $11-10=1$. $10-,5=9,5$ and $11+,5=11,5$.

Proof, As $10^2 : 11^2 :: 9,5 : 11,5$ nearly.

For a Triplicate Ratio.

RULE.—Subtract the *difference* of the assumed numbers from the

the leaſt, and add it to the greateſt, and the numbers, thus obtained, will be in the ſame proportion nearly as the cubes of the aſſumed numbers.

Let the numbers be 164 and 165: Then 165—164 = 1. 164—1 = 163 & 165+1 = 166. *Proof*, As $164^3 : 165^3 :: 163 : 166$ nearly.

For a quadruplicate proportion ſubtract, and add once and a half the difference, and ſo on, for each higher power, increaſing the number to be ſubtracted and added by ,5.

To reduce a Ratio, conſiſting of large numbers, to its leaſt Terms, and very nearly of the ſame value.

RULE 1.—Divide the greater of the terms by the leſs, and the laſt diviſor by the remainder, and ſo on continually, till nothing remain, in the ſame manner as we get the greateſt common meaſure for reducing a vulgar fraction: This will give a number of ratios, from which we can chooſe one that will ſuit our purpoſe.

2. Place the firſt quotient under unit for the firſt ratio; multiply that by the next quotient, adding nothing to the numerator, and 1 to the product of the denominator for a new denominator, and it will give a ſecond ratio, nearer than the firſt: Then, multiply the laſt ratio by the next quotient, adding the preceding ratio, and ſo on continually till you have gone through.

EXAMPLES.

1. Sir Iſaac Newton has demonſtrated, in his Principia, that the velocity of a comet, moving in a parabola, is to that of a planet, moving in a circular orb, at the ſame diſtance from the ſun, as $\sqrt{2}$ to 1. Let this be taken for an example.

$\sqrt{2} = 1{,}4142$; Thoſe motions, then, are as 1,4142 to 1; or as 14142 to 10000?

10000)14142(1
10000
4142)10000(2
8284
1716)4142(2
3432
710)1716(2
1420
296)710(2
592
118)296(2
236
60)118(1
60
58)60(1
58
2)58(29
58

Then $\frac{1}{1}$ = firſt ratio.

$\frac{1\times2+0=2}{1\times2+1=3}$ = ſecond.

$\frac{2\times2+1=5}{3\times2+1=7}$ = third.*

$\frac{5\times2+2=12}{7\times2+3=17}$ = fourth.

$\frac{12\times2+5=29}{17\times2+7=41}$ = fifth, &c.

2. Geometers

* The late Profeſſor Winthrop choſe 7 to 5 for a proportion.

2. Geometers have found the proportion of the circumference of a circle to its diameter, to be as 3,1416 to 1 : Let this ratio be reduced.

```
10000)31416(3
      30000
      -----
       1416)10000(7
             9912
             ----
               88)1416(16
                   88
                   ---
                   536
                   528
                   ---
                     8)88(11
                       88
```

Then $\frac{1}{3}$ = firſt ratio.

$\frac{1\times7+0}{3\times7+1} = \frac{7}{22}$ = ſecond.

$\frac{7\times16+1}{22\times16+3} = \frac{113}{355}$ = third :—This is the ratio generally made uſe of, and is ſufficiently exact for very nice calculations.

3. The area of a circle is to its circumſcribing ſquare, as ,7854 to 1, very nearly : Let this be reduced.

```
7854)10000(1
      7854
      ----
      2146)7854(3
           6438
           ----
           1416)2146(1
                1416
                ----
                 730)1416(1
                      730
                      ---
                      686)730(1
                          686
                          ---
                           44)686(15
                              44
                              ---
                              246
                              220
                              ---
                               26 &c.
```

Then $\frac{1}{1}$ = firſt ratio.

$\frac{1\times3+0}{1\times3+1} = \frac{3}{4}$ = ſecond.

$\frac{3\times1+1}{4\times1+1} = \frac{4}{5}$ = third.

$\frac{4\times1+3}{5\times1+4} = \frac{7}{9}$ = fourth.

$\frac{7\times1+4}{9\times1+5} = \frac{11}{14}$ = fifth.—This is very exact, and the proportion generally uſed.

Therefore, As 14 : 11 : : the ſquare of the diameter of a circle to its area.

To eſtimate the Diſtance of Objects on level ground, or at ſea, having only the height given.

RULE 1.—To the earth's diameter (viz. 42056462 feet,) add the height of the eye, and multiply the ſum by that height, then the ſquare root of the product is the diſtance, at which an object on the ſurface of the earth or water, can be ſeen by an eye ſo elevated.

2. As objects are ſeen in a ſtraight line, and that line is a tangent to the earth's ſurface ; therefore, *To find the diſtance of two elevated objects, when the right line joining them touches the earth's ſurface between thoſe objects, (for inſtance, the line from the eye of the obſerver to the diſtance found by the firſt part of the rule, and from thence to the object ;)* work for each object ſeparately, and the ſum of the ſquare roots of the products is the diſtance of the two objects from each other.

How

How far may a mountain be ſeen on level ground, or at ſea, which is a mile high, ſuppoſing the eye of the obſerver elevated 5 feet above the ſurface?

$$\sqrt{42056462 + 5 \times 5} = 2{,}746 \text{ miles.}$$
$$\sqrt{42056462 + 5280 \times 5280} = 89{,}253 \text{ miles.}$$

Anſ. 91,999 *miles.*

To eſtimate the Height of Objects on level ground, or at ſea, having only the diſtance given.

RULE 1.—From the given diſtance take the diſtance, which the elevation of your eye above the ſurface will give, found by the laſt Problem.

2. Divide the ſquare of the remainder in feet by 42056462 feet, and the quotient will be the height required.

Being on my return from a foreign voyage, and finding by my reckoning I was about $5\frac{1}{2}$ leagues from Boſton lighthouſe, it being in the duſk of the evening, with my teleſcope I deſcried the lamp of the light houſe in the horizon, at which time my eye was elevated 6 feet above the ſurface of the water: Now, ſuppoſing my reckoning to be true; What is the height of the lighthouſe above the water?

$5\frac{1}{2}$ leagues $=$ 16,5 miles; then $16{,}5 - \sqrt{42056462 + 6 \times 6} =$ 13,943 miles $=$ 73619 feet nearly, and $73619 \times 73619 \div 42056462$ $=$ 129 feet nearly, *Anſ.*

MISCELLANEOUS QUESTIONS, *with the* METHOD *of* SOLUTION.

1. What part of 9*d.* is $\frac{2}{5}$ of 7*d.*?

$$\frac{2}{5} \text{ of } \frac{7}{1} = \frac{14}{5}, \text{ and } \frac{9}{1} \div \frac{14}{5} = \frac{1 \times 14}{9 \times 5} = \frac{14}{45} \quad \textit{Anſ.}$$

2. What number is that, from which $\frac{3}{7}$ being taken, the remainder will be $\frac{1}{5}$?

$$\frac{1}{5} + \frac{3}{7} = \frac{1 \times 7 + 3 \times 5}{5 \times 7} = \frac{22}{35} \quad \textit{Anſ.}$$

3. What number is that, to which if $\frac{3}{7}$ of $\frac{12}{5}$ of $\frac{129}{313}$ be added, the total will be 1?

$$\frac{3}{7} \text{ of } \frac{12}{5} \text{ of } \frac{129}{313} = \frac{4644}{10955}, \text{ and } \frac{1}{1} - \frac{4644}{10955} = \frac{1 \times 10955 - 1 \times 4644}{1 \times 10955}$$
$$= \frac{6311}{10955} \quad \textit{Anſ.}$$

4. What number is that, of which $19\frac{3}{13}$ is $\frac{5}{7}$?

$19\frac{3}{13} = \frac{250}{13}$; then, As $\frac{5}{1} : \frac{250}{13} :: \frac{7}{1} : 26\frac{12}{13}$ *Anſ.*

5. In

5. In an orchard of fruit trees, $\frac{1}{2}$ of them bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plumbs, 60 of them peaches, and 40 cherries; How many trees does the orchard contain?

$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{11}{12}$, and $\frac{12}{12}-\frac{11}{12}=\frac{1}{12}$; therefore, As $\frac{1}{12}:\frac{60+40}{1}::\frac{12}{12}:1200$ *Anſ.*

6. A perſon, who was poſſeſſed of $\frac{2}{5}$ of a veſſel, ſold $\frac{5}{8}$ of his intereſt for 375*l.*; What was the ſhip worth at that rate?

$\frac{5}{8}$ of $\frac{2}{5}=\frac{1}{4}$. As $\frac{1}{4}:\frac{375}{1}::\frac{1}{1}:$ £1500 *Anſ.*

7. If $\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{4}{5}$ of a ſhip be worth $\frac{2}{9}$ of $\frac{7}{8}$ of $\frac{12}{13}$ of the cargo, valued at 1000*l.*; What did both ſhip and cargo coſt?

$\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{4}{5}=\frac{6}{28}$, and $\frac{2}{9}$ of $\frac{7}{8}$ of $\frac{12}{13}$ of $\frac{1000}{1}=\frac{7000}{39}$, then,

As $\frac{6}{28}:\frac{7000}{39}::\frac{28}{28}:\frac{28\times7000\times28}{6\times39\times28}=$ £837 12*s.* $1\frac{25}{39}$*d.* the coſt of the ſhip; and £1000+£837 12*s.* $1\frac{25}{39}$*d.* = £1837 12*s.* $1\frac{25}{39}$*d.* value of the ſhip and cargo, *Anſ.*

8. Two ſhips, A and B, ſailed from a certain port at the ſame time; A ſailed north 8 miles an hour, and B eaſt 6 miles an hour; Required, by an eaſy method, their diſtance aſunder at every hour's end?

$\sqrt{8\times8+6\times6}=10$ miles diſtant in 1 hour, and $10\times2=20$ miles in 2 hours, &c. *Anſ.*

9. *If a body be weighed in each ſcale of a balance, whoſe beam is unequally divided, and thoſe different weights of the body be multiplied together, the ſquare root of the product will be the true weight of that body.*

Suppoſe the weight of a bar of ſilver, in one ſcale, to be 10*oz.* and in the other ſcale 12*oz.*; Required the true weight of the bar?

$\sqrt{12\times10}=10,954=$ 10 *oz.* 19 *pwt.* 1,92 *gr.* *Anſ.*

10. A younger brother received 1560*l.* which was juſt $\frac{7}{12}$ of his elder brother's fortune; and $5\frac{3}{8}$ times the elder's money, was $1\frac{2}{3}$ the value of the father's eſtate; Pray, what was the father worth?

As $7:1560::12:2674\frac{2}{7}$ the elder brother's fortune; then, $2674\frac{2}{7}\times5\frac{3}{8}\div1\frac{2}{3}=$ £8624 11*s.* $5\frac{1}{7}$*d.* *Anſ.*

11. A gentleman divided his fortune among his ſons, giving A 9*l.* as often as B 5*l.* and to C but 3*l.* as often as to B 7*l.* and yet C's dividend was $1537\frac{5}{8}$*l.*; What did the whole eſtate amount to?

As $7:5::3:2\frac{1}{7}$; then, As $2\frac{1}{7}:1537\frac{5}{8}::\overline{9+5+2\frac{1}{7}}:$ £11583 8*s.* 10*d.* *Anſ.*

12. A gentleman left his ſon a fortune: $\frac{5}{16}$ of which he ſpent in 3 months; $\frac{3}{4}$ of $\frac{5}{6}$ of the remainder laſted him 9 months longer, when he had only 537*l.* left; Pray, what did his father bequeath him?

$\frac{16}{16}=$ whole legacy, $\frac{16}{16}-\frac{5}{16}=\frac{11}{16}$ left at 3 months, then, $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{11}{16}=\frac{165}{384}$, and $\frac{11}{16}-\frac{165}{384}=\frac{1584}{6144}=$ £537, therefore, As $\frac{1584}{6144}:\frac{537}{1}::\frac{1}{1}:$ £2082 18*s.* $2\frac{2}{11}$*d.* *Anſ.*

13. A gay young fellow ſoon got the better of $\frac{2}{7}$ of his fortune; he then gave 1500*l.* for a commiſſion, and his profuſion continued till he had but 450*l.* left, which he found to be juſt $\frac{6}{16}$ of his money, after he had purchaſed his commiſſion; What was his fortune at firſt?

As $6 : 450 :: 16 : 1200$, and $1200+1500 = £2700 = \frac{5}{7}$ of his fortune, and, As $5 : 2700 :: 7 : £3780$ *Anſ.*

14. A merchant begins the world with 1500*l.* and finds that by his diſtillery he clears 1500*l.* in 7 years; by his navigation 1500*l.* in 9 years, and that he ſpends in gaming 1500*l.* in $3\frac{1}{2}$ years; How long will his eſtate laſt?

$$\text{As}\left\{\begin{array}{l} 7 : 1500 :: 1 : 214\frac{2}{7} \\ 9 : 1500 :: 1 : 166\frac{2}{3} \\ 3\frac{1}{2} : 1500 :: 1 : 428\frac{4}{7} \end{array}\right\}\ \text{As } 428\frac{4}{7} - \overline{214\frac{2}{7}+166\frac{2}{3}} : 1 :: 1500 : 31\frac{1}{2}$$

(£. over the fourth column; Y. £. Y. over 1, 1500, $31\frac{1}{2}$)

15. A has 100*l.* of B's money in his hands, for the remittance of which B allows him 9 per cent.; What ſum muſt he remit, to diſcharge himſelf of the 100*l.*?

As $100+9 : 100 :: 100 : £91\frac{81}{109}$; or, $\frac{100 \times 100}{100+9} = £91\frac{81}{109}$ to be remitted, and $\frac{100 \times 9}{100+9} = £8\frac{28}{109}$ his commiſſion.

16. Said Harry to Edmund, I can place four 1s ſo that, when added, they ſhall make preciſely 12; Can you do ſo too?

17. A and B are on oppoſite ſides of a circular field 268 poles about; they begin to go round it, both the ſame way, at the ſame inſtant of time; A goes 22 rods in 2 minutes, and B 34 rods in 3 minutes; How many times will they go round the field, before the ſwifter overtakes the ſlower?

Pol. min. Pole. min. *Pol. min. Pol. min.*

As $22 : 2 :: 1 : \frac{1}{11}$ and, As $34 : 3 :: 1 : \frac{3}{34}$, then, $\frac{1}{11} - \frac{3}{34} = \frac{1}{374}$ minutes.

Pol. min. Pol. min. Time. Round. Time.

As $1 : \frac{1}{374} :: 22 : \frac{11}{187}$, As $\frac{11}{187} : \frac{1}{1} :: \frac{1}{1} : 17$ times round, *Anſ.*

18. If 15 men can perform a piece of work in 11 days; How many men will accompliſh another piece of work four times ſo large, in a fifth part of the time?

Work. Men. Works. Men. Time. Men. Time. Men.

As $1 : 15 :: 4 : 60$ As $\frac{1}{1} : \frac{60}{1} :: \frac{1}{5} : 300$ *Anſ.*

19. If A can do a piece of work alone in 7 days, and B in 12; ſet them both about it together; In what time will they finiſh it?

Days. Work. Day. Work. *Work. Work. Work.* *Work. Day. Work. Day.*

$$\text{As}\left\{\begin{array}{l} 7 : 1 :: 1 : \frac{1}{7} \\ 12 : 1 :: 1 : \frac{1}{12} \end{array}\right\}\ \text{Then, } \frac{1}{7}+\frac{1}{12} = \frac{19}{84}.\ \text{As } \frac{19}{84} : \frac{1}{1} :: \frac{1}{1} : 4\frac{8}{19}\ \textit{Anſ.}$$

20. A and B together can build a boat in 20 days; with the aſſiſtance of C they can do it in 12; In what time would C do it by himſelf?

As

$$\begin{array}{l} \quad D.\ \ W.\ \ D.\ \ W. \qquad\qquad W.\ \ W.\ \ W. \qquad W.\ D.\ \ W.\ \ D. \\ \text{As}\left\{\begin{array}{l} 20:1::1:\frac{1}{20} \\ 12:1::1:\frac{1}{12} \end{array}\right\} \text{Then, } \frac{1}{12}-\frac{1}{20}=\frac{8}{240}, \text{ \& As } 8:1::240:30 \textit{ Ans.} \end{array}$$

21. A can do a piece of work alone in 13 days, and A and B together in 8 days; In what time can B do it alone?

$$\begin{array}{l} \quad D.\ \ W.\ \ D.\ \ W. \qquad\qquad W.\ \ W.\ \ W. \qquad W.\ D.\ \ W.\ \ D. \\ \text{As}\left\{\begin{array}{l} 13:1::1:\frac{1}{13} \\ \ \ 8:1::1:\frac{1}{8} \end{array}\right\} \text{Then, } \frac{1}{8}-\frac{1}{13}=\frac{5}{104}, \text{ and, As } 5:1::104:20\frac{4}{5} \end{array}$$

22. A, B, and C, can complete a piece of work in 12 days; A can do it alone in 23 days, and B in 37 days; In what time can C do it by himself?

$$\begin{array}{l} \quad D.\ \ W.\ \ D. \\ \text{As}\left\{\begin{array}{l} 12:1::1:\frac{1}{12} \\ \hline 23:1::1:\frac{1}{23} \\ 37:1::1:\frac{1}{37} \end{array}\right\} \quad \begin{array}{l} \qquad\quad W. \\ \text{Then, } \frac{1}{12}-\overline{\frac{1}{23}+\frac{1}{37}}=\frac{131}{10212} \\ \quad W.\quad D.\qquad W. \\ \text{As } 131:1::10212:77\frac{125}{131} \text{ days, } \textit{Ans.} \end{array} \end{array}$$

23. A ciftern, for water, has two cocks to fupply it; by the firft it may be filled in 45 minutes, and by the fecond, in 55 minutes; it has likewife a difcharging cock, by which it may, when full, be emptied in 30 minutes; Now, if thefe three cocks be all left open when the water comes in, in what time will the ciftern be filled?

$$\begin{array}{l} \begin{array}{llll} \textit{Min.} & \textit{Cift.} & \textit{Min.} & \textit{Cift.} \\ 45 : & 1 :: & 60 : & 1{,}3333 \\ 55 : & 1 :: & 60 : & 1{,}0909 \\ \hline & & & 2{,}4242 \\ 30 : & 1 :: & 60 : & 2 \end{array} \qquad \begin{array}{l} \textit{Cift. Hour. Cift. h. m. s.} \\ \text{As } ,4242 : 1 :: 1 : 2\ \ 21\ \ 26\tfrac{1}{2}\ \textit{Anf.} \end{array} \end{array}$$

Gains in an hour ,4242 of a ciftern.

24. A water tub holds 73 gallons; the pipe which conveys the water to it, ufually admits 7 gallons in 5 minutes; and the tap difcharges 20 gallons in 17 minutes; Now, fuppofing thefe both to be carelefsly left open, and the water to be turned on at 4 o'clock in the morning; a fervant, at 6, finding the water running, puts in the tap; In what time, after this accident, will the tub be full?

$$\begin{array}{l} \quad \textit{Min. Gal. Min. Gal.} \qquad\qquad\qquad\qquad h. \\ \text{As}\left\{\begin{array}{l} \ \ 5:\ \ 7::60:84 \\ 17:20::60:70\frac{10}{17} \end{array}\right\} \begin{array}{l} \overline{84-70\frac{10}{17}}\times 2=26\frac{14}{17}\textit{gal.} \ \& \ 73-26\frac{14}{17}= \\ 46\frac{3}{17}\textit{gal.} \text{ which now remain to be filled.} \end{array} \end{array}$$

$$\begin{array}{l} \qquad\qquad\qquad\qquad \textit{Gal. Min. Gal.}\quad M.\quad s. \\ \text{Therefore, As } 7:5::46\frac{3}{17}:32\ \ 58\frac{118}{119}, \text{ and therefore the tub} \end{array}$$

will be full at $32^{m.}\ 58\frac{118}{119}{}^{s.}$ after 6.

25. A has a cheft of tea, weighing $3\frac{1}{2}$*cwt.* the prime coft of which is 60*l.*; now, allowing intereft at 6 per cent. per annum,

Ho,

How muſt he rate it per lb to B, ſo that by taking his note of hand, payable at 6 months, he may clear 50 dollars by the bargain?

Intereſt £2 5s. Then, As $3\frac{1}{2}$cwt. : £$60\frac{1}{4}$+£15+£2 5s. : : 1lb : 3s. $11\frac{23}{98}$d. *Anſ.*

26. Suppoſe the American continental debt to be 18 millions; What annuity, at 6 per cent. per annum, will diſcharge it in 25 years?

By Table 5, of annuities, p. 326, ,07823 is the annuity, which 1*l*, will purchaſe in 25 years, then, ,07823 × 18000000 = £1408140 *Anſ.*

The annual intereſt of the debt = 1080000

Therefore, there muſt be a ſinking fund of £328140 *pr. ann.*

27. The hour and minute hands of a watch are exactly together at 12 o'clock; When are they next together?

The velocities of the two hands of a watch, or clock, are to each other, As 12 to 1; therefore, the difference of velocities is 12 − 1 = 11,

$$\text{As } 11 : 1 : : \left\{\begin{array}{lllll} 12\times1 & : & 1 & 5 & 27\frac{3}{11} \\ 12\times2 & : & 2 & 10 & 54\frac{6}{11} \\ 12\times3 & : & 3 & 16 & 21\frac{9}{11} \end{array}\right\} \text{\&c. } Anſ.$$

(h. m. s.)

28. A hare ſtarts 12 rods before a hound; but is not perceived by him till ſhe has been up 45 ſeconds; ſhe ſcuds away at the rate of 10 miles an hour, and the dog, on view, makes after, at the rate of 16 miles an hour; How long will the courſe hold, and what ſpace will be run over, from the ſpot where the dog ſtarted?

$\frac{16-10}{16}=\frac{6}{16}=\frac{3}{8}$, or, as 8 to 3 againſt the hare, 1 hour = 3600 ſeconds.

10 miles = 52800 feet.

Sec.	Feet.	Sec.	Feet.
As 3600 :	52800 : :	45 :	660 diſtance the hare had run before the dog diſcovered her.

Add 12 rods = 198

858 = the diſtance of the hare when the dog ſtarted.
8

3)6864

Feet 2288 = the ground run over by the dog.

	Miles.	Feet.	Sec.	Feet.	Sec.
Now, As	16 =	84480 :	3600 : :	2288 :	$97\frac{1}{2}$

29. In a ſeries of proportional numbers, the firſt is 4, the third 12, and the product of the ſecond and third is 112,8; What is the difference of the ſecond and fourth?

112,8 ÷ 12 = 9,4 the ſecond. As 4 : 9,4 : : 12 : 28,2, and 28,2 − 9,4 = 18,8 *Anſ.*

30. A fellow ſaid that when he counted his nuts, two by two, three by three, four by four, five by five, and ſix by ſix, there was

was ſtill an odd one ; but when he told them ſeven by ſeven, they came out even ; How many had he ?

$2\times3\times4\times5\times6=720$, and $\overline{720+1}\div7=103$ even, *Anſ.* 721.

$\frac{721}{2.3.4.5 \text{ and } 6}$ reſpectively, will leave an odd one.

31. There is an iſland, 50 miles in circumference, and 3 men ſtart together to travel the ſame way about it : A goes 7 miles per day, B 8, and C 9 ; when will they all come together again, and how far will each travel ?

$\overline{50\times7}+\overline{50\times8}+\overline{50\times9}\div\overline{7+8+9}=50$ days.—A 350 miles, B 400, and C 450, *Anſ.*

32. Suppoſe A leaves Newburyport at 6 o'clock on Monday morning, and travels towards Providence, at the rate of 4 miles per hour without intermiſſion ; and that, at 3 in the afternoon, B ſets out from Providence for Newburyport, and travels conſtantly at the rate of 7 miles an hour ; Now, ſuppoſe the diſtance between the two towns to be 90 miles ; whereabout on the road will they meet ?

$6+3=9$ hours, and $9\times4=36$ miles, the time and diſtance A had travelled before B ſtarted. Then $90-36=54$ miles remain to be travelled by both ; now, as both together leſſen the diſtance $7+4=11$ miles an hour, therefore $\frac{4}{11}$ of $\overline{54+36}=55\frac{7}{11}$ miles from Newburyport ; which is near Ames's at Dedham.

33. If, during ebb tide, a wherry ſhould ſet out from Haverhill to come down the river, and, at the ſame time, another ſhould ſet out from Newburyport, to go up the river, allowing the diſtance to be 18 miles ; ſuppoſe the current forwards one and retards the other $1\frac{1}{2}$ mile per hour ; the boats are equally laden, the rowers equally good, and, in the common way of working in ſtill water, would proceed at the rate of 4 miles per hour ; Where, in the river, will the two boats meet ?

$\overset{M.}{4}+\overset{M.}{1\frac{1}{2}}=\overset{M.}{5\frac{1}{2}}$, and $\overset{M.}{4}-\overset{M.}{1\frac{1}{2}}=\overset{M.}{2\frac{1}{2}}$, then, $\overset{M.}{5\frac{1}{2}}+\overset{M.}{2\frac{1}{2}}=\overset{M.}{8}$ in one hour by both. As $\overset{M.}{8}:\overset{H.}{1}::\overset{M.}{18}:\overset{H.}{2\frac{1}{4}}$, then $\overset{M.}{5\frac{1}{2}}\times\overset{H.}{2\frac{1}{4}}=\overset{M.}{12\frac{3}{8}}$ from Haverhill, and $\overset{M.}{2\frac{1}{2}}\times\overset{H.}{2\frac{1}{4}}=\overset{M.}{5\frac{5}{8}}$ from Newburyport.

34. A gentleman making his addreſſes in a lady's family, who had 5 daughters ; ſhe told him that their father had made a will, which imported that the firſt four of the girls' fortunes were, together, to make 50000*l.* ; the laſt four 66000*l.* ; the three laſt with the firſt 60000*l.* the three firſt with the laſt 56000*l.* and the two firſt with the two laſt 64000*l.* which, if he would unravel, and make it appear what each was to have, as he appeared to have a partiality for Harriet, her third daughter, he ſhould be welcome to her ; Pray, what was Miſs Harriet's fortune ?

A+

$$\left.\begin{array}{r}
A+B+C+D = 50000 \\
B+C+D+E = 66000 \\
A +C+D+E = 60000 \\
A+B+C +E = 56000 \\
A+B +D+E = 64000 \\
\hline
296000
\end{array}\right\}$$

Then, $296000 \div 4$ the number of combinations $= 74000$ the ſum of their fortunes.

Then, $A+B+C+D+E = 74000$
And $A+B +D+E = 64000$

Anſ. Harriet's fortune $= £10000$

35. Three perſons purchaſe a veſſel in company, towards the payment whereof A advanced $\frac{2}{5}$, B $\frac{3}{7}$, and C, 256*l.*; What did A and B pay, each, and what part of the veſſel had C?

$\frac{2}{5} + \frac{3}{7} = \frac{14+15}{35} = \frac{29}{35}$, and $\frac{35}{35} - \frac{29}{35} = \frac{6}{35}$ C's part of the veſſel.

As $\frac{6}{35} : \frac{256}{1} :: \begin{cases} \frac{14}{35} : £597\ 6s.\ 8d. \text{ A advanced.} \\ \frac{15}{35} : £640 \text{ B advanced.} \end{cases}$

36. A and B cleared, by an adventure at ſea. 45 guineas, which was 35*l.* per cent. upon the money advanced, and with which they agreed to purchaſe a genteel horſe and carriage, whereof they were to have the uſe in proportion to the ſums adventured, which was found to be 11 to A, as often as 8 to B; What money did each adventure?

As £35 : 100 :: 45 *Guin.* : £180 = the whole adventure.

As $11+8 : 180 :: \begin{cases} 11 : £104\ 4s.\ 2\frac{10}{19}d. \text{ A's.} \\ 8 : £75\ 15s.\ 9\frac{9}{19}d. \text{ B's.} \end{cases}$

37. A, B and C are to ſhare 100*l.* in the proportion of $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ reſpectively; but C dying, it is required to divide the whole ſum properly between the other two?

$$\text{As } \tfrac{1}{3}+\tfrac{1}{4}+\tfrac{1}{5} : 100 :: \left\{\begin{array}{l} \frac{1}{3} : 42\frac{26}{47} \text{ A's.} \\ \frac{1}{4} : 31\frac{43}{47} \text{ B's.} \\ \frac{1}{5} : 25\frac{25}{47} \text{ C's.} \end{array}\right\}$$

Again, As $\frac{1}{3}+\frac{1}{4} : 25\frac{25}{47} :: \frac{1}{3} : 14\frac{582}{947}$.

Then, $42\frac{26}{47}+14\frac{582}{947} = £57\ 2s.\ 10\frac{1}{4}d.$ A's ſhare.

And $£100 - £57\ 2s.\ 10\frac{1}{4}d. = £42\ 17s.\ 1\frac{3}{4}d.$ B's ſhare.

38. A, B and C have among them 135 guineas; A's+B's are to B's+C's, as 5 to 7, and C's—B's to C's+B's as 1 to 7; How many had each?

A+B. A+C.

Suppoſe A's+B's = 50; then, as 5 to 7 :: 50 : 70; As 7 : 1 :: 70 : 10 = C's—B's; then, 70—10 = 60, and $60 \div 2 = 30$ = B's, 30—10 = 20 = A's, and 30+10 = 40 = C's, by the ſuppoſition: Now 20+30+40 = 90, which ſhould have been 135, therefore,

$$\text{As } 90 : 135 :: \begin{cases} 20 : 30 = \text{A's.} \\ 30 : 45 = \text{B's.} \\ 40 : 60 = \text{C's.} \end{cases}$$

Sum = 135 proof.

39. There

39. There are three horfes, belonging to different men, employed as a team to draw a load of falt from Newburyport to Bofton for 2*l.* 10*s.* A and B are fuppofed to do $\frac{3}{11}$ of the work; A and C $\frac{5}{13}$, and B and C $\frac{4}{14}$ of it; they are to be paid proportionally; Can you divide it as it fhould be?

$$\left.\begin{array}{l} A+B=\frac{3}{11}=,2727 \\ A+C=\frac{5}{13}=,3846 \\ B+C=\frac{4}{14}=,2857 \\ \hline \text{Sum}=,943 \end{array}\right\}$$

And ,943÷2, the number combined $=,4715=A+B+C$
$-,2727=A+B$

Then, As ,4715 : 50 :: ,1988 : £1 1*s.* $0\frac{3}{4}$*d.* $=,1988=$ C.

And in the fame manner proceed for the reft.

40. I would put 20 hogfheads of London beer into 10 wine pipes, and defire to know what the cafk muft contain, which will receive the difference, 231 folid inches being the wine gallon, and 282 that of beer?

Beer hhd. = 54 gal. and $54\times282\times20=304560$ folid inches.
Wine pipe. = 126 gal. and $126\times231\times10=291060$ folid inches,
and $\frac{304560-291060}{282}=47\frac{41}{47}$ beer gallons, *Anf.*

41. Being about to plant 5292 trees equally diftant in rows, the length of the grove is to be 3 times the breadth; How many of the fhorter rows will there be?

$\sqrt{\frac{5292}{3}}\times3=126$ rows, *Anf.* viz. $\frac{1}{3}$ of the trees are to form an exact fquare, the fide whereof being 42, fhews how many come into a fhort row.

42. A general, difpofing his army into a fquare battalion, found he had 231 over and above; but increafing each fide with one foldier, he wanted 44 to fill up the fquare; How many men did his army confift of?

$231+44=275$, and $\overline{275-1}\div2=137$, then $\overline{137\times137}+231=19000$ *Anf.*

43. I want the length of a fhoar, the bottom of which, being fet 9 feet from the perpendicular fide of a houfe, will fupport a weak place in the wall $22\frac{1}{2}$ feet from the ground?

$\sqrt{\overline{22,5\times22,5}+\overline{9\times9}}=24$ feet, $2\frac{3}{4}$ inches, *Anf.*

44. A line 35 yards long will exactly reach from the top of a fort, ftanding on the brink of a river, known to be 27 yards broad, to the oppofite bank; What is the height of the wall?

$\sqrt{\overline{35\times35}-\overline{27\times27}}=22$ yards, $9\frac{3}{4}$ inches, nearly.

45. Suppofe a light houfe built on the top of a rock; the diftance between the place of obfervation and that part of the rock level

level with the eye 620 yards; the diſtance from the top of the rock to the place of obſervation 846 yards, and from the top of the light houſe 900 yards: The height of the light houſe is required?

$\sqrt{900\times900-620\times620}-\sqrt{846\times846-620\times620}=76{,}77$ *yds. Anſ.*

46. *The ſum and difference of the ſquares of two numbers given, to find thoſe numbers.*

Rule.—From the ſum take the difference, and half the remainder is the ſquare of the leſs, which, taken from the ſum of the ſquares, will give the ſquare of the greater.

A and B have between them a number of guineas, which are to be ſo divided, that the ſum of their ſquares may be 208, and the difference of their ſquares 80; ſuppoſing A's the greater number, How many has he more than B?

$\overline{208-80}\div2=64$ the ſquare of B's, and $208-64=144$ the ſquare of A's; therefore $\sqrt{144}-\sqrt{64}=4$ *Anſ.*

47. *Having the ſum of two numbers, and the ſum of their ſquares given, to find thoſe numbers.*

Rule.—From the ſquare of their ſum take the ſum of their ſquares: Then from the ſum of their ſquares take this remainder, and the ſquare root of the difference will be the difference of the two numbers. To half their ſum add half their difference, and the ſum will be the greater. From half the ſum take half their difference, and the remainder will be the leſs.

A and B have 50 guineas between them, which are to be ſo divided, as that the ſum of the ſquares of the two numbers ſhall be 1300; How many had each, ſuppoſing A to have the greater number?

$\overline{50\times50}-1300=1200$; Then $\sqrt{1300-1200}=10$ difference. Now $50\div2+\overline{10\div2}=30=$ A's. And $50\div2-10\div2=20=$ B's, *Anſ.*

48. *Having the difference of two numbers, and the ſum of their ſquares given, to find thoſe numbers.*

Rule.—From the ſum of their ſquares take the ſquare of their difference; To the ſum of the ſquares add the remainder, and the ſquare root of this ſum will be the ſum of the required numbers; then, with the half ſum and half difference proceed as in the laſt queſtion.

A number of guineas are to be divided between A and B, in ſuch a manner that A may have 50 more than B, and that the ſum of the ſquares of the reſpective ſhares may be 12500; What number had each?

$12500-\overline{50\times50}=10000$, and $\sqrt{12500+10000}=150=$ ſum of their ſhares. Then, $\overline{150\div2}+\overline{50\div2}=100$ A's; and $\overline{150\div2}-\overline{50\div2}=50$ B's, *Anſ.*

49. *Having the ſum of the ſquares of two numbers, and the ſquare of their half ſum given, to find thoſe numbers.*

Rule.

RULE.—From the sum of the squares take *twice* the square of the half sum, and the square root of half the remainder will be their half difference, with which and the half sum proceed as before directed.

Let the sum of the squares of two numbers be 3161, and the square of their half sum 1560,25; Required those numbers?

$3161 - \overline{1560,25 \times 2} = 40,5$ $\quad \overline{40,5 \div 2} = 20,25$, and $\sqrt{20,25} = 4,5 = \frac{1}{2}$ difference, and $\sqrt{1560,25} = 39,5 = \frac{1}{2}$ sum; then, $39,5 + 4,5 = 44$ the greater, and $39,5 - 4,5 = 35$ the less, *Ans.*

50. 1. *If the quantity of matter, (or weights) of any two bodies, put in motion, be equal, the force by which they are moved will be in proportion to their velocities, or swiftness of motion.*

2. *If the velocities of these bodies be equal, their forces will be directly as the quantities of matter contained in them, that is, as their weights.*

3. *If both the quantities of matter and the velocities be unequal, the forces, with which the bodies are moved, will be in a proportion compounded of their quantities of matter and velocities.*

Suppose the battering ram of Vespasian weighed 60000 lb; that it was moved at the rate of 24 feet in one second, and that this was sufficient to demolish the walls of Jerusalem; With what velocity must a cannon ball, which weighs 42 lb, be moved, to do the same execution?

The velocity of the ram being 24, and the weight of the ball 42, compounded, will make a fraction $= \frac{24}{42} = \frac{4}{7}$, and $\frac{4}{7} \times 60000 = 34285\frac{5}{7}$ feet in a second, *Ans.*

51. A body weighing 30 lb is impelled by such a force as to send it 20 rods in a second; With what velocity would a body weighing 12 lb move, if it were impelled by the same force?

$$\frac{30 \times 20}{12} = 50 \text{ rods in a second, } \textit{Ans.}$$

Of GRAVITY.

52. *The gravity of bodies above the surface of the earth decreases in a duplicate ratio (or as the squares of their distances) in semidiameters of the earth, from the earth's centre.*

Supposing a body to weigh 400 lb at 2000 miles above the earth's surface; What would it weigh at the surface, estimating the earth's semidiameter at 4000 miles?

From the centre to the given height being $1\frac{1}{2}$ semidiameter; multiply the square of $1\frac{1}{2}$ by the weight, and the product will be the answer. $1,5 \times 1,5 \times 400 = 900$ lb, *Ans.*

53. If a body weigh 900 lb at the surface of the earth; What will it weigh at 2000 miles above the surface?

This being the reverse of the last, therefore, $1 + ,5 = 1,5$ and $900 \div \overline{1,5 \times 1,5} = 400$ lb, *Ans.*

54. A

54. A certain body on the ſurface of the earth, weighs 180 ℔; How high muſt it be carried, to weigh but 20 ℔?

$\sqrt{180 \div 20} = 3$, *Anſ.* 3 ſemidiameters from the earth's centre, that is 8000 above its ſurface.

55. How high muſt a ball be raiſed, to loſe *half* its weight?

As $1 : 4000 \times 4000 :: 2 : 32000000$, and $\sqrt{32000000} = 5656{,}85$, and $5656{,}85 - 4000 = 1656{,}85$ miles, *Anſ.*

56. If the attraction of the moon raiſe a tide on the earth 5 feet high; What will be the height of a tide, raiſed by the earth on the ſurface of the moon under ſimilar circumſtances?

The attraction of one of thoſe bodies to the other's ſurface is directly as its quantity of matter, and inverſely as its diameter; therefore, As $2182 \times 2182 \times 2182 \times 494 : 5 :: 8000 \times 8000 \times 8000 \times 400 : 199{,}3$, And as $2182 : 199{,}3 :: 8000 : 54{,}8$ feet, inverſely, *Anſ.*

57. 1. *If the diameters of two globes be equal, and their denſities, (compactneſs, or cloſeneſs) different; the weight of a body on their ſurfaces will be as their denſities.*

2. *If their denſities be equal, and their diameters different; the weight of a body will be as $\frac{1}{3}$ of their circumferences.*

3. *If their diameters and denſities be both different, the weight will be as $\frac{2}{3}$ of their ſemidiameters multiplied by their denſities.*

If a ſtone weigh 100 ℔ at the ſurface of the earth; What will it weigh at the ſurfaces of the ſun, and the ſeveral planets, whoſe denſities are known, reſpectively?

	Sun.	*Jupiter.*	*Saturn.*	*Earth.*	*Moon.*
Their denſities	100.	94,5.	67.	400.	494
Diam. in geog. miles	776970.	135079.	98566.	6875.	1869,35
Semidiam.	388485.	67539,5.	49283.	3437,5.	934,67
$\frac{2}{3}$ Sem. Diam.	258990.	44693.	32855,33.	2295.	623,11

Then, As $2295 \times 400 : 100 ::$
- $258990 \times 100 : 2821.24$ ℔ at the Sun.
- $44693 \times 94{,}5 : 460$ ℔ at Jupiter.
- $32855{,}33 \times 67 : 239{,}8$ ℔ at Saturn.
- $623{,}11 \times 494 : 33{,}5$ at the Moon.

Of the Fall of Bodies.

58. Heavy bodies near the ſurface of the earth, fall one foot the firſt quarter of a ſecond; three feet the ſecond quarter; five feet in the third, and ſeven feet in the fourth quarter; that is, 16 feet in the firſt ſecond.*

The velocities, acquired by bodies in falling, are in proportion to the ſquares of the times in which they fall; for inſtance, Let

* The exact velocity in *vacuo* is 16,1 in the ſecond; but in the air it will be ſcarcely 16 feet.

Let go three bullets together ; ſtop the firſt at one ſecond, and it will have fallen 16 feet. Stop the next at the end of the ſecond ſecond, and it will have fallen ($2 \times 2 = 4$) four times 16, or 64 feet ; and ſtop the laſt at the end of the third ſecond, and the diſtance fallen will be ($3 \times 3 = 9$) nine times 16, or 144 feet, and ſo on.

Or, which is the ſame, the ſpace fallen through (in feet) is always equal to the ſquare of the time in 4ths of a ſecond.

Or, By multiplying 16 feet by ſo many of the odd numbers, beginning at unity, as there are ſeconds in any given time ; viz. by 1 for the firſt ſecond, by 3 for the ſecond, by 5 for the third, and ſo on, theſe ſeveral products will give the ſpaces fallen through, in each of the ſeveral ſeconds, and their ſum will be the whole diſtance fallen.

The velocity given, to find the ſpace fallen through.

Rule 1.—The ſquare root of the feet, in the ſpace fallen through, will ever be equal to one eighth of the velocity acquired at the end of the fall ; therefore,

2. Divide the velocity by 8, and the ſquare of the quotient will be the diſtance fallen through, to acquire that velocity.

Suppoſe the velocity of a cannon ball to be about $\frac{1}{8}$ of a mile, or 660 feet per ſecond ; From what height muſt a body fall, to acquire the ſame velocity per ſecond ?

$660 \div 8 = 82{,}5$ and $82{,}5 \times 82{,}5 = 6806\frac{1}{4}$ feet, $= 1\frac{37}{128}$ mile, *Anſ.*

59. *The time given, to find the ſpace fallen through.*

Rule 1.—The ſquare root of the feet, in the ſpace fallen through, will ever be equal to four times the number of ſeconds the body has been falling : Therefore,

2. Multiply the time by 4, and the ſquare of the products will be the ſpace fallen through in the given time.

How many feet will a body fall in 5 ſeconds ?

$5 \times 4 = 20$, and $20 \times 20 = 400$ feet, *Anſ.*

60. A bullet is dropped from the top of a building, and found to reach the ground in $1\frac{3}{4}$ ſecond ; Required its height ?

$1{,}75 \times 4 = 7$, and $7 \times 7 = 49$ feet, *Anſ.* Or, $1\frac{3}{4} = 7$ *qrs.* and $7 \times 7 = 49$. Or, $1{,}75 \times 1{,}75 \times 16 = 49$ feet, *Anſ.*

61. What is the difference between the depth of two wells, into each of which ſhould a ſtone be dropped in the ſame inſtant, one would reach the bottom in 5 ſeconds, and the other in 3 ?

$5 \times 4 = 20$, and $20 \times 20 = 400$ feet.
$3 \times 4 = 12$, and $12 \times 12 = 144$ feet.

Anſ. 256 feet.

62. Aſcending bodies are retarded in the ſame ratio that deſcending bodies are accelerated ; therefore, if a ball, diſcharged from

from a gun, returned to the earth in 12 ſeconds ; How high did it aſcend ?

The ball being half of the time, or 6 ſeconds, in its aſcent, therefore, $6 \times 4 = 24$, and $24 \times 24 = 576$ feet, *Anſ.*

63. *The velocity per ſecond given, to find the time.*

Rule 1.—Four times the number of ſeconds, in which a body has been falling, is equal to one eighth of the velocity, in feet, per ſecond, acquired at the end of the fall : Therefore,

2. Divide the given velocity by 8, and one fourth part of the quotient will be the anſwer.

How long muſt a bullet be falling to acquire a velocity of 160 feet per ſecond ?

$160 \div 8 = 20$, and $20 \div 4 = 5$ ſeconds, *Anſ.*

64. *The ſpace, through which a body has fallen, given, to find the time it has been falling.*

Rule 1.—Four times the number of ſeconds, in which the body has been falling, will ever be equal to the ſquare root of the ſpace, in feet, through which it has fallen : Therefore,

2. Divide the ſquare root of the ſpace fallen through by 4, and the quotient will be the time, in which it was falling.

In how many ſeconds will a bullet fall through a ſpace of 10125 feet ?

$\sqrt{10125} = 100{,}6$, and $100{,}6 \div 4 = 25{,}1\dot{5}$ ſeconds, $= 25'' \; 9'''$ *Anſ.*

65. In what time will a muſket ball, dropped from the top of a ſteeple 484 feet high, come to the ground ?

$\sqrt{484} = 22$, and $22 \div 4 = 5\frac{1}{2}$ ſeconds, *Anſ.*

66. *To find the velocity per ſecond, with which a heavy body will begin to deſcend, at any diſtance from the earth's ſurface.*

Rule.—As the ſquare of the earth's ſemidiameter is to 16 feet : So is the ſquare of any other diſtance from the earth's centre inverſely, to the velocity with which it begins to deſcend per ſecond.

With what velocity per ſecond will an iron ball begin to deſcend, if raiſed 3000 miles above the earth's ſurface ?

As $4000 \times 4000 : 16 :: \overline{4000+3000} \times \overline{4000+3000} : 5{,}22449$ feet, *Anſ.*

67. How high muſt a ball be raiſed above the earth's ſurface, to begin to deſcend with a velocity of 5,22449 feet per ſecond ?

As $16 : 4000 \times 4000 :: 5{,}22449 : 49000000$, and $\sqrt{49000000} = 7000$,
Wherefore, $7000 - 4000 = 3000$ miles, *Anſ.*

68. *To find the mean velocity of a falling body.*

Rule.—Divide the ſpace fallen through by the number of ſeconds it was falling, and the quotient will be the mean velocity.

A muſket ball dropped from the top of a ſteeple 484 feet high in $5\frac{1}{2}$ ſeconds ; Required its mean velocity ?

$484 \div 5{,}5 = 88$ feet per ſecond, *Anſ.*

69. *To*

69. *To find the velocity acquired by a falling body per ſecond (or by a ſtream of water. having the perpendicular deſcent given) at the end of any given period of time.*

Rule 1.—The velocity acquired at the end of any period is equal to twice the mean velocity, with which it paſſed during that period.

Or, 2. Multiply the perpendicular ſpace fallen through by 64, and the ſquare root of the product is the velocity required.

If a ball fall through a ſpace of 484 feet in $5\frac{1}{2}$ ſeconds; with what velocity will it ſtrike?

By the former part of the rule.
$484 \div 5{,}5 = 88$, and
$88 \times 2 = 176$, *Anſ.*

By the latter part, without regarding the time.
$\sqrt{484 \times 64} = 176$, *Anſ.*

70. There is a ſluice, (or flume) one end of which is $2\frac{1}{2}$ feet lower than the other; What is the velocity of the ſtream per ſecond? $2{,}5 \times 64 = 160$, and $\sqrt{160} = 12{,}649$ feet, *Anſ.*

71. *The velocity with which a falling body ſtrikes, given, to find the ſpace fallen through.*

Rule.—Divide the ſquare of the velocity by 64, and the quotient wil be the height required.

If a ball ſtrike the ground with a velocity of 56 feet per ſecond; from what height did it fall?

$\overline{56 \times 56} \div 64 = 49$ feet, *Anſ.*

72. The mean velocity of a fluid, or ſtream, is 12,649 feet per ſecond; What is the perpendicular fall of the ſtream?

$\overline{12{,}649 \times 12{,}649} \div 64 = 2\frac{1}{2}$ feet, *Anſ.*

73. *The weight of a body, and the ſpace fallen through, given, to find the force with which it will ſtrike.*

Rule.—The momentum, or force, with which a falling body ſtrikes, is equal to its weight multiplied by its velocity; therefore, find the velocity by Problem 69th, and multiply it by the weight, which will produce the force required.

If the rammer, uſed for driving the piles of Charleſtown bridge, weighed $2\frac{1}{4}$ tons, or 4500lb, and fell through a ſpace of 10 feet, with what force did it ſtrike the pile?

$\sqrt{10 \times 64} = 25{,}3$ = velocity, and $25{,}3 \times 4500 = 113850$lb, momentum, *Anſ.*

74. *The weight and momentum, or ſtriking force, given, to find the ſpace fallen through.*

Rule.—Divide the momentum by the weight, and the quotient will be the velocity; then divide the ſquare of the velocity by 64, and the quotient will be the ſpace fallen through.

If the aforementioned rammer weighed 4500lb, and ſtruck with a force of 113850 lb; From what height did it fall?

$113850 \div 4500 = 25{,}3$, and $\overline{25{,}3 \times 25{,}3} \div 64 = 10$ feet, *Anſ.*

75. If it were required to know with what quantity of motion, momentum, or force, a fluid, moving with a given velocity, ſtrikes upon a fixed obſtacle.

RULE.—By Problem 71ſt, find the fall, which will produce the given velocity: Multiply that height by 62,5℔, Avoird. for clean river water, by 63℔, for dirty water, and by 64 for ſea water.

Suppoſe a ſtream of clear water to move at the rate of 5 feet per ſecond, and to meet with a fixed obſtacle (or bulk head) 15 feet wide and 4 feet high; What is the momentary, inſtantaneous preſſure of the ſtream?

$\overline{5\times5}\div64=\frac{25}{64}$ and $25\div64=,39$ of a foot, for the perpendicular fall of the water. Now $62,5\times,39=24,375$℔, the preſſure upon each ſquare foot, which, multiplied by 60 (the number of ſquare feet in the obſtacle) gives 1462,5℔, going with the given velocity of 5 feet per ſecond; therefore, $1462,5\times5=7312,5$℔, *Anſ.**

76. The velocity of water, ſpouting through a ſluice, or aperture in a reſervoir, or a bulk head, is the ſame that a body would acquire by falling through a perpendicular ſpace equal to that between the top of the water in the reſervoir, and the aperture.

What is the velocity of water iſſuing from a head of 5 feet deep?

By Problem 69*th*. $64\times5=320$, and $\sqrt{320}=18$ feet, nearly, *Anſ.*

77. If the velocity of a ſtream iſſuing through the bulk head of a mill, be 16 feet per ſecond; What head of water is there?

$\overline{16\times16}\div64=4$ feet, *Anſ.*

78. The quantity of water diſcharged from a hole in a veſſel, is as the ſquare root of the height of water above the aperture.

A miller has a head of water 4 feet above the ſluice; How high muſt the water be raiſed above the opening, ſo that half as much again water may be diſcharged from the ſluice in the ſame time?

$\sqrt{4}=2$, and half as much again as 2, is $2+1=3$, for the ſquare root of the required depth; therefore, $3\times3=9$ feet high, *Anſwer.*

Of PENDULUMS.

79. The time of a vibration, in the cycloid, is to the time of a heavy body's deſcent through half its length, as the circumference of a circle to its diameter, that is, As 3,1416 to 1: Therefore, (as a body deſcends freely, by gravity, through about 193,5 inches in the firſt ſecond) To find the length of a pendulum, vibrating ſeconds.

RULE.

* Water being a yielding ſubſtance, loſes two thirds of its power in producing effects.

RULE.—As $3,1416 \times 3,1416 : 1 \times 1 :: 193,5 : 19,6$ inches, the half length, and $19,6 \times 2 = 39,2$ inches, the length.

80. *To find the length of a pendulum, that will ſwing any given time.*

RULE.—Multiply the ſquare of the ſeconds in any given time by 39,2, and the product will be the length required, in inches.

Required the lengths of ſeveral pendulums, which will reſpectively ſwing $\frac{1}{4}$ ſeconds, $\frac{1}{2}$ ſeconds, ſeconds, minutes, and hours?

$,25 \times ,25 \times 39,2 = 2,45$ inches for $\frac{1}{4}$ ſeconds. $,5 \times ,5 \times 39,2 = 9,8$ inches for $\frac{1}{2}$ ſeconds. $1 \times 1 \times 39,2 = 39,2$ inches for ſeconds, as above; $60 \times 60 \times 39,2 =$ the inches in 2 miles and 1200 feet, for minutes; and 1 hour $= 3600$ ſeconds, therefore $3600 \times 3600 \times 39,2 =$ the inches in 8018 miles and 96 feet, for hours, *Anſ.*

81. What is the difference between the length of a pendulum, which vibrates half ſeconds, and one which ſwings 3 ſeconds?

$\overline{3 \times 3 \times 39,2} - \overline{,5 \times ,5 \times 39,2} = 28\frac{7}{12}$ feet, *Anſ.*

82. *To find the time which a pendulum of any given length will ſwing.*

RULE.—Divide the given length by 39,2, and the quotient will be the ſquare of the time in ſeconds.

Or, As 6,261 (the ſquare root of 39,2) is to the ſquare root of the given length: So is 1 ſecond, to the time of one oſcillation: That is, divide the ſquare root of the given length by 6,261, and the quotient will be the time of one vibration of that pendulum.

How often will a pendulum of 9,8 inches vibrate in a ſecond?

By the former part of the rule, $9,8 \div 39,2 = ,25$ of a ſecond, and $\sqrt{,25} = ,5$ of a ſecond, the time of one vibration, that is, it vibrates half ſeconds, or $60 \div ,5 = 120$ times in a minute.

By the latter part. $\sqrt{9,8} = 3,13$, and $\sqrt{39,2} = 6,261$, therefore, $3,13 \div 6,261 = ,5$ of a ſecond.

83. I obſerved, that while a ſtone was falling from a precipice, a ſtring, (with a bullet at the end) which meaſured 25 inches, (to the middle of the ball) had made 5 vibrations; What was the height of the precipice?

$25 \div 39,2 = ,6377$, and $\sqrt{,6377} = ,7985$ of a ſecond, the time of one vibration, and $,7985 \times 5 = 4$ ſeconds. nearly, the time of the ſtone's deſcent; then $4 \times 4 = 16$, and $16 \times 16 = 256$ feet, *Anſ.*

84. *To find the true depth of a well, by dropping a ſtone into it, alſo the time of the ſtone's deſcent, and of the ſound's aſcent.*

RULE 1.—Take a line of any length, and by the laſt Problem find the time from the dropping of the ſtone till you hear it ſtrike the bottom.

2. Multiply 73088 ($= 16 \times 4 \times 1142$; 1142 feet being the diſtance, which ſound moves in a ſecond) by the number of ſeconds till you hear the ſtone ſtrike the bottom.

3. To

3. To this product add 1304164 (= the square of 1142) and from the square root of the sum take 1142.

4. Divide the square of the remainder by 64 (= 16×4) and the quotient will be the depth of the well in feet.

5. Divide the depth by 1142, and the quotient will be the time of the sound's ascent, which, being taken from the whole time, will leave the time of the stone's descent in seconds.

Suppose I drop a stone into a well, and a string with a plummet, which measured to the middle of the ball 25 inches, made 5 vibrations before I heard the stone strike the bottom; Required the depth, time of the stone's descent, and of the sound's ascent?

$25 \div 39{,}2 = {,}6377$, and $\sqrt{,6377} = {,}7985$, and ${,}7985 \times 5 = 4$ seconds to the hearing of it strike; then $\sqrt{73088 \times 4 + 1304164} - 1142 = 121{,}53$ and $121{,}53 \times 121{,}53 \div 64 = 230{,}77$ feet, the depth, and $230{,}77 \div 1142 = {,}2$ of a second, the time of the sound's ascent, and $4 - {,}2 = 3{,}8$ seconds, the time of the stone's descent.

Of the LEVER *or* STEELYARD.

85. It is a principle in mechanics, that the power is to the weight, as the velocity of the weight, to the velocity of the power. Therefore, to find what weight may be raised or balanced by any given power, say;

As the distance between the body to be raised or balanced, and the fulcrum, or prop, is to the distance between the prop and the point where the power is applied: So is the power to the weight which it will balance.

If a man, weighing 160 ℔, rest on the end of a lever 10 feet long; What weight will he balance on the other end, supposing the prop one foot from the weight?

The distance between the weight and prop being 1 foot, the distance from the prop to the power is 10—1 = 9 feet; therefore, As 1 foot : 9 feet : : 160 ℔ : 1440 ℔, *Ans.*

86. If a weight of 1440 ℔ were to be raised with a lever 10 feet long, and the prop fixed 1 foot from the weight; What power, or weight, applied to the other end of the lever, would balance it? As 9 : 1 : : 1440 : 160 ℔, *Ans.*

87. If a weight of 1440 ℔ be placed 1 foot from the prop; at what distance from the prop must a power of 160 ℔ be applied, to balance it? As 160 : 1440 : : 1 : 9 feet, *Ans.*

88. At what distance from a weight of 1440 ℔ must a prop be placed, so as that a power of 160 ℔ applied 9 feet from the prop, may balance it? As 1440 : 160 : : 9 : 1 foot, *Ans.*

89. In giving directions for making a chaise, the length of the shafts, between the axletree and backband, being settled at 9 feet, a dispute arose whereabout on the shafts the centre of the body should

ſhould be fixed. The chaiſemaker adviſed to place it 30 inches before the axletree; others ſuppoſed 20 inches would be a ſufficient incumbrance for the horſe: Now, ſuppoſing two paſſengers to weigh 3 *Cwt.* and the body of the chaiſe $\frac{3}{4}$ *Cwt.* more; What will the beaſt in both theſe caſes bear, more than his harneſs?

Weight of the chaiſe and paſſengers $3\frac{3}{4}$ *Cwt.* = 420 ℔, and 9 feet = 108 inches.

$$\text{Then, As } \overset{In.}{108} : \overset{℔}{420} :: \left\{ \begin{array}{l} \overset{in.}{30} : \overset{℔}{116\frac{2}{3}} \\ 20 : 77\frac{7}{9} \end{array} \right\} \textit{Anſ.}$$

Of the WHEEL *and* AXLE.

90. The proportion for the wheel and axle (in which the power is applied to the circumference of the wheel, and the weight is raiſed by a rope, which coils about the axle as the wheel turns round) is, As the diameter of the axle is to the diameter of the wheel; ſo is the power applied to the wheel, to the weight ſuſpended by the axle.

A mechanic would make a windlaſs in ſuch a manner, as that 1 ℔ applied to the wheel, ſhould be equal to 10 ℔ ſuſpended from the axle; now, ſuppoſing the axle to be 6 inches diameter; Required the diameter of the wheel?

As $\overset{℔}{10} : \overset{in.}{6} :: \overset{℔}{1} : \overset{in.}{60}$ inverſely, the diameter required.

91. Suppoſe the diameter of the wheel to be 60 inches; Required the diameter of the axle, ſo as that 1 ℔ on the wheel may balance 10 ℔ on the axle?

Inverſely, As $\overset{℔}{1} : \overset{in.}{60} :: \overset{℔}{10} : \overset{in.}{6}$ diameter required.

92. Suppoſe the diameter of the axle 6 inches, and that of the wheel 60 inches; What power at the wheel will balance 10 ℔ at the axle?

Inverſely, As $\overset{in.}{6} : \overset{℔}{10} :: \overset{in.}{60} : \overset{℔}{1}$ *Anſ.*

93. Suppoſe the diameter of the wheel 60 inches, and that of the axle 6 inches; What weight at the axle will balance 1 ℔ at the wheel?

Inverſely, As $\overset{in.}{60} : \overset{℔}{1} :: \overset{in.}{6} : \overset{℔}{10}$ *Anſ.*

Of the SCREW.

94. The power is to the weight, which is to be raiſed, as the diſtance between two threads of the ſcrew, is to the circumference of a circle deſcribed by the power applied at the end of the lever.

RULE.—Find the circumference of the circle deſcribed by the end of the lever; then, As that circumference is to the diſtance between the ſpiral threads of the ſcrew; ſo is the weight to be raiſed, to the power which will raiſe it, abating the friction, which

 is

is not proportional to the quantity of ſurface; but to the weght of the incumbent part; and, at a medium, $\frac{1}{3}$ part of the effect of the machine is deſtroyed by it, ſometimes more and ſometimes leſs.

There is a ſcrew, whoſe threads are an inch aſunder; the lever by which it is turned, 30 inches long, and the weight to be raiſed a ton, or 2240 ℔; What power or force muſt be applied to the end of the lever, ſufficient to turn the ſcrew—that is, to raiſe the weight?

The lever being the ſemidiameter of the circle, the diameter is 60 inches; then, $3,1416\times 60 = 188,496$ inches, the circumference:

℔ ℔ *in.* *in.*

Therefore, As 188,496 : 1 : : 2240 : 11,88, *Anſ.*

95. Let the lever be 30 inches, (the circumference of which is found to be 188,496) the threads 1 inch aſunder, and the power 11,88 ℔; Required the weight to be raiſed?

in. *in.* ℔ ℔

As 1 : 188,496 : : 11,88 : 2240 nearly, *Anſ.*

96. Let the weight be 2240 ℔, the power 11,88 ℔, and the lever 30 inches; Required the diſtance between the threads?

℔ ℔ *in.* *in.*

As 2240 : 11,88 : : 188,496 : 1 nearly, *Anſ.*

97. Let the power be 11,88 ℔, the weight 2240 ℔, and the threads an inch aſunder, to find the length of the lever?

℔ ℔ *in.* *in.*

As 11,88 : 2240 : : 1 : 188,5; Then, As 355 : 113 : : 188,5 : 60 inches nearly, the diameter, and $60\div 2 = 30$ inches, *Anſ.*

98. Suppoſe one of thoſe meteors, called fire balls, to move parallel to the earth's ſurface, and 50 miles from it, at the rate of 20 miles per ſecond; In what time would it move round the earth?

Suppoſe the earth's diameter 8000 miles, then $8000+\overline{50\times 2} =$ 8100 the diameter of the circle deſcribed by the ball: Then, As 113 : 355 : : 8100 : 25446,9 miles nearly, its circumference, and $25446,9\div 20 = 1272,345$ ſeconds, $= 21'\ 12''\ 19'''$ *Anſ.*

99. Sound, uninterrupted, moves about 1142 feet in a ſecond; How long, then, after firing of a cannon at Newburyport, before it will be heard at Ipſwich, eſtimating the diſtance at 10 miles in a right line?

10 miles $= 52800$ feet, and $52800\div 1142 = 46\frac{134}{571}$ ſeconds, *Anſ.*

100. In a thunder ſtorm I obſerved by my clock that it was 6 ſeconds between the lightning and thunder; at what diſtance was the exploſion? $1142\times 6 = 6852$ feet $= 1\frac{13}{120}$ mile, *Anſ.*

101. Tubes may be made of gold, weighing not more than at the rate of $\frac{1}{1625}$ of a grain per foot; What would be the weight of ſuch a tube, which would extend acroſs the Atlantic, from Boſton to London, eſtimating the diſtance at 1000 leagues?

$1000\times 3 = 3000$ miles, and $3000\times 5280 = 15840000$ feet, and $15840000\times\frac{1}{1625} = 9747\frac{69}{100}$*gr.* or rather, 1℔ 8*oz.* 6*pwt.* $3\frac{69}{100}$*gr.* *Anſ.*

102. The

102. The mean diſtances of the planets from the Sun in Engliſh miles are as follows; viz. Mercury 36841468; Venus 68891486; the Earth 95173000; Mars 145014148; Jupiter 494990976; Saturn 907956130, and the Moon 240000 from the Earth: Now, as a cannon ball, at its firſt diſcharge, flies about a mile in 8 ſeconds, and ſound, 1142 feet per ſecond; How long would a bullet, at the aforementioned rate, be in paſſing from the Earth to the Sun, and ſound in moving from the Sun to Saturn?

95173000 × 8″ = 24 years, 52 days, 7 hours, 33 minutes, 20 ſeconds, for the paſſage of the ball; and 907956130 × 5280 = 4794008366400 feet, and 4794008366400 ÷ 1142 = 133*yrs.* 41*d.* 20*h.* 55*m.* $49\frac{521}{571}$*s.* So long would ſound be in paſſing from the Sun to Saturn.

103. Light paſſes from the Sun to the Earth in about 8 minutes; How long would it be in paſſing from the Sun to *Herſchell's* Planet, or the *Georgium Sidus*, ſuppoſing it to be 5000000000 miles?

As 95173000 : 8′ :: 5000000000 : 7*h.* 0*m.* 17*s.* 7‴, *Anſ.*

104. The diameter of the Sun is 890000 miles; Mercury's diameter 3000; Venus's 7924; the Earth's 7970; Mars's 7338; Jupiter's 94000; Saturn's 78000, and the Moon's 2182; What is the comparative magnitude between the Sun and the Earth, and between the Earth and all the others?

∴ The Sun = 890000 × 890000 × 890000 ÷ 7970 × 7970 × 7970 = 1392499,52 times larger than the Earth. The Earth = 7970 × 7970 × 7970 ÷ 3000 × 3000 × 3000 = 18,75 larger than Mercury. = 7970 × 7970 × 7970 ÷ 7924 × 7924 × 7924 = 1,0175 times larger than Venus. = 7970 × 7970 × 7970 ÷ 5400 × 5400 × 5400 = 3,21 times larger than Mars. = 7970 × 7970 × 7970 ÷ 2182 × 2182 × 2182 = 48,82 times larger than the Moon. Jupiter = 94000 × 94000 × 94000 ÷ 7970 × 7970 × 7970 = 1640,62 times larger than the Earth. Saturn = 78000 × 78000 × 78000 ÷ 7970 × 7970 × 7970 = 937,36 times larger than the Earth.

105. The denſity of the Moon is to that of the Earth, as 123,5 to 100: What is the proportion between the quantity of matter in the Earth and that of the Moon, allowing the Earth's diameter to be 7970, and the Moon's 2182 miles, and ſuppoſing the Earth to be a complete ſphere, which however it is not?

There is $\frac{7970 \times 7970 \times 7970 \times 100}{2182 \times 2182 \times 2182 \times 123,5} = 39,534$ times the quantity of matter in the Earth more than is in the Moon. In other words, the Earth weighs ſo much more than the Moon.

106. The mean diameter of the Earth's orbit (or annual path round the Sun) is 191263000 miles; Required its mean motion, (or the ſpace through which it moves,) per minute?

191263000 × 3,1416 = 600871840,8 miles circumference; then,

Days.	*Miles.*	*Miles.*
As 365,25 :	600871840,8 :: 1′ :	1142,44 *Anſ.*

N. B. The Earth's diurnal motion round its axis is $17\frac{1}{4}$ miles per minute, at the equator.

Qr

Of the SPECIFIC GRAVITIES *of* BODIES.

The ſpecific gravities of bodies are as their denſities, or weights, bulk for bulk; thus, a body is ſaid to have two or three times the ſpecific gravity of another, when it contains two or three times as much matter in the ſame ſpace.

A body, immerſed in a fluid, will ſink, if it be heavier than its bulk of the fluid. If it be ſuſpended therein, it will loſe ſo much of what it weighed in the air, as its bulk of the fluid weighs. Hence, all bodies of equal bulks, which will ſink in fluids, loſe equal weights when ſuſpended therein, and unequal bodies loſe in proportion to their bulks.

The *hydroſtatic balance* differs very little from a common balance that is nicely made; only it has a hook at the bottom of each ſcale, on which ſmall weights may be hung by horſe hairs, ſo that a body ſuſpended by the hair, may be immerſed in water without wetting the ſcales.

How to find the Specific Gravities of Bodies.

If the body thus ſuſpended under the ſcale, at one end of the balance, be firſt counterpoiſed in air by weights in the oppoſite ſcale, and then immerſed in water, the equilibrium will be immediately deſtroyed; then, if as much weight be put into the ſcale, to which the body is ſuſpended, as will reſtore the equilibrium, (without altering the weights in the oppoſite ſcale) that weight, which reſtores the equilibrium, will be equal to a quantity of water as big as the immerſed body; and if the weight of the body in air be divided by what it loſes in water, the quotient will ſhew how much that body is heavier than its bulk of water. Thus, if a guinea, ſuſpended in air, be counterbalanced by 129 grains in the oppoſite ſcale, and then, upon being immerſed in water, it becomes ſo much lighter, as to require $7\frac{1}{4}$ grains to be put into the ſcale over it, to reſtore the equilibrium, it ſhews that a quantity of water, of equal bulk with the guinea, weighs 7,25 grains; by which divide 129 (the weight of the guinea in air) and the quotient will be 17,793; which ſhews that the guinea is 17,793 times as heavy as its bulk of water.

Thus may any piece of gold be tried, by weighing it firſt in air, and then in water; and if, upon dividing the weight in air by the loſs in water, the quotient comes out 17,793, the gold is good: If the quotient be 18, or between 18 and 19, the gold is very fine; but, if it be leſs than 17, the gold is too much alloyed, by being mixed with ſome other metal.

If ſilver be tried in this manner and found to be 11 times as heavy as water, it is very fine: If it be $10\frac{1}{2}$ times as heavy, it is ſtandard; but if it be of any leſs weight, compared with water, it is mixed with ſome lighter metal, ſuch as tin, &c.

If a piece of braſs, glaſs, lead, or ſilver, be immerſed and ſuſpended in different ſorts of fluids, the different loſſes of weight therein will ſhew how much heavier it is than its bulk of the

fluid;

fluid ; that fluid being lighteſt, in which the immerſed body loſes leaſt of its ærial weight.

Common clear water, for common uſes, is generally made a ſtandard for comparing bodies by, whoſe gravity may be repreſented by unity, or 1, or, in caſe great accuracy be required, by 1,000, where 3 cyphers are annexed to give room to expreſs the ratios of other gravities in larger numbers in the table. In doing this there is a twofold advantage ; the firſt is, that, by this mean, the ſpecific gravities of bodies may be expreſſed to a much greater degree of accuracy. The ſecond is, that the numbers of the Table, conſidered as whole numbers, do alſo expreſs the ounces Avoirdupois contained in a cubic foot of every ſort of matter therein ſpecified ; becauſe a cubic foot of common water is found by experiment to weigh very nearly 1000 ounces Avoirdupois, or $62\frac{1}{2}$ pounds.

A TABLE *of the Specific Gravities of ſeveral ſolid and fluid Bodies ; where the ſecond Column contains their Abſolute Weight, and the third, their Relative Weight, in Avoirdupois Ounces.*

A Cubic Foot of	*Abſo. wt.*	*Rela. wt.*	*A Cubic Foot of*	*Abſo. wt.*	*Rela. wt.*
Platina rendered malleable and hammered	20170	20,170	Brick	2000	2,000
			Live Sulphur	2000	2,000
Very fine Gold	19637	19,637	Nitre	1900	1,900
Standard Gold	18888	18,888	Alabaſter	1875	1,875
Guinea Gold	17793	17,793	Dry Ivory	1825	1,825
Moidore Gold	17140	17,140	Brimſtone	1800	1,800
Quickſilver	13600	13,600	Solid ſubſ. of Gun Pow.	1745	1,745
Lead	11325	11,325	Alum	1714	1,714
Fine Silver	11087	11,087	Ebony	1117	1,117
Standard Silver	10535	10,535	Human Blood	1054	1,054
Roſe Copper	9000	9,000	Amber	1030	1,030
Copper	8843	8,843	Cows' Milk	1030	1,030
Plate Braſs	8000	8,000	Sea Water	1030	1,030
Steel	7852	7,852	Pure Water	1000	1,000
Caſt Braſs	7850	7,850	Red Wine	993	0,993
Iron	7645	7,645	Oil of Amber	978	0,978
Block Tin	7321	7,321	Proof Spirits*	925	0,925
Caſt Iron	7135	7,135	Dry Oak	925	0,925
Lead Ore	6800	6,800	Olive Oil	913	0,913
Copper Ore	3775	3,775	Looſe Gun Powder	872	c,872
Diamond	3400	3,400	Spirit of Turpentine	864	0,864
Chryſtal Glaſs	3150	3,150	Alcohol or pure Spirit	850	0,850
White Marble	2707	2,707	Elm and Aſh	800	0,800
Black ditto	2704	2,704	Oil of Turpentine	772	0,772
Rock Chryſtal	2658	2,658	Dry Crab Tree	765	0,765
Green Glaſs	2620	2,620	Æther	732	0,732
Clear Glaſs	2600	2,600	White Pine	569	0,569
Stone: Flint	2582	2,582	Saſſafras Wood	482	0,482
Stone: Paving	2570	2,570	Cork	240	0,240
Stone: Cornelian	2568	2,568	Common Air	$1\frac{25}{100}$	0,00125
Stone: Free	2352	2,352	Inflammable Air	$0\frac{12}{100}$	0,00012

Specific

* Although the ſpecific gravity be 925 according to theory, yet, in fact, it will amount to about 927.

Specific Gravities of the Solar Syſtem.

A cubic Foot of	*Abſo. wt.*	*Relat. wt.*
The Sun	11333	11,333
Mercury	9166	9,166
Venus	5733	5,733
Earth	4500	4,500
Mars	3286	3,286
Moon	3092	3,092
Jupiter	1042	1,042
Saturn	406	0,406

The uſe of the Table of Specific Gravities will beſt appear by ſeveral Examples.

How to diſcover the quantity of adulteration in metals.

Suppoſe a body be compounded of gold and ſilver, and it be required to find the quantity of each metal in the compound.

Firſt find the ſpecific gravity of the compound, by weighing it in air and in water, and dividing its aerial weight by what it loſes thereof in water, and the quotient will ſhew its ſpecific gravity, or how many times it is heavier than its bulk of water. Then, ſubtract the ſpecific gravity of ſilver (found in the Table) from that of the compound, and the ſpecific gravity of the compound from that of the gold: The firſt remainder will ſhew the bulk of gold, and the latter, the bulk of ſilver in the whole compound; and if theſe remainders be multiplied by the reſpective ſpecific gravities, the products will ſhew the proportional weights of each metal in the body.

Suppoſe the ſpecific gravity of the compounded body be 14; that of ſtandard ſilver (by the Table) is 10,535, and that of ſtandard gold 18,888; therefore, 10,535 from 14, remains 3,465, the proportional *bulk* of the gold in the compound; and 14 from 18,888, remains 4,888, the proportional *bulk* of ſilver in the compound: Then 18,888, the ſpecific gravity of gold, multiplied by the firſt remainder 3,465, produces 65,447 for the proportional *weight* of gold; and 10,535, the ſpecific gravity of ſilver, multiplied by the laſt remainder, produces 51,495 for the proportional weight of ſilver in the whole body: So that for every 65,447 ounces or pounds of gold, there are 51,495 ounces or pounds of ſilver in the body.

Hence it is eaſy to know whether any ſuſpected metal be genuine, or alloyed or counterfeit, by finding how much heavier it is than its bulk of water, and comparing the ſame with the table; if they agree, the metal is good; if they differ, it is alloyed or counterfeited.

How to try Spirituous Liquors.

A cubic inch of good brandy, rum, or other proof ſpirits, weighs 234 grains; therefore, if a true inch cube of any metal weighs 234 grains leſs in ſpirits than in air, it ſhews the ſpirits are proof: if it loſe leſs of its aerial weight in ſpirits, they are above proof; if it loſe more, they are under proof; for the better the ſpirits are, the lighter they are, and the worſe, the heavier.

Or, let any ſolid, of ſufficient ſpecific gravity, be weighed firſt in air, then in water, and then in another liquid; from its weight

weight in the air take its weight in water, and the remainder is the weight of its bulk of water. From its weight in air take its weight in the other liquid, and the remainder is the weight of the ſame quantity of that liquid. Divide the weight of this quantity of liquid by the weight of the ſame quantity of water, and the quotient will be the ſpecific gravity of the liquid.

All bodies expand with heat, and contract with cold; but ſome more, and ſome leſs than others; therefore, the ſpecific gravities of bodies are not preciſely the ſame in ſummer as in winter.

The four following Problems, relating to Spirituous Liquors, are wrought by Alligation.

107. What proportion of rectified ſpirits of wine muſt be mixed with water, to make proof ſpirit; the ſpecific gravity of the rectified ſpirits being 850, that of proof ſpirit 925, and of water 1000?

$$925\left\{\begin{matrix}1000\\850\end{matrix}\right\}\begin{matrix}75\\75\end{matrix}\Big\}\text{ Or equal meaſures.}$$

108. What proportional *weight* of rectified ſpirits of wine and water muſt be mixed, to make proof ſpirit, the ſpecific gravities as before?

Anſ. $\frac{1000}{850}=\frac{20}{17}$, or as 20 to 17.

109. What is the ſpecific gravity of beſt French brandy, conſiſting of 5 parts, meaſure, of rectified ſpirits of wine, and 3 parts water?

$$\begin{array}{rr} & 850\times5=4250\\ & 1000\times3=3000\\ 5+3= & 8\;)\;7250\\ & 906,25=\text{ſpecific gravity.}\end{array}$$

110. A retailer has 30 gallons of rum, whoſe ſpecific gravity is 900; How much water muſt he add, to reduce it to ſtandard proof?

$$925\left\{\begin{matrix}1000\\900\end{matrix}\right\}\begin{matrix}25\\75\end{matrix}\Big\}\quad\begin{matrix}\textit{G. rum.} & \textit{G. wat.} & \textit{G. rum.} & \textit{G. wat.}\\ \text{As }75 & :\;25\;:: & 30 & :\;10\text{ to be added.}\end{matrix}$$

111. The cubic inch of common glaſs, weighs about 1,36*oz.* Troy; ditto of ſalt water ,5427*oz.* ditto of brandy ,48927*oz.* Suppoſe then, a ſeaman has a gallon of brandy in a bottle, which weighs $4\frac{1}{2}$ ℔ Troy, out of water, and, to conceal it, throws it overboard into ſalt water: Pray, will it ſink or ſwim, and by how much is it heavier or lighter than the ſame bulk of ſalt water?

$4\frac{1}{2}$ ℔ $=54$*oz.* $=$ weight of bot. $\frac{54}{1,36}=39,7059$ cub. in. in the bot.

Add 231 $=$ do. in the brandy.

270,7059 $=$ ditto in both.

Then, $270,7059\times,5427=146,9120z.$ $=$ weight of ſalt water occupied by the bottle and brandy. And ,48927 ($=$ weight of a cubic

cubic inch of brandy) $\times 231 = 113{,}02$ oz. and $113{,}02+54=167{,}02$ oz. = weight of the bottle and brandy. From this take the weight of the salt water, viz. 146,912 oz. and it leaves 20,11 oz. *Anſ.* Suppoſing the bottle full, it is 20,11 oz. heavier than the ſame bulk of ſalt water, and therefore will ſink.

Given the weight to be raiſed by a balloon, to find its diameter.

Rule 1.—As the ſpecific difference between common and inflammable air, is to one cubic foot; ſo is any weight to be raiſed, to the cubic feet contained in the balloon.

2. Divide the cubic feet by ,5236, and the cube root of the quotient will be the diameter required, to balance it with common air; but, to raiſe it, the diameter muſt be ſomewhat greater, or the weight ſomewhat leſs.

112. I would conſtruct a ſpherical balloon of ſufficient capacity to aſcend with 4 perſons weighing one with another 160 lb, and the balloon and a bag of ſand weighing 60 lb; Required the diameter of the balloon?

By the Table of Specific Gravities, page 373d, I find a cubic foot of common air weighs 1,25 ounce Avoirdupois, and a cubic foot of inflammable air ,12 of an ounce Avoirdupois; therefore,

$1{,}25-{,}12=1{,}13$ oz. difference. And $160\text{ lb}\times 4+60\text{ lb}=700\text{ lb}=11200$ oz.

$$\text{As } \overset{\text{oz.}}{1{,}13} : \overset{\text{Cub. foot.}}{1} :: \overset{\text{oz.}}{11200} : \overset{\text{Cub. feet.}}{9911{,}5044}. \text{ And } \sqrt[3]{\frac{9911{,}5044}{{,}5236}} = 26{,}65 \text{ feet, } Anſ.$$

Given the diameter of a balloon, to find what weight it is capable of raiſing.

Rule 1.—Multiply the cube of the diameter by ,5236, and the product will be the content in cubic feet.

2. As one cubic foot is to the ſpecific difference between common and inflammable air; ſo is the content of the balloon to the weight it will raiſe.

113. The diameter of a balloon is 26,65 feet; What weight is it capable of raiſing?

$26{,}65\times 26{,}65\times 26{,}65\times{,}5236 = 9911{,}4+$ cubic feet. And

$$\text{As } \overset{\text{Cub. foot.}}{1} : \overset{\text{oz.}}{1{,}13} :: \overset{\text{Cub. feet.}}{9911{,}4+} : \overset{\text{oz.}}{11199{,}882} = 700 \text{ lb, nearly.}$$

If the magnitude of any body be multiplied by its ſpecific gravity, the product will be its abſolute weight.

114. What weight of lead will cover a houſe, the area of whoſe roof is 6000 feet, and the thickneſs of the lead $\frac{1}{120}$ of a foot?

$6000\times\frac{1}{120}=50$ cub. feet, and its ſpecif. grav. $11325\times 50=566250$ ounces = 15 Tons. 15 Cwt. 3 qrs. 26 lb 10 oz. *Anſ.*

To

To find the magnitude of any thing, when the weight is known.

Divide the weight by the ſpecific gravity in the Table, and the quotient will be the magnitude ſought.

116. What is the magnitude of ſeveral fragments of clear glaſs, whoſe weight is 13 ounces?

$13 \div 2600 = ,005$ of a cubic foot, and $,005 \times 1728 = 8,640$ cubic inches, *Anſ.*

Having the magnitude and weight of any body given, to find its ſpecific gravity.

Divide the weight by the magnitude, and the quotient will be the ſpecific gravity.

117. Suppoſe a piece of marble contains 8 cubic feet, and weighs $1353\frac{1}{2}$℔, or 21656 ounces; What is the ſpecific gravity?

$21656 \div 8 = 2707$. the ſpecific gravity required, as by the Table.

To find the quantity of preſſure againſt the ſluice or bank, which pens water.

Multiply the area of the ſluice, under water, by the depth of the centre of gravity, (which is equal to half the depth of the water) in feet, and that product again by $62\frac{1}{2}$ (the number of pounds Avoirdupois in a cubic foot of freſh water) or by 64,4℔, (the Avoirdupois weight of a cubic foot of ſalt water) and the product will be the number of pounds required.

118. Suppoſe the length of a ſluice or flume be 30 feet, and the depth of the water 4 feet; What is the preſſure againſt the ſide of the ſluice?

$30 \times 4 = 120$ feet, the area of the ſide, and 120×2 (the depth of the centre of gravity) gives 240 cubic feet, and $240 \times 62,5 =$ 15000℔ $=$ 6*T.* 13*cwt.* 3*qrs.* 20℔, *Anſ.*

The perpendicular preſſure of fluids on the bottoms of veſſels is eſtimated by the area of the bottom multiplied by the altitude of the fluid.

119. Suppoſe a veſſel 3 feet wide, 5 feet long, and 4 feet high; What is the preſſure on the bottom, it being filled with water to the brim?

$3 \times 5 = 15$ ſquare feet, the area of the bottom, and $15 \times 4 = 60$ cubic feet, and $60 \times 62,5 = 3750$℔ $=$ 33*cwt.* 1*qr.* 26℔.

The USE *of the* BAROMETER.

The barometer is ſo formed, that a column of quickſilver is ſupported within it, to ſuch a height as to counterbalance the weight of a column of air, of an equal diameter, extending from the barometer to the top of the atmoſphere.

120. At the ſurface of the earth, the height of this column of quickſilver is, at an average, almoſt 30 inches; when the barometer is at that height, What is the preſſure of atmoſphere on a ſquare foot, and on the ſurface of a man's body, eſtimated at 14 ſquare feet?

 As

As the cubic foot of quickſilver is 13600 ounces, Avoirdupois, and as the height in the barometer is 2,5 feet, therefore $13600 \times 2,5 = 34000$ ounces, $= 2125$ pounds on a ſquare foot; and $2125 \times 14 = 29750$ pounds on a man's body.

121. If the mercury in a barometer, at the bottom of a tower, be obſerved to ſtand at 30 inches, and, on being carried to the top of it, be obſerved at 29,9 inches; What is the height of the tower?

Divide 13600, the ſpecific gravity of quickſilver, by 1,25, the ſpecific gravity of air, and the quotient will be the height of the tower, in tenths of an inch.

$\frac{13600}{1,25} = 10880$ tenths, and $\frac{10880}{10} = 1088$.inch. $= 90\frac{2}{3}$ *ft. Anſ.*

The number of feet, in height, of the atmoſphere, correſponding with $\frac{1}{10}$ of an inch on the barometer is variable, depending on the temperature and denſity of the atmoſphere.

The variation, depending on the temperature, is ſhewn in the following Table, calculated for every 5 degrees, from 32 to 80, Farenheit's Thermometer, from whence it may be eaſily calculated for the intermediate degrees, by allowing $\frac{21}{100}$ of a foot for each degree.

TABLE.

Thermo.	*feet.*
32	86,86
35	87,49
40	88,54
45	89,60
50	90,66
55	91,72
60	92.77
65	93,82
70	94,88
75	95,93
80	96,99

The altitude, thus found, will be to the altitude corrected for the denſity of the air, inverſely, as the mean height of the barometer, at the two ſtations, is to 30 inches; therefore,

RULE.—Multiply the mean height correſponding to the mean temperature of the two barometers (found in the Table) by the *tenths* of an inch in the difference of the two barometers, and this product by 30; divide this laſt product by the mean height of the two barometers, and the quotient will be the anſwer, or height required, with the error of a few feet only, if the height be leſs than a mile.*

122. At the 1ſt ſtation, ſuppoſe the barometer to ſtand at 29, and the thermometer at 60; at the 2d ſtation, the barometer at 28, and the thermometer at 40: What is the height of the 2d ſtation, or the diſtance between the two places of obſervation?

Barometer.

* Let $h =$ mean height of the barometer at its two ſtations, (or of two barometers, one at each ſtation) in inches. $d =$ difference of the two barometers in *tenths* of an inch; and $n =$ number from the Table anſwering to the mean temperature of the two thermometers accompanying the barometer, then $\frac{30dn}{h} =$ the altitude required nearly.

Barometer.

Add $\begin{cases} \text{First station} = 29 \\ \text{Second station} = 28 \end{cases}$

$\frac{1}{2})57$

$\frac{1}{2}$ sum $= 28,5 =$ mean height of the two barometers.

$$\begin{array}{r} 29 \\ 28 \end{array}$$

Difference $=$ 1 $=$ 10 tenths of an inch.

Thermometer.

First station $= 60$
Second station $= 40$

$\frac{1}{2})100$

50 = mean height of the two thermometers, against which, in the Table, you will find 90,66, the mean temperature of the two barometers. Now according to the rule $90,66 \times 10 \times 30 \div 28,5 = 954,3$ feet, *the Answer, nearly.*

T A B L E S.

Value of English and Portuguese gold, in dollars, cents and mills, throughout the United States.

Gr.	*Cts.*	*m.*	*Pwts.*	*Dols. cts.*
1	3	7	1	0 89
2	7	4	2	1 77$\frac{3}{4}$
3	11	1	3	2 66$\frac{2}{3}$
4	14	8	4	3 55$\frac{1}{2}$
5	18$\frac{1}{2}$	0	5	4 44$\frac{1}{2}$
6	22	2	6	5 33$\frac{1}{3}$
7	25	9	7	6 22
8	29	6	8	7 11
9	33$\frac{1}{3}$	0	9	8 0
10	37	0	10	8 89
11	40	7	11	9 77$\frac{3}{4}$
12	44	4	12	10 66$\frac{2}{3}$
13	48	1	13	11 55$\frac{1}{2}$
14	51	8	14	12 44$\frac{1}{2}$
15	55	5	15	13 33$\frac{1}{3}$
16	59$\frac{1}{4}$	0	16	14 22
17	63	0	17	15 11
18	66$\frac{2}{3}$	0	18	16 0
19	70	4	19	16 89
20	74	0	1oz.	17 77$\frac{3}{4}$
21	77$\frac{3}{4}$	0		
22	81$\frac{1}{2}$	0		
23	85	2		

Note. 89 cents is the value of 1 penny-weight of English and Portuguese Gold.

Value of French and Spanish gold, in dollars, cents and mills, throughout the United States.

Gr.	*Cts.*	*m.*	*Pwts.*	*Dols. cts.*	*m.*
1	3	6	1	0 87	6
2	7	3	2	1 75	2
3	11	0	3	2 62$\frac{3}{4}$	0
4	14	6	4	3 50	3
5	18	2	5	4 38	0
6	21	9	6	5 25	5
7	25	5	7	6 13	1
8	29	2	8	7 0	7
9	32	8	9	7 88	3
10	36$\frac{1}{9}$	0	10	8 76	0
11	40	1	11	9 63$\frac{1}{2}$	0
12	43	8	12	10 51	1
13	47	4	13	11 38	7
14	51	1	14	12 26	3
15	54$\frac{3}{4}$	0	15	13 13	9
16	58	4	16	14 1$\frac{1}{2}$	0
17	62	0	17	14 89	0
18	65	7	18	15 76	6
19	69	3	19	16 64	2
20	73	0	1oz.	17 51	8
21	76	6			
22	80	3			
23	83	9			

Note. 87 cents 6 mills, the value of 1 penny-weight French and Spanish gold.

Federal Money.	N. Hampshire, Massachusetts, R. Island, Connecticut, & Virginia.	Newyork and N. Carolina.	N. Jersey, Pennsylvania, Delaware & Maryland.	S. Carolina & Georgia.	Canada and Novascotia.	French.
Dol. d. c.	£. s. d.	£. s. d.	£. s. d.	£. s. d.	£. s. d.	Liv. Tour. } Sous.
0,0 1	$\frac{18}{25}$	$\frac{24}{25}$	$\frac{9}{10}$	$\frac{14}{25}$	$\frac{3}{5}$	$1\frac{1}{20}$
0,0 2	$1\frac{11}{25}$	$1\frac{23}{25}$	$1\frac{8}{10}$	$1\frac{3}{25}$	$1\frac{1}{5}$	$2\frac{1}{10}$
0,0 3	$2\frac{4}{25}$	$2\frac{22}{25}$	$2\frac{7}{10}$	$2\frac{17}{25}$	$1\frac{4}{5}$	$3\frac{3}{20}$
0,0 4	$2\frac{22}{25}$	$3\frac{21}{25}$	$3\frac{6}{10}$	$2\frac{6}{25}$	$2\frac{2}{5}$	$4\frac{1}{5}$
0.0 5	$3\frac{3}{5}$	$4\frac{4}{5}$	$4\frac{5}{10}$	$3\frac{4}{25}$	3	$5\frac{1}{4}$
0,0 6	$4\frac{3}{25}$	$5\frac{19}{25}$	$5\frac{4}{10}$	$3\frac{9}{25}$	$3\frac{3}{5}$	$6\frac{3}{10}$
0,0 7	$5\frac{1}{25}$	$6\frac{18}{25}$	$6\frac{3}{10}$	$3\frac{23}{25}$	$4\frac{1}{5}$	$7\frac{7}{20}$
0,0 8	$5\frac{19}{25}$	$7\frac{17}{25}$	$7\frac{2}{10}$	$4\frac{12}{25}$	$4\frac{4}{5}$	$8\frac{3}{5}$
0,0 9	$6\frac{12}{25}$	$8\frac{16}{25}$	$8\frac{1}{10}$	$5\frac{1}{25}$	$5\frac{2}{5}$	$9\frac{9}{20}$
0,1 0	7 $\frac{1}{5}$	$9\frac{3}{5}$	9	5 $\frac{3}{5}$	6	10 $\frac{1}{2}$
0,2 0	1 2 $\frac{2}{5}$	1 $7\frac{1}{5}$	1 6	11 $\frac{1}{5}$	1 0	1 1
0,3 0	1 9 $\frac{3}{5}$	2 $4\frac{4}{5}$	2 3	1 4 $\frac{4}{5}$	1 6	1 11 $\frac{1}{2}$
0,4 0	2 4 $\frac{4}{5}$	3 $2\frac{2}{5}$	3 0	1 10 $\frac{2}{5}$	2 0	2 2
0,5 0	3 0	4 0	3 9	2 4	2 6	2 12 $\frac{1}{2}$
0,6 0	3 7 $\frac{1}{5}$	4 $9\frac{1}{5}$	4 6	2 9 $\frac{3}{5}$	3 0	3 3
0,7 0	4 2 $\frac{2}{5}$	5 $7\frac{2}{5}$	5 3	3 3 $\frac{1}{5}$	3 6	3 13 $\frac{1}{2}$
0,8 0	4 9 $\frac{3}{5}$	6 $4\frac{3}{5}$	6 0	3 8 $\frac{4}{5}$	4 0	4 4
0,9 0	5 4 $\frac{4}{5}$	7 $2\frac{4}{5}$	6 9	4 2 $\frac{2}{5}$	4 6	4 14 $\frac{1}{2}$
1,0 0	6 0	8 0	7 6	4 8	5 0	5 5
2,0 0	12 0	16 0	15 0	9 4	10 0	10 10
3,0 0	18 0	1 4 0	1 2 6	14 0	15 0	15 15
4,0 0	1 4 0	1 12 0	1 10 0	18 8	1 0 0	21 0
5,0 0	1 10 0	2 0 0	1 17 6	1 3 4	1 5 0	26 5
6,0 0	1 16 0	2 8 0	2 5 0	1 8 0	1 10 0	31 10
7,0 0	2 2 0	2 16 0	2 12 6	1 12 8	1 15 0	36 15
8,0 0	2 8 0	3 4 0	3 0 0	1 17 4	2 0 0	42 0
9,0 0	2 14 0	3 12 0	3 7 6	2 2 0	2 5 0	47 5
10,0 0	3 0 0	4 0 0	3 15 0	2 6 8	2 10 0	52 10
20,0 0	6 0 0	8 0 0	7 10 0	4 13 4	5 0 0	105 0
30,0 0	9 0 0	12 0 0	11 5 0	7 0 0	7 10 0	157 10
40,0 0	12 0 0	16 0 0	15 0 0	9 6 8	10 0 0	210 0
50,0 0	15 0 0	20 0 0	18 15 0	11 13 4	12 10 0	262 10
60,0 0	18 0 0	24 0 0	22 10 0	14 0 0	15 0 0	315 0
70,0 0	21 0 0	28 0 0	26 5 0	16 6 8	17 10 0	367 10
80,0 0	24 0 0	32 0 0	30 0 0	18 13 4	20 0 0	420 0
90,0 0	27 0 0	36 0 0	33 15 0	21 0 0	22 10 0	472 10
100,0 0	30 0 0	40 0 0	37 10 0	23 6 8	25 0 0	525 0
200,0 0	60 0 0	80 0 0	75 0 0	46 13 4	50 0 0	1050 0
300,0 0	90 0 0	120 0 0	112 10 0	70 0 0	75 0 0	1575 0
400,0 0	120 0 0	160 0 0	150 0 0	93 6 8	100 0 0	2100 0
500,0 0	150 0 0	200 0 0	187 10 0	116 13 4	125 0 0	2625 0
600,0 0	180 0 0	240 0 0	325 0 0	140 0 0	150 0 0	3150 0
700,0 0	210 0 0	280 0 0	262 10 0	163 6 8	175 0 0	3675 0
800,0 0	240 0 0	320 0 0	300 0 0	186 13 4	200 0 0	4200 0
900,0 0	270 0 0	360 0 0	337 10 0	210 0 0	225 0 0	4725 0
1000,0 0	300 0 0	400 0 0	375 0 0	233 6 8	250 0 0	5250 0

V. Hamp. Maſſachu. R. Iſland, Connecticut, nd Virgin.			Federal Coin.	Newyork and Northcarolina.			Newjerſey, Pennſylvania, Delaware & Maryland.			S. Carolina and Georgia.			Eng. Money.			French Money.	
£.	s.	d.	Dol. d. c.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	Livr. Tour.	Sous.
		1	0,01$\frac{7}{18}$			$1\frac{1}{3}$			$1\frac{1}{4}$			$\frac{7}{9}$			$\frac{3}{4}$		$1\frac{11}{24}$
		2	0,02$\frac{7}{9}$			$2\frac{2}{3}$			$2\frac{1}{2}$			$1\frac{5}{9}$			$1\frac{1}{2}$		$2\frac{11}{12}$
		3	0,04$\frac{1}{6}$			4			$3\frac{3}{4}$			$2\frac{3}{9}$			$2\frac{1}{4}$		$4\frac{3}{8}$
		4	0,05$\frac{5}{9}$			$5\frac{1}{3}$			5			$3\frac{1}{9}$			3		$5\frac{5}{6}$
		5	0,06$\frac{8}{9}$			$6\frac{2}{3}$			$6\frac{1}{4}$			$3\frac{8}{9}$			$3\frac{3}{4}$		$7\frac{7}{24}$
		6	0,08$\frac{1}{3}$			8			$7\frac{1}{2}$			$4\frac{6}{9}$			$4\frac{1}{2}$		$8\frac{3}{4}$
		7	0,09$\frac{2}{3}$			$9\frac{1}{3}$			$8\frac{3}{4}$			$5\frac{4}{9}$			$5\frac{1}{4}$		$10\frac{5}{24}$
		8	0,11$\frac{1}{9}$			$10\frac{2}{3}$			10			$6\frac{2}{9}$			6		$11\frac{2}{3}$
		9	0,12$\frac{1}{2}$		1	0			$11\frac{1}{4}$			7			$6\frac{3}{4}$		$13\frac{1}{8}$
		10	0,13$\frac{8}{9}$		1	$1\frac{1}{3}$		1	$0\frac{1}{2}$			$7\frac{7}{9}$			$7\frac{1}{2}$		$14\frac{7}{12}$
		11	0,15$\frac{1}{3}$		1	$2\frac{2}{3}$		1	$1\frac{3}{4}$			$8\frac{5}{9}$			$8\frac{1}{4}$		$16\frac{1}{24}$
	1	0	0,16$\frac{2}{3}$		1	4		1	3			$9\frac{3}{9}$			9		$17\frac{1}{2}$
	2	0	0,33$\frac{1}{3}$		2	8		2	6		1	$6\frac{6}{9}$		1	6	1	15
	3	0	0,50		4	0		3	9		2	4		2	3	2	$12\frac{1}{2}$
	4	0	0,66$\frac{2}{3}$		5	4		5	0		3	$1\frac{3}{9}$		3	0	3	10
	5	0	0,83$\frac{1}{3}$		6	8		6	3		3	$10\frac{6}{9}$		3	9	4	$7\frac{1}{2}$
	6	0	1,00		8	0		7	6		4	8		4	6	5	5
	7	0	1,16$\frac{2}{3}$		9	4		8	9		5	$4\frac{3}{9}$		5	3	6	$2\frac{1}{2}$
	8	0	1,33$\frac{1}{3}$		10	8		10	0		6	$2\frac{6}{9}$		6	0	7	0
	9	0	1,50		12	0		11	3		7	0		6	9	7	$17\frac{1}{2}$
	10	0	1,66$\frac{2}{3}$		13	4		12	6		7	$9\frac{3}{9}$		7	6	8	15
1	0	0	3,33$\frac{1}{3}$	1	6	8	1	5	0		15	$6\frac{6}{9}$		15	0	17	10
2	0	0	6,66$\frac{2}{3}$	2	13	4	2	10	0	1	11	$1\frac{3}{9}$	1	10	0	35	0
3	0	0	10,00	4	0	0	3	15	0	2	6	8	2	5	0	52	10
4	0	0	13,33$\frac{1}{3}$	5	6	8	5	0	0	3	2	$2\frac{6}{9}$	3	0	0	70	0
5	0	0	16,66$\frac{2}{3}$	6	13	4	6	5	0	3	17	$9\frac{3}{9}$	3	15	0	87	10
6	0	0	20,00	8	0	0	7	10	0	4	13	4	4	10	0	105	0
7	0	0	23,33$\frac{1}{3}$	9	6	8	8	15	0	5	8	$10\frac{6}{9}$	5	5	0	122	10
8	c	0	26,66$\frac{2}{3}$	10	13	4	10	0	0	6	4	$5\frac{3}{9}$	6	0	0	140	0
9	0	0	30,00	12	0	0	11	5	0	7	0	0	6	15	0	157	10
10	0	0	33,33$\frac{1}{3}$	13	6	8	12	10	0	7	15	$6\frac{6}{9}$	7	10	0	175	0
20	0	0	66,66$\frac{2}{3}$	26	13	4	25	0	0	15	11	$1\frac{3}{9}$	15	0	0	350	0
30	0	0	100,00	40	0	0	37	10	0	23	6	8	22	10	0	525	0
40	0	0	133,33$\frac{1}{3}$	53	6	8	50	0	0	31	2	$2\frac{6}{9}$	30	0	0	700	0
50	0	0	166,66$\frac{2}{3}$	66	13	4	62	10	0	38	17	$9\frac{3}{9}$	37	10	0	875	0
60	0	0	200,00	80	0	0	75	0	0	46	13	4	45	0	0	1050	0
70	0	0	233,33$\frac{1}{3}$	93	6	8	87	10	0	54	8	$10\frac{6}{9}$	52	10	0	1225	0
80	0	0	266,66$\frac{2}{3}$	106	13	4	100	0	0	62	4	$5\frac{3}{9}$	60	0	0	1400	0
90	0	0	300,00	120	0	0	112	10	0	70	0	0	67	10	0	1575	0
100	0	0	333,33$\frac{1}{3}$	133	6	8	125	0	0	77	15	$6\frac{6}{9}$	75	0	0	1750	0
200	0	0	666,66$\frac{2}{3}$	266	13	4	250	0	0	155	11	$1\frac{3}{9}$	150	0	0	3500	0
300	0	0	10,0000	400	0	0	375	0	0	233	6	8	225	0	0	5250	0
400	0	0	1333,33	533	6	8	500	0	0	311	2	$2\frac{6}{9}$	300	0	0	7000	0
500	0	0	1666,66	666	13	4	625	0	0	388	17	$9\frac{3}{9}$	375	0	0	8736	0

TABLE *of the Value of several Pieces of Coin, in the Federal Coin, and the several Currencies of the United States.*

			Federal Coin.	N. Hampshire, Massachusetts, Rhode Island, Connecticut, Virginia.			Newyork and North-carolina.			N. Jersey, Pennsylvania, Delaware and Maryland.			Southcarolina and Georgia.		
			Cents.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
$\frac{1}{16}$ of a Dollar			0,06 $\frac{1}{4}$			$4\frac{1}{2}$			6			$5\frac{5}{8}$			$3\frac{1}{2}$
$\frac{1}{2}$ a Pistareen			0,10			$7\frac{1}{5}$			$9\frac{3}{5}$			9			$5\frac{3}{5}$
				Vir.		8									
$\frac{1}{9}$ of a Dollar			0,11 $\frac{1}{9}$			8			$10\frac{2}{3}$			10			$6\frac{2}{9}$
$\frac{1}{8}$ of ditto			0,12 $\frac{1}{2}$			9		1	0			$11\frac{1}{5}$			7
A Pistareen			0,20		1	$2\frac{2}{5}$		1	$4\frac{1}{2}$		1	6			$11\frac{1}{5}$
				Vir.	1	4									
An Eng. Shill.			0,22 $\frac{2}{9}$		1	4		1	$7\frac{2}{3}$		1	8		1	$0\frac{4}{9}$
$\frac{1}{4}$ of a Dollar			0,25		1	6		2	0		1	$10\frac{1}{2}$		1	2
Half ditto			0,50		3	0		4	0		3	9		2	4
A Dollar			1,00		6	0		8	0		7	6		4	8
En. or Fr. Crown			1,11 $\frac{1}{9}$		6	8	N. York	9	0		8	4		5	$2\frac{2}{9}$
							N. Caro.	8	9						
	pwt.	gr.													
Fr. Guin.	5	5	4,62 $\frac{26}{27}$	1	7	6	1	16	0	1	14	6	1	1	5
In Massa.	5	6	4,55 $\frac{1}{9}$	1	7	4									
En. Guin.	5	6	4,66 $\frac{2}{3}$	1	8	0	1	17	0	1	15	0	1	1	9
In S. Caro.	5	7											1	1	10
$\frac{1}{2}$ Johann.	9	9	4,00	2	8	0	3	4	0	3	0	0	1	17	4
Pistole	4	5	3,66 $\frac{2}{3}$	1	2	0	1	8	0	1	7	0		17	6
In Massa.	4	3													
Moidore	6	18	6,00	1	16	0	2	8	0	2	5	0	1	8	0
Doubloon	17	0	14,66 $\frac{2}{3}$	4	8	0	5	16	0	5	12	0	3	10	0

The standard weight of an eagle 11*pwt.* $4\frac{2}{33}$*gr.* Half ditto 5*pwt.* $14\frac{1}{33}$*gr.* A dollar 17*pwt.* $1\frac{3}{48}$*gr.* Half ditto 8*pwt.* $12\frac{7}{85}$*gr.* A double dime 3*pwt.* $9\frac{4}{55}$*gr.* A dime 1*pwt.* $16\frac{9}{100}$*gr.*

TABLE *of Refiner's Weight.*

Blanks
24 = 1 Perrot.
480 = 20 = 1 Mite.
9600 = 400 = 20 = 1 Grain.

Note, What they denominate a carat, is the $\frac{1}{24}$ of a ℔, an *oz.* or any other weight.

DUTCH WEIGHTS *for* GOLD *and* SILVER.

Note, 32 aces = 1 engel, 20 engels = 1 ounce, 8 ounces = 1 mark, for gross gold. Also, 24 parts = 1 grain, 12 grains = 1 carat, 24 carats = 1 mark, for fine gold.

The mark weights are 1 per cent. lighter than our Troy weight.

A TABLE of Commission or Brokage.

Goods or stock sold	at ½ per cent.	at 1 per cent.	at 1½ per cent.	at 2 per cent.	at 2½ per cent.	at 3 per cent.
Shill. 1	£. s. d.	0 0 0	0 0 0	0 0 0	0 0 0¼	0 0 0¼
2	0 0 0	0 0 0¼	0 0 0¼	0 0 0¼	0 0 0½	0 0 0½
3	0 0 0¼	0 0 0¼	0 0 0½	0 0 0½	0 0 0¾	0 0 1
4	0 0 0¼	0 0 0½	0 0 0½	0 0 0¾	0 0 1	0 0 1¼
5	0 0 0¼	0 0 0½	0 0 0¾	0 0 1	0 0 1½	0 0 1¾
6	0 0 0¼	0 0 0¾	0 0 1	0 0 1¼	0 0 1¾	0 0 2
7	0 0 0½	0 0 0¾	0 0 1¼	0 0 1½	0 0 2	0 0 2½
8	0 0 0½	0 0 1	0 0 1¼	0 0 1¾	0 0 2¼	0 0 2¾
9	0 0 0½	0 0 1	0 0 1½	0 0 2	0 0 2½	0 0 3
10	0 0 0½	0 0 1¼	0 0 1¾	0 0 2¼	0 0 3	0 0 3½
11	0 0 0½	0 0 1½	0 0 1¾	0 0 2½	0 0 3¼	0 0 3¾
12	0 0 0½	0 0 1½	0 0 2	0 0 2¾	0 0 3½	0 0 4¼
13	0 0 0¾	0 0 1½	0 0 2¼	0 0 3	0 0 3¾	0 0 4½
14	0 0 0¾	0 0 1¾	0 0 2½	0 0 3¼	0 0 4	0 0 5
15	0 0 0¾	0 0 1¾	0 0 2½	0 0 3½	0 0 4½	0 0 5¼
16	0 0 0¾	0 0 2	0 0 2¾	0 0 3¾	0 0 4¾	0 0 5¾
17	0 0 1	0 0 2	0 0 3	0 0 4	0 0 5	0 0 6
18	0 0 1	0 0 2¼	0 0 3	0 0 4¼	0 0 5¼	0 0 6¼
19	0 0 1	0 0 2¼	0 0 3¼	0 0 4½	0 0 5½	0 0 6¾
Pounds 1	0 0 1¼	0 0 2½	0 0 3½	0 0 4¾	0 0 6	0 0 7
2	0 0 2½	0 0 5	0 0 7	0 0 9½	0 1 0	0 1 2½
3	0 0 3¾	0 0 7	0 0 10¾	0 1 2¼	0 1 6	0 1 9½
4	0 0 5	0 0 9½	0 1 2¼	0 1 7	0 2 0	0 2 4¾
5	0 0 6	0 1 0	0 1 6	0 2 0	0 2 6	0 3 0
6	0 0 7¼	0 1 2¼	0 1 9½	0 2 4¾	0 3 0	0 3 7
7	0 0 8½	0 1 4¾	0 2 1	0 2 9½	0 3 6	0 4 2¼
8	0 0 9¾	0 1 7	0 2 4¾	0 3 2¼	0 4 0	0 4 9½
9	0 0 10¾	0 1 9½	0 2 8¼	0 3 7	0 4 6	0 5 4¾
10	0 1 0	0 2 0	0 3 0	0 4 0	0 5 0	0 6 0
20	0 2 0	0 4 0	0 6 0	0 8 0	0 10 0	0 12 0
30	0 3 0	0 6 0	0 9 0	0 12 0	0 15 0	0 18 0
40	0 4 0	0 8 0	0 12 0	0 16 0	1 0 0	1 4 0
50	0 5 0	0 10 0	0 15 0	1 0 0	1 5 0	1 10 0
60	0 6 0	0 12 0	0 18 0	1 4 0	1 10 0	1 16 0
70	0 7 0	0 14 0	1 1 0	1 8 0	1 15 0	2 2 0
80	0 8 0	0 16 0	1 4 0	1 12 0	2 0 0	2 8 0
90	0 9 0	0 18 0	1 7 0	1 16 0	2 5 0	2 14 0
100	0 10 0	1 0 0	1 10 0	2 0 0	2 10 0	3 0 0
200	1 0 0	2 0 0	3 0 0	4 0 0	5 0 0	6 0 0
300	1 10 0	3 0 0	4 10 0	6 0 0	7 10 0	9 0 0
400	2 0 0	4 0 0	6 0 0	8 0 0	10 0 0	12 0 0
500	2 10 0	5 0 0	7 10 0	10 0 0	12 10 0	15 0 0
600	3 0 0	6 0 0	9 0 0	12 0 0	15 0 0	18 0 0
700	3 10 0	7 0 0	10 10 0	14 0 0	17 10 0	21 0 0
800	4 0 0	8 0 0	12 0 0	16 0 0	20 0 0	24 0 0
900	4 10 0	9 0 0	13 10 0	18 0 0	22 10 0	27 0 0
1000	5 0 0	10 0 0	15 0 0	20 0 0	25 0 0	30 0 0

A TABLE *of the Returns of the Neat Proceeds of an Account of Sales from a Factor to his Employer, reserving his Commissions for Remittance.*

Neat Proceeds.	Sum to be remitted, reserving 2½ pr cent. Commission.	Sum to be remitted, reserving 5 pr cent. Commission.	Neat Proceeds.	Sum to be remitted, reserving 2½ per cent. Commission.	Sum to be remitted, reserving 5 per cent. Commission.
£. s. d.	£. s. d.	£. s. d.	£. s. d.	£. s. d.	£. s. d.
3	3	2¾	6 0 0	5 17 0¾	5 14 5½
4	4	3¾	7 0 0	6 16 7	6 13 4
5	5	4¾	8 0 0	7 16 1¼	7 12 4½
6	5¾	5¾	9 0 0	8 15 7¼	8 11 5¼
7	6¾	6¾	10 0 0	9 15 1½	9 10 5¾
8	7¾	7½	20 0 0	19 10 3	19 0 11½
9	8¾	8½	30 0 0	29 5 4¼	28 11 5¼
10	9¾	9½	40 0 0	39 0 5¾	38 1 10¼
11	10¾	10¼	50 0 0	48 15 7¼	47 12 4½
1 0	11¾	11¼	60 0 0	58 10 8¾	57 2 10¼
2 0	1 11½	1 10¾	70 0 0	68 5 10	66 13 4
3 0	2 11¼	2 10¼	80 0 0	78 0 11½	76 3 9¾
4 0	3 10¾	3 9¾	90 0 0	87 16 1	85 14 3½
5 0	4 10½	4 9¼	100 0 0	97 11 2¾	95 4 9
6 0	5 10¼	5 8½	200 0 0	195 2 5¼	190 9 6¼
7 0	6 10	6 8	300 0 0	292 13 8	285 14 3¼
8 0	7 9¾	7 7½	400 0 2	390 4 10½	380 19 0½
9 0	8 9¼	8 6¾	500 0 0	487 16 1¼	476 3 9½
10 0	9 9	9 6¼	600 0 0	585 7 3¾	571 8 6¾
1 0 0	19 6¼	19 0½	700 0 0	682 18 4¾	666 13 4
2 0 0	1 19 0¼	1 18 1¼	800 0 0	780 9 9	761 18 1
3 0 0	2 18 6½	2 17 1¾	900 0 0	878 0 11¾	857 2 10
4 0 0	3 18 0½	3 16 2¼	1000 0 0	975 12 2¼	952 7 7¼
5 0 0	4 17 6¾	4 15 2¾			

Suppose I have the neat proceeds, or balance of an account of sales 325*l.* 17*s.* 9*d.* in my hands, and would make remittance to my employer, reserving my commission at 2½ per cent. What sum must be remitted, so that my employer's account may be closed?

	£. s. d.		£. s. d.
Against	300 0 0	stands	292 13 8
	20 0 0		19 10 3
	5 0 0		4 17 6¾
	10 0		9 9
	7 0		6 10
	9		8¾

To be remitted £317 18 9½ *Answer.*

A

A Table, *shewing the number of Days from any Day in any Month to the same Day in any other Month through the Year.*

	From *January*	*February*	*March*	*April*	*May*	*June*	*July*	*August*	*September*	*October*	*November*	*December*
	Feb. 3	*Mar.* 28	*April* 3	*May* 3	*Jun.* 31	*July* 30	*Aug.* 31	*Sept.* 31	*Oct.* 30	*Nov.* 31	*Dec.* 30	*Jan.* 31
	Mar. 59	*April* 59	*May* 61	*June* 61	*July* 61	*Aug.* 61	*Sept.* 62	*Oct.* 61	*Nov.* 61	*Dec.* 61	*Jan.* 61	*Feb.* 62
	April 90	*May* 89	*June* 92	*July* 91	*Aug.* 92	*Sept.* 92	*Oct.* 92	*Nov.* 92	*Dec.* 91	*Jan.* 92	*Feb.* 92	*Mar.* 90
	May 120	*June* 120	*July* 122	*Aug.* 122	*Sept.* 123	*Oct.* 122	*Nov.* 123	*Dec.* 122	*Jan.* 122	*Feb.* 123	*Mar.* 120	*April* 121
	June 151	*July* 150	*Aug.* 153	*Sept.* 153	*Oct.* 153	*Nov.* 153	*Dec.* 153	*Jan.* 153	*Feb.* 153	*Mar.* 151	*April* 151	*May* 151
	July 181	*Aug.* 181	*Sept.* 181	*Oct.* 183	*Nov.* 184	*Dec.* 183	*Jan.* 181	*Feb.* 184	*Mar.* 181	*April* 182	*May* 181	*June* 182
To	*Aug.* 212	*Sept.* 212	*Oct.* 214	*Nov.* 214	*Dec.* 214	*Jan.* 214	*Feb.* 215	*Mar.* 212	*April* 21	*May* 212	*June* 212	*July* 212
	Sept. 243	*Oct.* 242	*Nov.* 245	*Dec.* 244	*Jan.* 245	*Feb.* 245	*Mar.* 243	*April* 243	*May* 242	*June* 243	*July* 242	*Aug.* 243
	Oct. 273	*Nov.* 273	*Dec.* 275	*Jan.* 275	*Feb.* 276	*Mar.* 273	*April* 274	*May* 273	*June* 273	*July* 273	*Aug.* 273	*Sept.* 274
	Nov. 304	*Dec.* 303	*Jan.* 306	*Feb.* 306	*Mar.* 304	*April* 304	*May* 304	*June* 304	*July* 303	*Aug.* 304	*Sept.* 304	*Oct.* 304
	Dec. 334	*Jan.* 334	*Feb.* 337	*Mar.* 334	*April* 335	*May* 331	*June* 335	*July* 334	*Aug.* 334	*Sept.* 335	*Oct.* 334	*Nov.* 335
	Jan. 365	*Feb.* 365	*Mar.* 365	*April* 365	*May* 365	*June* 365	*July* 365	*Aug.* 365	*Sept.* 365	*Oct.* 365	*Nov.* 365	*Dec.* 365

The use of the preceding Table of Number of Days, will easily appear from the following Examples.

Suppose the number of days between the 1st, or 10th, or 30th, &c. of *January*. and the 1st, or 10th, or 30th &c. of *October*, were required? Look in the column under *January* for *October*, and against that month you will find 273, which is the number of days between the said times; and so for the days between any other two months.

If the *given days* be *different*, it is only adding or subtracting their inequality to or from the *tabular number*.

How many days from the 6th of April to the 12th of January? From the 6th of April to the 6th of January is 275, and adding the 6 overplus days, it makes 281 days. And from the 5th of June to the 1st of February is 240 days.

Note, After *February* 31, (in leap years) increase each number with an unit or 1.

A TABLE *of the Measure of Length of the principal places in Europe, compared with the American yard.*

100 Aunes or Ells of England, - -	=	125
100 —— of Holland or Amsterdam, Hærlem, Leyden, the Hague, Rotterdam, Nuremberg, and other cities of Holland,	=	75
100 —— of Brabant or Antwerp, - -	=	76
100 —— of France and Oznaburg, - -	=	128½
100 —— of Hamburg, Francfort, Leipsic, Bern and Basil,	=	62½
100 —— of Breslau, - - -	=	60
100 —— of Dantzick, - -	=	66¾
100 —— of Bergen and Drontheim, -	=	68¼
100 —— of Sweden and Stockholm, - -	=	65¾
100 —— of St. Gall. for Linens, - -	=	87½
100 —— of ditto for Cloths, - -	=	67
100 —— of Geneva, - - -	=	124¾
100 Canes of Marseilles and Montpelier, -	=	214½
100 —— of Thoulouse and High Languedoc, -	=	200
100 —— of Genoa, of 9 palms, - -	=	245¼
100 —— of Rome, - - -	=	227¼
100 Varas of Spain, - - -	=	93¾
100 —— of Portugal, - - -	=	123
100 Cavidos of Portugal, - - -	=	75
100 Brasses of Venice, - - -	=	73½
100 —— of Bergamo, - - -	=	71¼
100 —— of Florence and Leghorn, -	=	64
100 —— of Milan, - - -	=	58½

The use of the following TABLE, *directing how to buy and sell by the hundred.*

If you buy or sell any thing by the great hundred (112℔) and desire to know, by the pound, what the hundred is valued at, observe the following Examples.

1. If you buy sugar at 6¾*d.* per ℔, look for 6¾*d.* in the left hand column of the Table, and against it, in the second column, you will find £3 3*s.* which is the value of 1 *cwt.* at that rate.

2. If 1 *cwt.* (112℔.) cost £9 4*s.* 4*d.* to know how much it is per ℔, look £9 4*s.* 4*d.* in the fourth column, and against it, in the next left hand column, you will find 1*s.* 7¾*d.* which is the price per ℔.

Again, if you buy one hundred weight of goods for 9*l.* 4*s.* 4*d.* and retail it at 1*s.* 9¾*d.* per ℔ it comes at that rate, to 10*l.* 3*s.*; then take 9*l.* 4*s.* 4*d.* from 10*l.* 3*s.* and by the remainder, you will find that you have gained 18*s.* 8*d.*

And in this manner, you may, with ease, calculate any quantity by the following Table.

A TABLE *directing how to buy and sell by the* hundred.

d.	£.	s.	d.	s.	d.	£.	s.	d.	s.	d.	£.	s.	d.
¼	0	2	4	1	0¼	5	14	4	2	0¼	11	6	4
½	0	4	8	1	0½	5	16	8	2	0½	11	8	8
¾	0	7	0	1	0¾	5	19	0	2	0¾	11	11	0
1	0	9	4	1	1	6	1	4	2	1	11	13	4
1¼	0	11	8	1	1¼	6	3	8	2	1¼	11	15	8
1½	0	14	0	1	1½	6	6	0	2	1½	11	18	0
1¾	0	16	4	1	1¾	6	8	4	2	1¾	12	0	4
2	0	18	8	1	2	6	10	8	2	2	12	2	8
2¼	1	1	0	1	2¼	6	13	0	2	2¼	12	5	0
2½	1	3	4	1	2½	6	15	4	2	2½	12	7	4
2¾	1	5	8	1	2¾	6	17	8	2	2¾	12	9	8
3	1	8	0	1	3	7	0	0	2	3	12	12	0
3¼	1	10	4	1	3¼	7	2	4	2	3¼	12	14	4
3½	1	12	8	1	3½	7	4	8	2	3½	12	16	8
3¾	1	15	0	1	3¾	7	7	0	2	3¾	12	19	0
4	1	17	4	1	4	7	9	4	2	4	13	1	4
4¼	1	19	8	1	4¼	7	11	8	2	4¼	13	3	8
4½	2	2	0	1	4½	7	14	0	2	4½	13	6	0
4¾	2	4	4	1	4¾	7	16	4	2	4¾	13	8	4
5	2	6	8	1	5	7	18	8	2	5	13	10	8
5¼	2	9	0	1	5¼	8	1	0	2	5¼	13	13	0
5½	2	11	4	1	5½	8	3	4	2	5½	13	15	4
5¾	2	13	8	1	5¾	8	5	8	2	5¾	13	17	8
6	2	16	0	1	6	8	8	0	2	6	14	0	0
6¼	2	18	4	1	6¼	8	10	4	2	6¼	14	2	4
6½	3	0	8	1	6½	8	12	8	2	6½	14	4	8
6¾	3	3	0	1	6¾	8	15	0	2	6¾	14	7	0
7	3	5	4	1	7	8	17	4	2	7	14	9	4
7¼	3	7	8	1	7¼	8	19	8	2	7¼	14	11	8
7½	3	10	0	1	7½	9	2	0	2	7½	14	14	0
7¾	3	12	4	1	7¾	9	4	4	2	7¾	14	16	4
8	3	14	8	1	8	9	6	8	2	8	14	18	8
8¼	3	17	0	1	8¼	9	9	0	2	8¼	15	1	0
8½	3	19	4	1	8½	9	11	4	2	8½	15	3	4
8¾	4	1	8	1	8¾	9	13	8	2	8¾	15	5	8
9	4	4	0	1	9	9	16	0	2	9	15	8	0
9¼	4	6	4	1	9¼	9	18	4	2	9¼	15	10	4
9½	4	8	8	1	9½	10	0	8	2	9½	15	12	8
9¾	4	11	0	1	9¾	10	3	0	2	9¾	15	15	0
10	4	13	4	1	10	10	5	4	2	10	15	17	4
10¼	4	15	8	1	10¼	10	7	8	2	10¼	15	19	8
10½	4	18	0	1	10½	10	10	0	2	10½	16	2	0
10¾	5	0	4	1	10¾	10	12	4	2	10¾	16	4	4
11	5	2	8	1	11	10	14	8	2	11	16	6	8
11¼	5	5	0	1	11¼	10	17	0	2	11¼	16	9	0
11½	5	7	4	1	11½	10	19	4	2	11½	16	11	4
11¾	5	9	8	1	11¾	11	1	8	2	11¾	16	13	8
12	5	12	0	2	0	11	4	0	3	0	16	16	0

A Comparison of the American Foot with the Feet of other Countries.

The American foot being divided into 1000 parts, or into 12 inches, the feet of several other countries will be as follow.

	Parts.	Inch.	lin.	points.
America,	1000	12	0	0 dec.
London,	1000	12	0	0
Antwerp,	946	11	4	1,32
Bologna,	1204	14	5	2,25
Bremen,	964	11	6	4,89
Cologne,	954	11	5	2,25
Copenhagen,	965	11	6	5,76
Amsterdam,	942	11	3	3,88
Dantzick,	944	11	3	5,61
Dort,	1184	14	2	2,97
Frankfort on the Main,	9[illegible]8	11	4	3,07
The Greek,	1007	12	1	0,04
Lorrain,	958	11	5	5,71
Mantua,	1569	18	9	5,61
Mecklin,	919	11	0	2,01
Middleburg,	991	11	10	4,22
France,	938	11	3	0,43
Prague,	1026	12	3	4,46
Rhyneland or Leyden,	1033	12	4	4,51
Riga,	1831	21	11	3,98
Roman,	967	11	7	1,48
Old Roman,	970	11	8	0
Scotch,	1005	12	0	4,32
Strasburgh,	920	11	0	2,88
Toledo,	899	10	9	2,73
Turin,	1062	12	8	5,66
Venice,	1162	13	11	1,96

A TABLE *representing the Conformity of the Weights of the principal trading Cities of Europe with those of America.*

℔	of America.	
	Equal 100℔	oz.
100 of England, Scotland and Ireland,	100	0
100 of Amsterdam, Paris, Bourdeaux, &c.	109	8
100 of Antwerp, or Brabant,	103	12
100 of Roüen, the Viscounty,	113	14
100 of Lyons, the city,	94	3
100 of Rochelle,	110	9
100 of Thoulouse, and Upper Languedoc,	92	6
100 of Marseilles and Provence,	88	11
100 of Geneva,	123	
100 of Hamburg,	107	5
100 of Francfort,	111	11
100 of Leipsic,	104	5

A

A TABLE *representing the Conformity of the Weights of the principal trading Cities of Europe with those of America.*

℔		of America.	
100 of Genoa,	Equal	73	
100 of Leghorn,	——	75	8
100 of Milan,	——	65	3
100 of Venice,	——	65	11
100 of Naples,	——	64	10
100 of Seville, Cadiz, &c.	——	103	7
100 of Portugal,	——	95	4
100 of Liege,	——	104	
100 of Spain,	——	97	dr
Note, The Spanish Arrobe is 25 Spanish pounds,	——	25	12 6

A TABLE *to cast up wages, or expenses, for a year, at so much per day, week, or month.*

Day. s.	d	by week. £.	s.	d.	by mon. £.	s.	d.	by year. £.	s.	d.
0	1	0	0	7	0	2	4	1	10	5
0	2	0	1	2	0	4	8	3	0	10
0	3	0	1	9	0	7	0	4	11	3
0	4	0	2	4	0	9	4	6	1	8
0	5	0	2	11	0	11	8	7	12	1
0	6	0	3	6	0	14	0	9	2	6
0	7	0	4	1	0	16	4	10	12	11
0	8	0	4	8	0	18	8	12	3	4
0	9	0	5	3	1	1	0	13	13	9
0	10	0	5	10	1	3	4	15	4	2
0	11	0	6	5	1	5	8	16	14	7
1	0	0	7	0	1	8	0	18	5	0
2	0	0	14	0	2	16	0	36	10	0
3	0	1	1	0	4	4	0	54	15	0
4	0	1	8	0	5	12	0	73	0	0
5	0	1	15	0	7	0	0	91	5	0
6	0	2	2	0	8	8	0	109	10	0
7	0	2	9	0	9	16	0	127	15	0
8	0	2	16	0	11	4	0	146	0	0
9	0	3	3	0	12	12	0	164	5	0
10	0	3	10	0	14	0	0	182	10	0
11	0	3	17	0	15	8	0	200	15	0
12	0	4	4	0	16	16	0	219	0	0
13	0	4	11	0	18	4	0	237	5	0
14	0	4	18	0	19	12	0	255	10	0
15	0	5	5	0	21	0	0	273	15	0
16	0	5	12	0	22	8	0	292	0	0
17	0	5	19	0	23	16	0	310	5	0
18	0	6	6	0	25	4	0	328	10	0
19	0	6	13	0	26	12	0	346	15	0
20	0	7	0	0	28	0	0	365	0	0

NOTE, In these two Tables the month is only 28 days.

A TABLE *to find wages or expenses, for a month, week, or day, at so much by the year.*

by yr. £.	by month. £.	s.	d.	by week. £.	s.	d.	by day. £.	s.	d.
1	0	1	6½	0	0	4½	0	0	0¾
2	0	3	0¾	0	0	9¼	0	0	1¼
3	0	4	7¼	0	1	1¾	0	0	2
4	0	6	1¾	0	1	6½	0	0	2¾
5	0	7	8	0	1	11	0	0	3¼
6	0	9	2½	0	2	3½	0	0	4
7	0	10	9	0	2	8¼	0	0	4½
8	0	12	3¼	0	3	0¾	0	0	5¼
9	0	13	9¾	0	3	5½	0	0	6
10	0	15	4	0	3	10	0	0	6½
11	0	16	10½	0	4	2¾	0	0	7¼
12	0	18	5	0	4	7¼	0	0	8
13	0	19	11¼	0	4	11¾	0	0	8½
14	1	1	5¾	0	5	4½	0	0	9¼
15	1	3	0¼	0	5	9	0	0	9¾
16	1	4	6½	0	6	1¾	0	0	10½
17	1	6	1	0	6	6¼	0	0	11¼
18	1	7	7½	0	6	10¾	0	0	11¾
19	1	9	1¾	0	7	3½	0	1	0½
20	1	10	8¼	0	7	8	0	1	1¼
30	2	6	0¼	0	11	6	0	1	7¾
40	3	1	4½	0	15	4	0	2	2¼
50	3	16	8½	0	19	2¼	0	2	9
60	4	12	0¾	1	3	0¼	0	3	3½
70	5	7	4¾	1	6	10¼	0	3	10
80	6	2	9	1	10	8¼	0	4	4½
90	6	18	1	1	14	6¼	0	4	11¼
100	7	13	5	1	18	4¼	0	5	5¾
200	15	6	10¼	3	16	8½	0	10	11½
300	23	0	3¼	5	15	0¾	0	16	5¼
400	30	13	8½	7	13	5	1	1	11
500	38	7	1½	9	11	9¼	1	7	4¾
1000	76	14	3	19	3	6¾	2	14	9½

SIMPLE INTEREST, *at* 6 *per cent. from* 1*s. to* £1000, *and from* 1 *day to* 1 *year.*

Princ. money.	1 day.			2 days.			3 days.			4 days.			5 days.			6 days.			7 days.			
s. 1	s.	d.	q.	s.	d.	q.	s.	d.	q.	s.	d.	q.	s.	d.	q.	s.	d.	q.	£.	s.	d.	q.
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
£. 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
2	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	2
3	0	0	0	0	0	0	0	0	1	0	0	1	0	0	2	0	0	2	0	0	0	3
4	0	0	0	0	0	1	0	0	1	0	0	2	0	0	3	0	0	3	0	0	1	0
5	0	0	0	0	0	1	0	0	2	0	0	3	0	0	3	0	1	0	0	0	1	1
6	0	0	0	0	0	1	0	0	2	0	0	3	0	1	0	0	1	1	0	0	1	2
7	0	0	1	0	0	2	0	0	3	0	1	0	0	1	1	0	1	2	0	0	1	3
8	0	0	1	0	0	2	0	0	3	0	1	1	0	1	2	0	1	3	0	0	2	0
9	0	0	1	0	0	2	0	1	0	0	1	1	0	1	3	0	2	0	0	0	2	1
10	0	0	1	0	0	3	0	1	0	0	1	2	0	1	3	0	2	1	0	0	2	3
20	0	0	3	0	1	2	0	2	1	0	3	0	0	3	3	0	4	2	0	0	5	2
30	0	1	0	0	2	1	0	3	2	0	4	2	0	5	2	0	7	0	0	0	8	1
40	0	1	2	0	3	0	0	4	2	0	6	1	0	7	3	0	9	1	0	0	11	0
50	0	1	3	0	3	3	0	5	3	0	7	3	0	9	3	0	11	3	0	1	1	3
60	0	2	1	0	4	2	0	7	0	0	9	1	0	11	3	1	2	0	0	1	4	2
70	0	2	3	0	5	2	0	8	1	0	11	0	1	1	3	1	4	2	0	1	7	1
80	0	3	0	0	6	1	0	9	1	1	0	2	1	3	3	1	6	2	0	1	10	0
90	0	3	2	0	7	0	0	10	2	1	2	0	1	5	2	1	9	1	0	2	0	3
100	0	3	3	0	7	3	0	11	3	1	3	3	1	7	2	1	11	2	0	2	3	2
200	0	7	3	1	3	3	1	11	2	2	7	2	3	3	1	3	11	1	0	4	7	0
300	0	11	3	1	11	2	2	11	1	3	11	1	4	11	0	5	10	3	0	6	10	3
400	1	3	3	2	7	2	3	11	1	5	3	0	6	6	3	7	10	2	0	9	2	1
500	1	7	2	3	3	1	4	11	0	6	6	3	8	2	2	9	10	1	0	11	6	0
600	1	11	2	3	11	1	5	10	3	7	10	2	9	10	1	11	9	3	0	13	9	2
700	2	3	2	4	7	0	6	10	3	9	2	1	11	5	3	13	9	2	0	16	1	0
800	2	7	2	5	3	0	7	10	2	10	6	0	13	1	2	15	9	1	0	18	4	3
900	2	11	1	5	10	3	8	10	1	11	9	0	14	9	1	17	8	3	1	0	8	1
1000	3	3	1	6	6	2	9	10	1	13	1	0	16	5	0	19	10	2	1	3	0	0

SIMPLE INTEREST at £6 per cent. from 1s. to £1000, and from 1 day to 1 year.

Princ. money.	8 days.				10 days.				15 days.				30 days, or 1 month.				2 such months.			
	£.	s.	d.	q.	£.	s.	d.	q.	£.	s.	d.	q.	£.	s.	d.	q.	£.	s.	d.	q.
s. 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	3
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	3
9	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2	0	0	1	0
10	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2	0	0	1	0
11	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2	0	0	1	1
12	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2	0	0	1	1
13	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	3	0	0	1	2
14	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	3	0	0	1	2
15	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	3	0	0	1	3
16	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	3	0	0	1	3
17	0	0	0	1	0	0	0	1	0	0	0	2	0	0	0	3	0	0	2	0
18	0	0	0	1	0	0	0	1	0	0	0	2	0	0	1	0	0	0	2	0
19	0	0	0	1	0	0	0	1	0	0	0	2	0	0	1	0	0	0	2	0
£. 1	0	0	0	1	0	0	0	1	0	0	0	2	0	0	1	0	0	0	2	1
2	0	0	0	2	0	0	0	3	0	0	1	0	0	0	2	1	0	0	4	2
3	0	0	0	3	0	0	1	0	0	0	1	3	0	0	3	2	0	0	7	0
4	0	0	1	1	0	0	1	2	0	0	2	1	0	0	4	2	0	0	9	1
5	0	0	1	2	0	0	1	3	0	0	2	3	0	0	5	3	0	0	11	3
6	0	0	1	3	0	0	2	1	0	0	3	2	0	0	7	0	0	1	2	0
7	0	0	2	0	0	0	2	3	0	0	4	0	0	0	8	1	0	1	4	2
8	0	0	2	2	0	0	3	0	0	0	4	2	0	0	9	1	0	1	6	3
9	0	0	2	3	0	0	3	2	0	0	5	1	0	0	10	2	0	1	9	1
10	0	0	3	0	0	0	3	3	0	0	5	3	0	0	11	3	0	1	11	2
20	0	0	6	1	0	0	7	3	0	0	11	3	0	1	11	2	0	3	11	1
30	0	0	9	1	0	0	11	3	0	1	5	2	0	2	11	1	0	5	11	0
40	0	1	0	2	0	1	3	3	0	1	11	2	0	3	11	1	0	7	10	2
50	0	1	3	2	0	1	7	2	0	2	5	2	0	4	11	0	0	9	10	1
60	0	1	6	3	0	1	11	2	0	2	11	1	0	5	10	3	0	11	0	0
70	0	1	10	0	0	2	3	2	0	3	5	1	0	6	10	3	0	13	9	2
80	0	2	1	0	0	2	7	2	0	3	11	0	0	7	10	2	0	15	9	1
90	0	2	4	1	0	2	11	1	0	4	5	0	0	8	10	1	0	17	9	0
100	0	2	7	1	0	3	3	1	0	4	11	0	0	9	10	1	0	19	8	2
200	0	5	2	3	0	6	6	3	0	9	10	0	0	19	8	2	1	19	5	1
300	0	7	10	1	0	9	10	1	0	14	9	0	1	9	6	3	2	19	2	0
400	0	10	5	3	0	13	1	2	0	19	8	0	1	19	5	1	3	18	10	2
500	0	13	1	0	0	16	5	0	1	4	7	0	2	9	3	2	4	18	7	2
600	0	15	8	2	0	19	8	2	1	9	6	0	2	19	1	3	5	18	4	1
700	0	18	4	0	1	3	0	0	1	14	5	1	3	9	0	0	6	18	0	3
800	1	0	11	2	1	6	3	1	1	19	4	1	3	18	10	2	7	17	9	2
900	1	3	6	3	1	9	6	3	2	4	4	2	4	8	9	0	8	17	6	1
1000	1	6	2	1	1	12	1	0	2	9	2	1	4	18	7	0	9	17	3	0

Simple Interest, at £6 per cent. from 1s. to £1000, and from 1 day to 1 year.

	three months.				four months.				five months.				six months.				seven months.			
	£.	s.	d.	q.	£.	s.	d.	q.	£.	s.	d.	q.	£.	s.	d.	q.	s.	s.	d.	q.
s. 1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1
2	0	0	0	1	0	0	0	1	0	0	0	2	0	0	0	2	0	0	0	3
3	0	0	0	2	0	0	0	2	0	0	0	3	0	0	1	0	0	0	1	0
4	0	0	0	2	0	0	0	3	0	0	1	0	0	0	1	1	0	0	1	2
5	0	0	0	3	0	0	1	0	0	0	1	1	0	0	1	3	0	0	2	0
6	0	0	1	0	0	0	1	1	0	0	1	3	0	0	2	0	0	0	2	1
7	0	0	1	0	0	0	1	2	0	0	2	0	0	0	2	1	0	0	2	3
8	0	0	1	1	0	0	1	3	0	0	2	1	0	0	2	3	0	0	3	1
9	0	0	1	2	0	0	2	0	0	0	2	2	0	0	3	0	0	0	3	2
10	0	0	1	3	0	0	2	1	0	0	2	3	0	0	3	2	0	0	4	0
11	0	0	1	3	0	0	2	2	0	0	3	1	0	0	3	3	0	0	4	2
12	0	0	2	0	0	0	2	3	0	0	3	2	0	0	4	1	0	0	4	3
13	0	0	2	1	0	0	3	0	0	0	3	3	0	0	4	2	0	0	5	1
14	0	0	2	1	0	0	3	1	0	0	4	0	0	0	4	3	0	0	5	3
15	0	0	2	2	0	0	3	2	0	0	4	1	0	0	5	1	0	0	6	0
16	0	0	2	3	0	0	3	3	0	0	4	2	0	0	5	2	0	0	6	2
17	0	0	2	3	0	0	4	0	0	0	5	0	0	0	6	0	0	0	7	0
18	0	0	3	0	0	0	4	1	0	0	5	1	0	0	6	1	0	0	7	1
19	0	0	3	1	0	0	4	1	0	0	5	2	0	0	6	2	0	0	7	3
£. 1	0	0	3	2	0	0	4	2	0	0	5	3	0	0	7	0	0	0	8	1
2	0	0	7	0	0	0	9	1	0	0	11	3	0	1	2	0	0	1	4	0
3	0	0	10	2	0	1	2	0	0	1	5	2	0	1	9	0	0	2	0	0
4	0	1	2	0	0	1	6	3	0	1	11	2	0	2	4	0	0	2	9	0
5	0	1	5	2	0	1	11	2	0	2	5	2	0	2	11	0	0	3	5	1
6	0	1	9	0	0	2	4	1	0	2	11	1	0	3	6	0	0	4	1	2
7	0	2	0	2	0	2	9	0	0	3	5	1	0	4	1	0	0	4	9	3
8	0	2	4	0	0	3	1	3	0	3	11	1	0	4	8	0	0	5	6	1
9	0	2	7	2	0	3	6	2	0	4	5	0	0	5	3	0	0	6	2	2
10	0	2	11	0	0	3	11	1	0	4	11	0	0	5	11	0	0	6	10	3
20	0	5	10	0	0	7	10	2	0	9	10	1	0	11	10	0	0	13	9	2
30	0	8	9	0	0	11	9	3	0	14	9	1	0	17	9	0	1	0	8	2
40	0	11	8	0	0	15	9	1	0	19	8	1	1	3	8	0	1	7	7	1
50	0	14	7	0	0	19	8	2	1	4	7	2	1	9	7	0	1	14	6	0
60	0	17	6	0	1	3	7	1	1	9	6	3	1	15	6	0	2	1	5	0
70	1	0	5	0	1	7	7	1	1	14	5	0	2	1	5	0	2	8	3	3
80	1	3	4	0	1	11	6	2	1	19	5	0	2	8	4	0	2	15	2	3
90	1	6	3	0	1	15	5	3	2	4	4	0	2	13	3	0	3	2	1	2
100	1	9	2	0	1	19	5	1	2	9	3	1	2	19	2	0	3	9	0	1
200	2	18	4	0	3	18	10	2	4	18	8	2	5	18	4	0	6	18	0	3
300	4	7	6	0	5	18	3	3	7	8	0	3	8	17	6	0	10	7	1	1
400	5	16	8	0	7	17	9	0	9	17	5	1	11	16	8	0	13	16	1	3
500	7	5	10	0	9	17	2	1	12	6	9	3	14	15	10	0	17	5	2	1
600	8	15	0	0	11	16	7	2	14	16	2	0	17	15	0	0	20	14	2	3
700	10	4	2	0	13	16	0	3	17	5	6	2	20	14	2	0	24	3	3	1
800	11	13	4	0	15	15	6	0	19	15	10	3	23	13	4	0	27	12	3	3
900	13	2	6	0	17	14	11	2	22	4	3	0	26	12	6	0	31	1	4	1
1000	14	11	8	0	19	14	4	3	24	13	7	2	29	11	8	0	34	10	4	3

SIMPLE INTEREST *at 6l. per cent. from 1s. to 1000l. and from 1 Day to 1 Year.*

Princ. Money	*8 Months.*				*9 Months.*				*10 Months.*				*11 Months, or 330 days.*				*1 Year, or 365 days.*			
	l.	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1	0	0	0	1	0	0	0	2	0	0	0	2	0	0	1	2	0	0	0	2
2	0	0	0	3	0	0	1	0	0	0	1	0	0	0	1	1	0	0	1	1
3	0	0	1	1	0	0	1	2	0	0	1	3	0	0	1	3	0	0	2	0
4	0	0	1	3	0	0	2	0	0	0	2	1	0	0	2	2	0	0	2	3
5	0	0	2	1	0	0	2	2	0	0	2	3	0	0	3	1	0	0	3	2
6	0	0	2	3	0	0	3	0	0	0	3	2	0	0	3	3	0	0	4	1
7	0	0	3	1	0	0	3	2	0	0	4	0	0	0	4	2	0	0	5	0
8	0	0	3	3	0	0	4	1	0	0	4	2	0	0	5	0	0	0	5	3
9	0	0	4	1	0	0	4	3	0	0	5	1	0	0	5	3	0	0	6	1
10	0	0	4	2	0	0	5	1	0	0	5	3	0	0	6	2	0	0	7	0
11	0	0	5	0	0	0	5	3	0	0	6	1	0	0	7	0	0	0	7	2
12	0	0	5	2	0	0	6	1	0	0	7	0	0	0	7	3	0	0	8	0
13	0	0	6	0	0	0	6	3	0	0	7	2	0	0	8	1	0	0	9	1
14	0	0	6	2	0	0	7	1	0	0	8	1	0	0	9	0	0	0	10	0
15	0	0	7	0	0	0	7	3	0	0	8	3	0	0	9	3	0	0	10	3
16	0	0	7	2	0	0	8	2	0	0	9	1	0	0	10	1	0	0	11	2
17	0	0	8	0	0	0	9	0	0	0	10	0	0	0	11	0	0	1	0	0
18	0	0	8	2	0	0	9	3	0	0	10	2	0	0	11	2	0	1	0	3
19	0	0	8	3	0	0	10	0	0	0	11	0	0	1	0	1	0	1	1	2
£. 1	0	0	9	1	0	0	10	2	0	0	11	3	0	1	1	0	0	1	2	1
2	0	1	6	3	0	1	9	1	0	1	11	2	0	2	2	0	0	2	4	3
3	0	2	4	1	0	2	7	3	0	2	11	1	0	3	3	0	0	3	7	0
4	0	3	1	3	0	3	6	2	0	3	11	0	0	4	4	0	0	4	9	2
5	0	3	11	1	0	4	2	0	0	4	11	0	0	5	5	0	0	6	0	0
6	0	4	8	2	0	5	3	3	0	5	10	3	0	6	6	0	0	7	2	1
7	0	5	6	0	0	6	2	2	0	6	10	2	0	7	7	0	0	8	4	[illegible]
8	0	6	3	2	0	7	1	0	0	7	8	3	0	8	8	0	0	9	7	0
9	0	7	1	0	0	7	11	3	0	8	10	0	0	9	9	0	0	10	9	2
10	0	7	10	2	0	8	10	2	0	9	10	0	0	10	10	0	0	12	0	0
20	0	15	9	0	0	17	9	0	0	19	8	0	1	1	8	0	1	4	0	0
30	1	3	7	2	1	6	7	2	1	9	6	0	1	12	6	0	1	16	0	0
40	1	11	6	0	1	15	6	0	1	19	4	1	2	3	4	0	2	8	0	0
50	1	19	4	3	2	4	4	2	2	9	2	1	2	14	2	0	3	0	0	0
60	2	7	3	1	2	13	3	0	2	19	0	1	3	5	0	0	3	12	0	0
70	2	15	1	3	3	2	1	2	3	8	10	2	3	15	10	0	4	4	0	0
80	3	3	0	1	3	11	0	0	3	18	8	2	4	6	8	0	4	16	0	0
90	3	10	11	0	3	19	10	2	4	8	6	2	4	17	6	0	5	8	0	0
100	3	18	9	2	4	8	11	0	4	18	4	2	5	8	4	0	6	0	0	0
200	7	17	7	0	8	17	10	0	9	16	9	2	10	6	8	0	12	0	0	0
300	11	16	4	0	13	6	9	0	14	15	2	1	16	5	0	0	18	0	0	0
400	15	15	2	1	17	15	8	0	19	13	7	0	21	13	4	0	24	0	0	0
500	29	14	0	0	22	4	7	0	24	12	0	0	27	1	8	0	30	0	0	0
600	23	12	9	2	26	13	6	0	29	10	4	3	32	10	0	0	36	0	0	0
700	37	11	7	0	31	2	5	0	34	8	5	2	37	18	4	0	42	0	0	0
800	31	10	4	3	35	11	4	0	39	7	3	1	43	6	8	0	48	0	0	0
900	35	9	2	1	40	0	4	0	44	5	7	0	48	15	0	0	54	0	0	0
1000	39	8	0	0	44	9	3	0	49	4	0	0	54	3	4	0	60	0	0	0

1787	Thursday
1788	*Saturday*+
1789	Sunday
1790	Monday
1791	Tuesday
1792	*Thursday*+
1793	Friday
1794	Saturday
1795	Sunday
1796	*Tuesday*+
1797	Wednesday
1798	Thursday
1799	Friday
1800	Saturday
1801	Sunday
1802	Monday
1803	Tuesday
1804	*Thursday*+
1805	Friday
1806	Saturday
1807	Sunday
1808	*Tuesday*+
1809	Wednesday
1810	Thursday
1811	Friday
1812	*Sunday*+
1813	Monday
1814	Tuesday
1815	Wednesday
1816	*Friday*+
1817	Saturday
1818	Sunday
1819	Monday

PERPETUAL ALMANACK.

February *March* *November*	*February** *August*	*May*	*January* *October*	*January** *April* *July*	*September* *December*	*June*
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

To find on what day of the week any given day in any month will fall, and the contrary.

EXAMPLE.

On what day of the week will the 31ſt day of January, 1787, fall?

Obſerve the day of the week annexed to the year in the outer column; then, in the Table, under the given month, in the upper row of figures, you will find the day of the month on which that day falls.—According to this direction, I find that, in January 1787, Thurſday is the 4th, 11th, 18th, and 25th, then reckoning on, Friday 26th, Saturday 27th, &c. I find the 31ſt day falls on Wedneſday; or, that the laſt Wedneſday in January is the 31ſt day.

NOTE, In leap years, January and February muſt be taken in the columns marked thus, *. Leap years are marked in the outer columns, thus, +.

The years 1800, 1900, and all other 100dth years, not to be leap years, except the years 2000, 2400, 2800, and every 400dth year following, which muſt be leap years.

1820	*Wedneſday*+
1821	Thurſday
1822	Friday
1823	Saturday
1824	*Monday*+
1825	Tueſday
1826	Wedneſday
1827	Thurſday
1828	*Saturday*+
1829	Sunday
1830	Monday
1831	Tueſday
1832	*Thurſday*+
1833	Friday
1834	Saturday
1835	Sunday
1836	*Tueſday*+
1837	Wedneſday
1838	Thurſday
1839	Friday
1840	*Sunday*+
1841	Monday
1842	Tueſday
1843	Wedneſday
1844	*Friday*+
1845	Saturday
1846	Sunday
1847	Monday
1848	*Wedneſday*+
1849	Thurſday
1850	Friday
1851	Saturday
1852	*Monday*+
1853	Tueſday

A TABLE for reducing Troy wt. to Avoirdupois.

Troy.	Avoirdupois.			Tr.	Avoir.
℔.	℔.	oz.	dr.	pw.	drams.
4000	3291	6	13,68	19	16,67
3000	2528	9	2,26	18	15,79
2000	1645	11	6,84	17	14,92
1000	822	13	11,42	16	14,04
900	740	9	2,28	15	13,16
800	658	4	9,14	14	12,29
700	576	0	0	13	11,41
600	493	11	6,85	12	10,53
500	411	6	13,71	11	9,65
400	329	2	4,57	10	8,78
300	246	13	11,42	9	7,9
200	164	9	2,28	8	7,02
100	82	4	9,15	7	6,14
90	74	0	13,62	6	5,27
80	65	13	4,11	5	4:39
70	57	9	9,6	4	3:51
60	49	5	15,08	3	2,63
50	41	2	4,57	2	1,75
40	32	14	10,05	1	0,88
30	24	10	15,54	gr.	
20	16	7	5,03	23	,84
10	8	3	10,52	22	,8
9	7	6	7,86	21	,77
8	6	9	5,21	20	,73
7	5	12	2,56	19	,69
6	4	14	15,9	18	,66
5	4	1	13,25	17	,62
4	3	4	10,6	16	,58
3	2	7	7,95	15	,55
2	1	10	5,3	14	,51
1	0	13	2,65	13	,47
oz				12	,44
11		12	1,09	11	,4
10		10	15,54	10	,36
9		9	13,99	9	,33
8		8	12,43	8	,29
7		7	10,88	7	,26
6		6	9,32	6	,22
5		5	7,77	5	,18
4		4	6,22	4	,15
3		3	4,66	3	,11
2		2	3,11	2	,07
1		1	1,55	1	,04

A TABLE for reducing Avoirdupois weight into Troy.

Avoir.	Troy.				Av	Troy.			
℔.	℔.	oz.	pw.	gr.	oz	℔.	oz.	pw.	gr.
6000	7291	8	0	0	15	1	1	13	10,5
5000	6076	4	13	8	14	1	0	15	5
4000	4861	1	6	16	13		11	16	23:5
3000	3645	10	0	0	12		10	18	18
2000	2430	6	13	8	11		10	0	12,5
1000	1215	3	6	16	10		9	2	7
900	1093	9	0	0	9		8	4	1,5
800	972	2	13	8	8		7	5	20
700	850	8	6	16	7		6	7	14,5
600	729	2	0	0	6		5	9	9
500	607	7	13	8	5		4	11	3,5
400	486	1	6	16	4		3	12	22
300	364	7	0	0	3		2	14	16,5
200	243	0	13	8	2		1	16	11
100	121	6	6	16	1		0	18	5,5
90	109	4	10	0	dr				
80	97	2	13	8	15			17	2,1
70	85	0	16	16	14			15	22,76
60	72	11	0	0	13			14	19,42
50	60	9	3	8	12			13	15,08
40	48	7	6	16	11			12	12,74
30	36	5	10	0	10			11	9,4
20	24	3	13	8	9			10	6,06
10	12	1	16	16	8			9	2,72
9	10	11	5	0	7			8	23,38
8	9	8	13	8	6			7	20,04
7	8	6	1	16	5			6	16,7
6	7	3	10	0	4			5	13,36
5	6	0	18	8	3			3	10,02
4	4	10	6	16	2			2	6,68
3	3	7	15	0	1			1	3,34
2	2	5	3	8	¾			0	20,51
1	1	2	11	16	½				13,67

An Account of the Gregorian *or* New Style, *together with some* Chronological Problems, *for finding the* Epact, Golden Number, Moon's Age, *&c.*

POPE GREGORY the XIIIth made a reformation of the calendar. The Julian calendar, or old style, had, before that time, been in general use all over Europe. The year, according to the Julian calendar, consists of three hundred and sixty five days and six hours; which six hours being one fourth part of a day, the common years consisted of three hundred and sixty five days, and every fourth year, one day was added to the month of February, which made each of those years three hundred and sixty six days, which are usually called leap years.

This computation, though near the truth, is more than the solar year by 11 minutes, which, in one hundred and thirty one years, amounts to a whole day. By which the Vernal Æquinox was anticipated ten days, from the time of the general council of Nice, held in the year 325 of the Christian Æra, to the time of Pope Gregory; who therefore caused ten days to be taken out of the month of October in 1582, to make the Æquinox fall on the 21st of March, as it did at the time of that Council. And, to prevent the like variation for the future, he ordered that three days should be abated in every four hundred years, by reducing the leap year at the close of each century, for three successive centuries, to common years, and retaining the leap year at the close of each fourth century only

This was at that time esteemed as exactly conformable to the true solar year; but Dr. Halley makes the solar year to be three hundred and sixty five days, five hours, forty eight minutes, fifty four seconds, forty one thirds, twenty seven fourths, and thirty one fifths: According to which, in four hundred years, the Julian year of three hundred and sixty five days and six hours will exceed the solar by three days, one hour and fifty five minutes, which is near two hours, so that in fifty centuries it will amount to a day.

Though the Gregorian calendar, or new style, had long been used throughout the greatest part of Europe, it did not take place in Greatbritain and America till the first of January 1752; and in September following, the eleven days were adjusted, by calling the third day of that month the fourteenth, and continuing the rest in their order.

CHRONOLOGICAL PROBLEMS.

PROBLEM I.

As there are three leap years to be abated in every four centuries: To shew how to find on which century the last year is to be a leap year, and in which it is not.

RULE.—Cut off two cyphers, and divide the remaining figures by 4; if nothing remain, the year is a leap year.

EXAMP.

EXAMP. 1. The year 18|00.

```
4)18(4
  16
  --
   2
```

EXAMP. 2. The year 19|00.

```
4)19(4
  16
  --
   3
```

EXAMP. 3. The year 20|00.

```
4)20(5
  20
  --
   0
```

EXAMP. 4. The year 40|00.

```
4)40(10
  4
  -
  0
```

The firſt and ſecond examples, having remainders, ſhew the years to be common years of three hundred and ſixty five days; but the third and fourth, having no remainders, are leap years of three hundred and ſixty ſix days.

PROBLEM II.

To find, with regard to any other years, whether any given year be leap year, and the contrary.

RULE.

Divide the propoſed year by 4, and if there be no remainder, after the diviſion, it is leap year; but if 1, 2 or 3 remain, it is the firſt, ſecond or third after leap year.

EXAMP. 1. For the year 1784.

```
4)1784(446
  16
  --
   18
   16
   --
    24
    24
    --
     0
```

EXAMP. 2. For the year 1786.

```
4)1786(446
  16
  --
   18
   16
   --
    26
    24
    --
     2 { ſecond after leap year.
```

PROBLEM III.

To find the Dominical Letter for any year, according to the Julian method of calculation.

RULE.

Add to the year its fourth part and 4, and divide that ſum by 7: If nothing remain, the Dominical Letter is G; but if there be any remainder, it ſhews the letter in a retrograde order from G, beginning the reckoning with F; or, if it be ſubtracted from 7, you will have the index of the letter from A, accounting as follows:

A

A	B	C	D	E	F	G
1	2	3	4	5	6	7

EXAMP. For the year 1786.

$$\text{Add}\begin{cases}\text{Given year} = 1786\\ \text{Its fourth} = 446\\ \text{And} \quad 4\end{cases}$$

```
7)2236(319
  21
  --
   13
    7
   --
    66
    63
    --
```

And $7-3=4=$ D, reckoning from A.

PROBLEM IV.

To find the Dominical Letter for any year, according to the Gregorian computation.

RULE.—Divide the year and its fourth part by 7; subtract the remainder, after the division, from 7, and this remainder will be the index of the Dominical Letter, as before; if nothing remain it is G.

EXAMP. 1. For the year 1785. EXAMP. 2. For the year 1788.*

$$\text{Add}\begin{cases}\text{Given year} = 1786\\ \text{Its fourth} = 446\end{cases}$$

```
7)2232(318        7)2235(319
  21                21
  --                --
   13                13
    7                 7
   --                --
    62                65
    56                63
    --                --
```

And $7-6=1=$ A. And $7-2=5=$ E.

PROBLEM

* Here it is to be observed, that every leap year has two Dominical Letters; that, found by this rule, is the Dominical Letter from the twenty fifth day of February to the end of the year; and the next in the order of the alphabet serves from the first of January to the twenty fourth of February.

In the 2d Example, E is the Dominical letter for the year; but F, the next in the order of the alphabet, is the Dominical Letter for January and February. From this interruption of the Dominical Letter every fourth year, it is twenty eight years before the Dominical Letter returns to the same order, which, were it not for the leap years, would return to the same every seven years.

This Cycle of twenty eight years is called the Cycle of the Sun.

PROBLEM V.

To find the Prime, or Golden Number.

RULE.—Add 1 to the given year; divide the ſum by 19, and the remainder, after the diviſion, will be the Prime; if nothing remain, then 19 will be the Golden Number.

EXAMP. For the year 1786.

To the given year 1786
Add 1

19)1787(94
171

77
76

1 Golden Number.

The Golden Number, or Lunar Cycle, is a period of nineteen years, invented by *Meton*, an *Athenian*, and from him called the *Metonic* Cycle. The uſe of this cycle is to find the change of the moon; becauſe, after nineteen years, the changes of the moon fall on the ſame days of the month as in the former 19 years; though not at the ſame time of the day, there being an anticipation of one hour, twenty ſeven minutes, forty one ſeconds, and thirty two thirds; which, in three hundred and twelve years, amount to a whole day. Hence, the Golden Number will not ſhew the true change of the moon for more than three hundred and twelve years, without being varied. But the Golden Number is not ſo well adapted to the *Gregorian*, as the *Julian* calendar: The Epact being more certain in the new ſtyle, to find which, the Golden Number is of uſe.

PROBLEM VI.

To find the Julian Epact.

RULE.—Firſt find the Golden Number, which multiply by 11, and the product, if leſs than 30, will be the number required; if the product exceed 30, then divide it by 30, and the remainder is the Epact.

EXAMP. 1. For the year 1786.

To the given year 1786
Add 1

19)1787(94
171

77
76

Golden Number = 1 and 1 × 11 = 11 the Julian Epact.

EXAMP.

EXAMP. 2. For the year 1791.

```
   1791
      1
   ----
19)1792(94
   171
   ---
     82
     76
     --
      6 = Golden Numb. and 6 × 11 = 66, therefore 30)66(2
                                                     60
                                                     --
                                                      6 Epact.
```

PROBLEM VII.

To find the Gregorian Epact.

RULE.—Subtract 11 from the Julian Epact: If the ſubtraction cannot be made, add 30 to the Julian Epact; then ſubtract, and the remainder will be the Gregorian Epact; if nothing remain, the Epact is 29.

Or, take 1 from the Golden Number, divide the remainder by 3; if 1 remain, add 10 to the dividend, which ſum will be the Epact; if 2 remain, add 20 to the dividend; but if nothing remain, the dividend is the Epact.

EXAMP. 1. For the year 1786.

```
The Julian Epact being 11
            Subtract 11
                     --
                      0
```

Becauſe nothing remains, the Epact is 29.

Or,

EXAMP. 2. For the year 1786.

```
The Golden Number being 1
           Take from it 1
                        -
         Divide by 3)0(0
```

There being no remainder, the Epact is 29, as before.

EXAMP. 3. For the year 1791.

```
The Julian Epact being but 6
               Add to it 30
                         --
                         36
                Subtract 11
                         --
       Gregorian Epact = 25
```

Or,

EXAMP. 4. For the year 1791.

```
The Golden Numb. being 6
          Take from it 1
                       -
                    3)5(1
                      3
                      -
                      2
```

Therefore, as 2 remains, add 20 to the dividend, and it gives the Epact 25, as before.

A general Rule for finding the Gregorian Epact forever.

Divide the *centuries* of any year of the Christian Æra by 4, (rejecting the subsequent numbers ;) multiply the remainder by 17, and to this product add the quotient multiplied by 43 ; divide this sum plus 86 by 25, multiplying the Golden Number by 11, from which subtract the last quotient, and rejecting the *thirties*, the remainder will be the Epact.

Examp. For the year 1786.

Rejecting the subsequent numbers 86, it will be 17.

4)17(4
16
—
1
Multiply by 17
—
17
Add 4×43 = 172
——
189
Add 86
——
25)275(11
25
——
25
25

Golden Number = 1
Multiply by 11
—
11
Subtract the last quotient = 11
—
00

Therefore, as nothing remains, the Epact is 29, as before.

A Table of *the nineteen Epacts for the* Julian *and* Gregorian *Accounts, by the Golden Number.*

G. N.	*Julian Epact.*	*Greg. Epact.*	G. N.	*Julian Epact.*	*Greg. Epact.*	G. N.	*Julian Epact.*	*Greg. Epact.*
1	11	29	7	17	6	13	23	12
2	22	11	8	28	17	14	4	23
3	3	22	9	9	28	15	15	4
4	14	3	10	20	9	16	26	15
5	25	14	11	1	20	17	7	26
6	6	25	12	12	1	18	18	7
						19	29	18

Problem VIII.

To calculate the Moon's Age on any given day.

Rule.

To the given day of the month, add the Epact and number of the month : If the sum be less than 30, it is the Moon's age ; but if it exceed 30, then take 30 from it, and the remainder will be the Moon's age.

Note, The numbers to be added to the following months, are as follow :

To	January	0	July	5
	February	2	Auguſt	6
	March	1	September	8
	April	2	October	8
	May	3	November	10
	June	4	December	10

Example. For January 25th, 1786.

Add	Given day	=	25
	Epact	=	29
	No. of the month	=	00
			54
	Subtract		30
			24 = Moon's age.

Problem IX.

To find the times of the New and Full Moon, and the firſt and laſt Quarters.

Rule.

Find the Moon's age on the given day, then, if it be 15, the Moon will be full on that day, and by counting $7\frac{1}{2}$ days backward and forward you will have the firſt and laſt quarters, and by counting backward and forward 15 days, you will have the times of the laſt and next change ; but if the age of the Moon be greater than 15, take 15 from it, and the remainder will ſhew how many days have paſt ſince the laſt full moon, and, counting theſe backward, you will have the day the laſt full Moon happened on, and by knowing that, we can find the change, or either of the quarters, as before. Again, if the age of the Moon, on the aſſumed day, be leſs than 15, then take that from 15, and the remainder will ſhew how many days are to run till the next full Moon, which you will have by adding the remainder to the aſſumed day ; and, proceeding as before, you will have the days of the change, and either quarter as above.

Examp.

Examp. For January 25th, 1786.

Add { Assumed day = 25
Epact = 29
Number of the month = 00 }

54
Subtract 30

Moon's age = 24
Subtract 15

Take the days since the last full Moon = 9
From the assumed day = 25

To the day of the full Moon = 16th.
Add 15

New Moon 31st.

From the full Moon 16
Take $7\frac{1}{2}$

First quarter 9th.

To the full Moon = 16
Add $7\frac{1}{2}$

Last quarter = 23d.

PROBLEM X.

The time of the Moon's coming to the South, after the Sun, being given, to find the Age of the Moon.

RULE.

As 24 hours, the whole difference of time, are to 30, the number of days from change to change: So is the difference of time, to the Moon's age.

Examp. I observed the Moon to be on the meridian, or due south, at 6 o'clock in the afternoon; What is the Moon's age?

24 : 30 :: 6 : $7\frac{1}{2}$ days, *Ans.*

PROBLEM XI.

To find the time of the Moon's Southing.

RULE.

Multiply the Moon's age, on the given day, by 48 minutes, and divide the product by 60, the minutes in an hour, (or multiply by 4, and divide by 5,) and the quotient will shew how many

ny hours and minutes the Moon is later, in coming on the meridian, than the Sun, and counting ſo many hours and minutes forward from 12 o'clock, we have the time of the Moon's ſouthing; if the hours and minutes, found as above, be leſs than 12, then, that will be the time of the Moon's ſouthing after noon; but, if greater than 12, then take 12 from them, and the remainder will be the time of the Moon's ſouthing in the morning.

Examp. 1. Required the time of the Moon's ſouthing on the 25th day of January 1786?

	Moon's age = 24	*Or,* 24
h. m.	48	4
From 19 12	192	5)96
Take 12 00	96 *h. m.*	*h. m.*
7 12	60)1152(19 12	$19\frac{1}{5}$ = 19 12 as before.
Hence, the Moon ſouths at 12 minutes paſt 7 in the morning.	60	
	552	
	540	
	12	

Examp. 2. For the 9th of February 1786?

Moon's age = 10
48
h. m.
60)480(8 0 afternoon, is the time of the Moon's
480 ſouthing.

Note. From the change to the full, the Moon comes to the ſouth afternoon; but from the full to the change, before noon.

Problem XII.

To find on what day of the week, any given day in any month will fall.

As one of the firſt ſeven letters of the alphabet is prefixed to every day in the year, beginning with A, which is always prefixed to the firſt day of January: And as, in common years, the letter, annexed to the firſt Sunday in January, ſhews the Dominical Letter for that year; but every leap year having two Dominical Letters, the firſt of which ſerving to the 24th of February, and the other for the reſt of the year, conſequently, in any common year, the Dominical Letter being known, the firſt of January may be eaſily found, reckoning from A according to the natural order of the letters: And in any leap year, the firſt of its two Dominical Letters will ſhew as above, counting from A 1, B 2, C 3, &c. and by counting backward, you may have the day of the week, on which the firſt of January will happen.

Rule.

RULE.—Find the day of the week anſwering to the firſt of January that year, then add together the days contained in each month from the beginning of the year to the propoſed day of the month incluſively ; divide this ſum by 7, and if any thing remain, after the diviſion, then, count ſo many forward, beginning with that day on which the firſt of January falls, and you will have the day of the week, on which the propoſed day will fall : But if nothing remain, then the day of the week, preceding that day on which the firſt of January falls, anſwers to the propoſed day.

EXAMPLE.

On what day of the week will the 5th day of May 1786 fall?

By the preceding obſervations, and by Prob. 4th, the firſt of January is found to fall on Sunday.

Jan.	31
Feb.	28
March	31
April	30
May	5th.

Now, counting forward ſix days from Sunday, the firſt of January (incluſively) and the 5th of May falls on Friday.

7)125(17
   7
  ——
  55
  49
  ——
   6 from Jan. 1.

PROBLEM XIII.

To find the Cycle of the Sun.

RULE.—Add 9 to the given year ; divide the ſum by 28, and the remainder, after diviſion, is the Cycle required ; but if nothing remain, the Cycle is 28.

EXAMPLE.

For the year 1786?

To 1786
Add   9
——
28)1795(64
    168
    ——
    115
    112
    ——
      3 = Cycle required.

The uſe of this Cycle is to find the Dominical Letter by the following Table.

A

A TABLE *of the Dominical Letters for the New Style, according to the Cycle of the Sun.*

Cycle.	Letter.	Cycle.	Letter.	Cycle.	Letter.	Cycle.	Letter.
1	D C	8	B	15	G	22	E
2	B	9	A G	16	F	23	D
3	A	10	F	17	E D	24	C
4	G	11	E	18	C	25	B A
5	F E	12	D	19	B	26	G
6	D	13	C B	20	A	27	F
7	C	14	A	21	G F	28	E

This Table, by the prefent rule, will ferve but to the end of this century. The leap year being to be omitted in the year 1800, will make it neceffary to add 25 to the date of the year, and then dividing by 28, it will give the Cycle right during the next century. And this is a general rule to be obferved, that when a leap year has been abated, add 16 to the number which was before added to the year, rejecting 28, when it exceeds it, and this number being added to the year, and the fum divided by 28, the remainder, after divifion, will be the Cycle for finding the Dominical Letter. Thus, in the nineteenth century, it will be $9+16=25$, and in the twentieth century $\overline{25+16}-28=13$, which number will ferve two centuries, for the year 2000 is a leap year.

PROBLEM XIV.

To find the year of the Dionyfian Period.

RULE.—Add to the given year 457; divide the fum by 532, and the remainder will be the number required.

EXAMPLE.

Required the year of the Dionyfian Period for the year 1786?

To 1786
Add 457

532)2243(4
2128

115 = Dionyfian Period.

PROBLEM XV.

To find the year of Indiction.

RULE.—Add 3 to the given year; divide the fum by 15, and the remainder, after divifion, will be the Indiction; if nothing remain, it will be 15.

EXAMPLE.

EXAMPLE.

Required the year of Indiction for 1786?

To 1786
Add 3

15)1789(119
15

28
15

139
135

4 = Indiction.

PROBLEM XVI.

To find the Julian Period.

RULE.—Add 4713 to the given year, and the ſum will be the Julian Period.

EXAMPLE.

What year of the Julian Period will anſwer to the year 1786?

To 1786
Add 4713

6499 *Anſ.*

PROBLEM XVII.

To find the Cycle of the Sun, Golden Number, and Indiction, for any current year.

RULE.—To the current year add 4713; divide the ſum by 28, 19 and 15, reſpectively, and the ſeveral remainders will be the numbers required; when nothing remains, the diviſor is the number required.

EXAMPLE.

What are the Cycle of the Sun, Golden Number, and Indiction, for the year 1786?

1786
4713

28)6499(232
56

89
84

59
56

3 Cycle of the Sun.

19)6499(342
57

79
76

39
38

Gold. Numb. = 1

15)6499(433
60

49
45

49
45

Indiction = 4

PROBLEM

PROBLEM XVIII.

Having the Cycle of the Sun, the Golden Number, and Indiction, to find the year of the Christian Æra.

RULE.—Multiply 4845 by the Cycle of the Sun; 4200 by the Golden Number, and 6916 by the Indiction: Add the ſeveral products together, and divide the ſum by 7980; the remainder, after diviſion, will be the Julian Period, from which ſubtract 4713, and the remainder will be the year required.

EXAMPLE.

The Cycle of the Sun being 3, Golden Number 1, and Indiction 4; What year of the Chriſtian Æra is it?

```
 4845       4200       6916          27664
    3          1          4           4200
 ----       ----       -----         14535
14535       4200       27664         -----
                               7980)46399(5
                                     39900
                                     -----
                                      6499
                             Subtract 4713
                                      ----
                                      1786 Anſ.
```

PROBLEM XIX.

To find the time of High Water.

RULE.—Find the Moon's ſouthing, to which add the point of the compaſs making full ſea, on the full and change days, for the place propoſed, and the ſum will be the time required.

EXAMPLE.

I demand the time of high water at Boſton, January 25th 1786, admitting the tide to flow and ebb N. W. and S. E. on the days of change and full?

We have before found the Moon's ſouthing to be *7h.* 12*m.* in the morning.

```
                 h. m.
Therefore to     7 12
           Add   4  0 = the point of the compaſs, and it
                 ----
         Gives  11 12 in the morning, for the time of high water.
```

PROBLEM XX.

To find on what day Eaſter will happen.

It was ordered by the Nicene Council, that Eaſter Sunday ſhould be kept on the firſt *Sunday* after the firſt full Moon, which happened upon or after the twenty firſt day of March, the day on which they thought the Vernal Æquinox happened. Though this was a miſtake, for the Vernal Æquinox, that year, fell on the twentieth of March. But yet, the full Moon, which fell on, or

or next after the twenty firſt of March, they called the Paſchal full Moon. And by the introduction of the Gregorian, or New Style, the Æquinox will now always happen on the twentieth or twenty firſt of March. And the feaſt of Eaſter is now to be kept on the next Sunday after the Paſchal full Moon, or the full Moon which happens after the twenty firſt of March; but, if the full Moon happens on a Sunday, Eaſter day is to be the next Sunday after.

Rule.—Find the age of the Moon on the 21ſt of March, in the given year, and if it be 14, then find the day of the week anſwering to it, and the Sunday following is Eaſter Sunday; but if the Moon's age on the 21ſt day of March be not 14, then reckon forward to the day on which the Moon's age is 14, and find the day of the week anſwering to that day; the Sunday following will be the day required.

N. B. On leap year take the 20th of March.

Examp. When does Eaſter happen in the year 1786?

21 of March
29 Epact.
1 No. of the month.
—
51
Subt. 30
—
21 Moon's age.
Add 23 $\left\{\begin{array}{l}\text{No. of days to the Moon's}\\ \text{being 14 days old.}\end{array}\right.$
—
44
Take 31 = days in March.
—
13th of April, the day of the full Moon, or Eaſter limit.

Jan. 31
Feb. 28
March 31
April 13th
——
7)103(14
7
—
33
28
—
5 Therefore, the firſt of January being Sunday, reckon forward 5 days, including Sunday, and you will find the 13th of April falls on Thurſday, conſequently the next Sunday is the 16th, which is Eaſter Sunday.

Eaſter may be found, for any future time, by the following Table, which is calculated from 1753, the time of the commencement of the New Style in America, and which ſhews, by the Golden Number, the days of the Paſchal full Moons; by which, and the Dominical Letter, the day on which Eaſter will fall, may be found.

The Uſe of the Table.

Firſt find the Golden Number as before taught, which ſeek in the column of Golden Numbers under the time in which the given year is included; right againſt the Golden Number of the year, in the laſt column but one, you have the day of the month on which the Paſchal full Moon happens, which is the limit of Eaſter; from thence run your eye down among the Dominical

Letters, till you come to the Letter of the given year, and againſt it you have the day of the month, on which Eaſter falls that year.

EXAMP. To know when Eaſter falls in 1786.

The Golden Number for the year being 1, and the Dominical Letter A; therefore ſeek in the firſt column (the given year being included between the years 1753 and 1899) for the Golden Number: Then caſt your eye along to the laſt column but one, under the title, Paſchal full ●, and you will find the thirteenth of April to be the day of the full Moon; againſt which, in the laſt column, ſtands E, which ſhews it to be Thurſday, therefore the next Sunday following is Eaſter Sunday, which, by going down the column of Letters to the next A, you will find to be the ſixteenth of April.

GOLDEN NUMBERS *from* 1753 *to* 1899, *and ſo on to* 4199, *incluſively.*

1753 *to* 1899	1900 *to* 2199	2200 *to* 2299	2300 *to* 2399	2400 *to* 2499	2500 *to* 2599	2600 *to* 2899	2900 *to* 3099	3100 *to* 3399	3400 *to* 3499	3500 *to* 3590	3600 *to* 3699	3700 *to* 3799	3800 *to* 4099	4100 *to* 4199	*Month*	*Paſchal full* ● *Days of the Month*	*Dom. Letter.*
14	—	6	17	6	17	—	9	—	1	12	1	12	—	4	*March*	21	*C*
3	14	—	6	—	6	17	—	9	—	9	—	1	12	—		22	*D*
—	3	14	—	14	—	6	17	—	9	—	9	—	1	12		23	*E*
11	—	3	14	3	14	—	6	17	—	9	—	9	—	1		24	*F*
—	11	—	3	—	3	14	—	6	17	—	17	—	9	—		25	*G*
19	—	11	—	11	—	13	14	—	6	17	6	17	—	9		26	*A*
8	19	—	11	—	11	—	3	14	—	6	—	6	17	—		27	*B*
—	8	19	—	19	—	11	—	3	14	—	14	—	6	17		28	*C*
16	—	8	19	8	19	—	11	—	3	14	3	14	—	6		29	*D*
5	16	—	8	—	8	19	—	11	—	3	—	3	14	—		30	*E*
—	5	16	—	16	—	8	19	—	11	—	11	—	3	14		31	*F*
13	—	5	16	5	16	—	8	19	—	11	—	11	—	3	*April*	1	*G*
2	13	—	5	—	5	16	—	8	19	—	19	—	11	—		2	*A*
—	2	13	—	13	—	5	16	—	8	19	8	19	—	11		3	*B*
10	—	2	13	2	13	—	5	16	—	8	—	8	19	—		4	*C*
—	10	—	2	—	2	13	—	5	16	—	16	—	8	19		5	*D*
18	—	10	—	10	—	2	13	—	5	10	5	16	—	8		6	*E*
7	18	—	10	—	10	—	2	13	—	5	—	5	16	—		7	*F*
—	7	18	—	18	—	10	—	2	13	—	13	—	5	16		8	*G*
15	—	7	18	7	18	—	10	—	2	13	2	13	—	5		9	*A*
4	15	—	7	—	7	18	—	10	—	2	—	2	13	—		10	*B*
—	4	15	—	15	—	7	18	—	10	—	10	—	2	13		11	*C*
12	—	4	15	4	15	—	7	18	—	10	—	10	—	2		12	*D*
1	12	—	4	—	4	15	—	7	18	—	18	—	10	—		13	*E*
—	1	12	—	12	—	4	15	—	7	18	7	18	—	10		14	*F*
9	—	1	12	1	12	—	4	15	—	7	—	7	18	—		15	*G*
—	9	—	1	—	1	12	—	4	15	—	15	—	7	18		16	*A*
17	17	9	—	9	—	1	12	12	4	15	4	15	15	7		17	*B*
6	6	17	9	17	9	9	1	1	12	4	12	4	4	15		18	*C*
																19	*D*
																20	*E*
																21	*F*
																22	*G*
																23	*A*
																24	*B*
																25	*C*

The USE of LOGARITHMS.

1. In MULTIPLICATION.

Given two numbers, viz. 275 and 12,6, to find their product.

RULE.—To the logarithm of 275, viz. - - 2,43933
Add the logarithm of 12,6, viz. - - - - - 1,10037

And their ſum is the logar. of their prod. viz. 3465 = 3,53970

2. In DIVISION.

Let it be required to find the quotient, which ariſes by dividing one number by another; ſuppoſe 1425 by 57.
From the logarithm of the dividend, viz. 1425 = 3,15381
Take the logarithm of the diviſor, viz. 57 = 1,75587

And the remainder is the logar. of the quot. viz. 25 = 1,39794

3. In the RULE of THREE.

Three numbers given, to find a fourth, in direct proportion.

RULE.—From the Tables take the logarithms of each of the propoſed numbers, then, add the logarithms of the ſecond and third together, and from the ſum take the logarithm of the firſt, and the remainder will be the logarithm of the fourth number.

Let the three propoſed numbers be 18, 24, and 33, and the operation will ſtand thus;
1,38021 = the logarithm of 24, the 2d term.
1,51851 = the logarithm of 33, the 3d term.

2,89872 = the logarithm of their product.
—1,25527 = the logarithm of the firſt term 18.

1,64345 = the logarithm of the fourth term required, which, by the Table, anſwers to the natural number 44, the 4th proportional to the three propoſed numbers.

4. In INVOLUTION or RAISING POWERS.

To find any power of any propoſed number, or to involve any number to any propoſed power, by logarithms.

RULE.—Multiply the logarithm of the given root by the power, viz. by 2 for the ſquare, by 3 for the cube, &c. and the product is the logarithm of the power ſought.
Required to find the cube of 12?
1,07918 = the logarithm of 12.
×3 = the third power, or cube.

3,23754 = 1728, the cube of 12.

5. *In* Evolution *or* Extracting Roots.

To extract any root of any proposed number.

Rule.—Divide the logarithm of the proposed number by the index of the required root, viz. by 2 for the square, by 3 for the cube, &c. and the quotient will be the logarithm of the root required.

Required to find the cube root of 1728?

3,23754 = the logarithm of 1728, and 3,23754÷3 = 1,07918 is the logarithm of the cube root of 1728, viz. 12.

6. *In* Compound Interest.

To find the amount of any sum for any time, and at any rate, at Compound Interest.

Rule.—Multiply the logarithm of the ratio (*i. e.* the amount of 1*l.* for 1 year) by the number of years, and to the product add the logarithm of the principal; the sum will be the logarithm of the amount.

What will 45*l.* amount to, forborne 12 years, at 6 per cent. per annum, compound interest?

Log. of 1,06, the ratio, is	,02533
Multiply by the time	12
	,30396
Add log. of 45, the princip.	1,65321

The sum is 1,95717 which is the logarithm of 90,7 = £90 14*s.* *Ans.*

7. *In* Discount *at* Compound Interest.

To find the present worth of any sum of money, due any time hence, at any rate, at Compound Interest.

Rule.—From the logarithm of the sum to be discounted, subtract the logarithm of the rate multiplied by the time; and the remainder is the logarithm of the present worth.

What present money will pay a debt of 90*l.* 14*s.* due 12 years hence, discounting at the rate of 6 per cent. per annum?

From the logarithm of £90 14 =	1,95717
Subt. prod. of the log. of the ratio × by the time =	,30396

The remainder 1,65321 is the logarithm of £45 *Ans.*

PLAIN GEOMETRY.

Definitions.

1. A *Point* in the Mathematics is considered only as a mark, without any regard to dimensions.

2. A *Line* is considered as length, without regard to breadth or thickness.

3. A *Plain* or *Surface* has two dimenſions, length and breadth, but is not conſidered as having thickneſs.

4. A *Solid* has three dimenſions, length, breadth and thickneſs, and is uſually called a *Body*.

5. A Line is either *ſtraight*, which is the neareſt diſtance between two Points ; or *crooked*, called a Curve Line, whoſe ends may be drawn further aſunder.

6. If two Lines are at equal diſtance from one another in every part, they are called *parallel* Lines, which, if continued infinitely, will never meet.

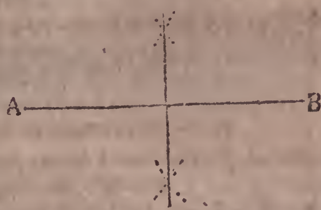
7. If two Lines incline one towards another, they will, if continued, meet in a point : By which meeting is formed an *Angle*.

8. If one Line fall directly upon another, ſo that the Angles on both ſides are equal, the Line, ſo falling, is called a *perpendicular*, and the Angles, ſo made, are called *right* Angles, and are equal to 90 degrees, each.

9. All Angles, except right Angles, are called oblique Angles, whether they are *acute*, that is, leſs than a right Angle ; or *obtuſe*, that is, greater than a right Angle.

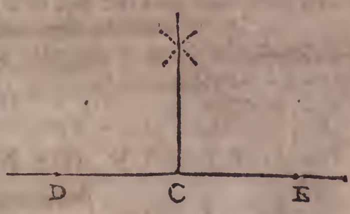
GEOMETRICAL PROBLEMS.

PROBLEM I. *To divide a Line* AB *into two equal parts.*



Set one foot of the compaſſes in the point A, and, opening them beyond the middle of the line, deſcribe arches above and below the line ; with the ſame extent of the compaſſes, ſet one foot in the point B, and deſcribe two arches croſſing the former : Draw a line from the interſection of the arches above the line, to the interſection below the line, and it will divide the line AB into two equal parts.

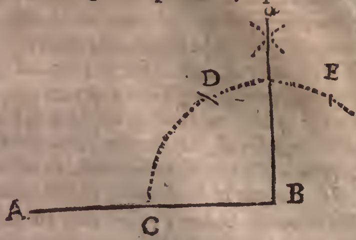
PROBLEM II. *To erect a perpendicular on the point* C *in a give line.*



Set one foot of the compaſſes in the given point C, extend the other foot to any diſtance at pleaſure, as to D, and with that extent make the marks D, and E. With the compaſſes, one foot in D, at any extent above half the diſtance of D and E, deſcribe an arch above the line, and with the ſame extent, and one foot in E, deſcribe an arch, croſſing the former ; draw a line from the interſection of the arches to the given point C, which will be perpendicular to the given line in the point C.

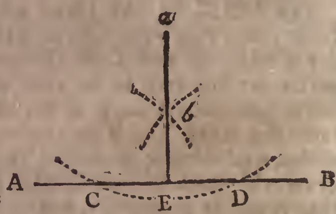
Problem III. *To erect a perpendicular upon the end of a line.*

Set one foot of the compaſſes in the given point B, open them to any convenient diſtance, and deſcribe the arch CDE; ſet one foot in C, and with the ſame extent, croſs the arch at D: With the ſame extent croſs the arch again, from D to E; then with one foot of the compaſſes in D, and with any extent above the half of DE, deſcribe an arch *a*; take the compaſſes from D, and, keeping them at the ſame extent with one foot in E, interſect the former arch *a* in *a*; from thence draw a line to the point B, which will be a perpendicular to AB.



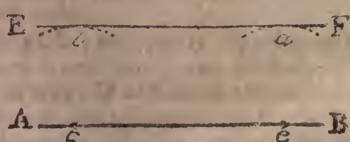
Problem IV. *From a given point, a, to let fall a perpendicular to a given line* AB.

Set one foot of the compaſſes in the point *a*, extend the other ſo as to reach beyond the line AB, and deſcribe an arch to cut the line AB in C and D; put one foot of the compaſſes in C, and, with any extent above half CD, deſcribe an arch *b*; keeping the compaſſes at the ſame extent, put one foot in D, and interſect the arch *b* in *b*; through which interſection, and the point *a*, draw *a* E, the perpendicular required.



Problem V. *To draw a Line parallel to a given Line* AB.

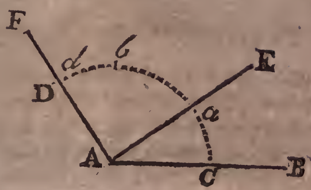
Set one foot of the compaſſes in any part of the line, as at *c*; extend the compaſſes at pleaſure, unleſs a diſtance be aſſigned, and deſcribe an arch *b*; with the ſame extent, in ſome other part of the line AB, as at *e*, deſcribe the arch *a*; lay a ruler to the extremities of the arches, and draw the line EF, which will be parallel to the line AB.



Problem VI. *To make an Angle equal to any number of Degrees.*

It is required to lay off an acute Angle of 35° on a given line AB.

Take

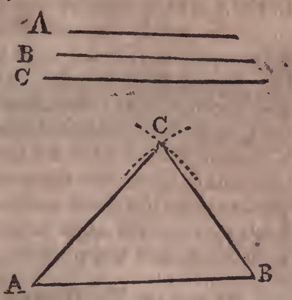


Take 60 degrees from the line of chords in the compaffes, fet one foot of the compaffes in the point A, defcribe an arch CD, at pleafure; then fet one foot of the compaffes in the brafs centre in the beginning of the line of chords, and bring the other to 35 on the line; with this extent, fet one foot in C, with the other interfect the arch CD, in *a*, and through *a* draw the line AE, fo will EAB be an angle of 35 degrees.

If the angle had been obtufe, fuppofe 125°, then take 90° from the line of chords; fet one foot in C, and interfect the arch in *b*; then take 35° from the fame line of chords, and fet them from *b* to *d*; a line drawn from A through *d* to F will make an angle, FAB, of 125°.

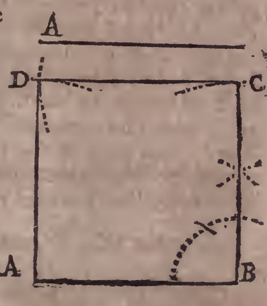
To meafure an angle by the line of chords is only to take the diftance on the arch between the lines AB and AE, or AB and AF, and laying it on the line of chords.

PROBLEM VII. *To make a Triangle, whofe fides fhall be equal to three given lines, provided any two of them be longer than the third.*



Let A,B,C, be the three given lines; draw a line, AB, at pleafure; take the line C in the compaffes, fet one foot in A, and with the other make a mark at B; then take the given line B in the compaffes, and fetting one foot in A, draw the arch C; then take the line A in the compaffes, and interfect the arch C in C; laftly, draw the lines AC and BC, and the triangle will be completed.

PROBLEM VIII. *To make a Square, having equal fides, equal to any given line.*



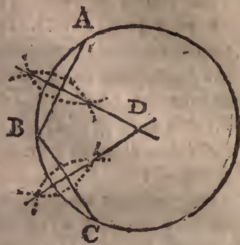
Let A be the given line; draw a line AB equal to the given line; from B raife a perpendicular to C equal to AB, with the fame extent, fet one foot in C, and defcribe the arch D; alfo with the fame extent, fet one foot in A and interfect the arch D; laftly, draw the line AD and CD, and the fquare will be completed.

In like manner may a Parallelogram be conftructed, only attending to the difference between the length and breadth.

PROBLEM

PROBLEM IX. *To describe a Circle, which shall pass through any three given Points, which are not in a straight line.*

Let the three given points be ABC, through which the circle is to pass. Join the points AB and BC with right lines, and bisect these lines; the point D, where the bisecting lines cross each other, will be the centre of the circle required. Therefore, place one point of the compasses in D, extending the other to either of the given points, and the circle, described by that radius, will pass through all the points.



Hence, it will be easy to find the centre of any given circle; for, if any three points are taken in the circumference of the given circle, the centre will be readily found as above. The same may also be observed, when only a part of the circumference is given.

PROBLEM X. *To describe an Ellipsis or Oval mechanically.*

Draw two parallel lines, as L and M, at a moderate distance, by Prob. 5; then draw two others at the same distance, across the former, as N and O; by the crossing of these lines will be made a figure, ABCD, of four sides; extend the compasses at pleasure, and setting one foot in D, describe the arch *cde*; with the same extent, set one foot in B, and describe the arch *fgh*; then, set one foot in C, and contract them so as to reach the point *e*, and describe the arch *lm*; with the same extent, and one foot in A, describe the arch *ik*, and the oval will be completed. In the same manner, with a greater or less extent of the compasses, may a greater or less oval be made by the same four sided figure ABCD.

Of PLAIN TRIGONOMETRY, RIGHT *and* OBLIQUE ANGLED.

Plain Trigonometry is that science, by which we measure the sides and angles of plain triangles.

SECTION I. *Of Rectangular Trigonometry.*

In a right triangle, the longest side is usually called the hypothenuse, the next longest, the base, and the shortest, the perpendicular.

Logarithmic sines, tangents, and secants, are called the *tabular* sides of a triangle, and are the sines, &c. of the opposite angles. The length of the sides are called the *natural* sides.

All

All the three angles of a triangle are equal to two right angles, or 180°.

The proportion ought to be made between ſides and ſides; and between angles and angles.

When a ſide is required, any ſide (whether known, or not) may be made radius; but when an angle is required, then a known ſide only, muſt be made radius.

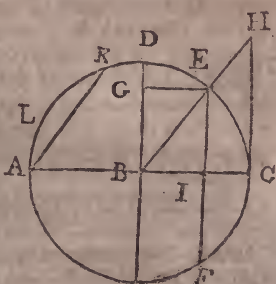
Note. A ſide is ſaid to be made radius, when one foot of the dividers is ſet in one end of the ſide, and ſuch a circle deſcribed, of which the ſide is the ſemidiameter: Alſo, that when the hypothenuſe is radius, it is the ſign of the right angle, or 90°, and the baſe and perpendicular, uſually called the legs, become ſines of their oppoſite angles: But when one of the legs is made radius, the other becomes the tangent of the oppoſite angle, and the hypothenuſe, the ſecant of the ſame angle.* Tangent's radius is 45°.

When a ſide is to be found, the two firſt terms of the proportion muſt be tabular ſides, and the laſt a real one; but when an angle is to be found, the two firſt terms muſt be real ſides, and the third, a tabular one.

The given parts, whether ſides or angles, are marked with —, and the part required, with O.

Angles are meaſured by the arch of a circle. The periphery of every circle, whether great or ſmall, is divided into 360 degrees, each degree into 60 minutes, every minute into 60 ſeconds, and ſo on, to thirds, fourths, &c.

Any portion of the periphery of a circle, as ECF, is called an *arch*, and a line drawn from the ends of an arch, as, EIF, is called the *chord* of the arch. Half the chord of any arch, as EI, is called the *ſine* of the arch EC, and IC is called the *verſed ſine* of the ſame arch EC: So alſo, EG is the *ſine* of the arch ED. A line drawn perpendicular to the diameter of a circle, ſo as to touch the circle, and not cut it, is called a tangent, as CH, which is the tangent of the arch EC, becauſe the line BH, drawn from the centre B, through E, called the *ſecant*, meets it in the point H.



The complement of an arch is the remainder, after the arch is taken from 90°; thus KD is the complement of the arch ALK, taken from the arch AD. The coſine or ſine complement of an arch is the ſine of the complement of that arch, as ED is the complement of EC.

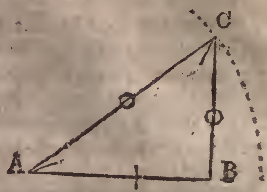
PROBLEM

* To work on the ſcale with a ſecant, you muſt take the ſines backwards, that is 80 ſines for 10 ſecants, &c.

Problem I. *The Angles and one of the Legs given, to find the Hypothenuſe and other Leg.*

Examp. In the triangle ABC, right angled at B, ſuppoſe the leg AB, 86 equal parts (as feet, yards, miles, &c.) the angle A = 33°, 40′, and the angle C = 56°, 20′; Required the length of the hypothenuſe AC, and the other leg BC?

Geometrically. Draw AB equal to 86, from any line of equal parts, then upon the point B, erect the perpendicular BC; laſtly, from the point A, draw the line AC, making with AB an angle = 33°, 40′, and that line produced will meet BC in C, and ſo conſtitute the triangle. The length of AC and BC may be found by taking them in your compaſſes, and applying them to the ſame line of equal parts that AB was taken from.



By Calculation. By making the hypothenuſe radius, the legs will become the ſines of the oppoſite angles; and as natural ſides are required, the proportions muſt begin with tabular ſides: Therefore, for the hypothenuſe,

As the ſine of C	56°,20′	9,92027
Is to radius	90 ,00	10,
So is the ſide AB	86	1,93450
		11,93450
		9,92027
To the ſide AC	103,3	2,01423

Here, I add the logarithms of the 2d and 3d terms, and from their ſum ſubtract the firſt, and the remainder is the logarithm of the ſide ſought, which gives 103,3. The ſame muſt be done in all the following caſes.

For the Leg BC.

As the ſine of the angle C	56°,20′	9,92027
Is to the ſine of the angle A	33 ,40	9,74380
So is the ſide AB	86	1,93450
To the ſide BC	57, 28	1,75803

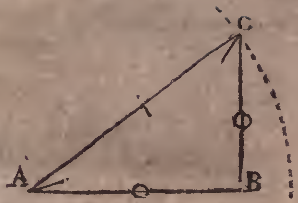
It might have been as eaſily found by the following proportion: As R : S,A : : AC : BC.

Problem II. *The Angles and Hypothenuſe given, to find the Legs.*

Examp. In the triangle ABC, ſuppoſe the hypothenuſe AC = 146, the angle A = 36°,25′, and the angle C = 53°,35′; Required the legs AB and BC?

Geometrically. Draw the line AB at pleaſure, and make the angle A = 36°,25′; then take AC = 146 from any line of equal parts

parts; laſtly, from the point C let fall the perpendicular CB on the line AB: So the triangle is conſtructed, and AB and BC may be meaſured from the line of equal parts.



By Calculation. Making AC radius, the legs become ſines, as before, and as the angles are given to find the ſides, we muſt begin the proportions with angles, or tabular ſides.

For the Leg AB.

As radius	90°,00′	10,
Is to the ſine of C	53 ,35	9,90565
So is ſide AC	146	2,16435
To ſide AB	117,5	2,07000

For the Leg BC.

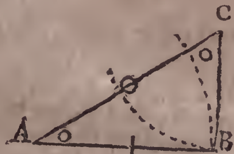
As radius	90°,00	10,
Is to the ſine of A	36 ,25	9,77353
So is ſide AC	146	2,16435
To ſide BC	86,67	1,93788

As we had before found AB, the proportion might have been As S,C : S,A : : AB : BC.

Problem III. and IV. *The two Legs given, to find the Angles and Hypothenuſe.*

Examp. In the triangle ABC, ſuppoſe the leg AB = 94, and BC = 56; Required the angles and hypothenuſe?

Geometrically. Draw AB = 94 from any line of equal parts, then, from the point B raiſe BC perpendicular to AB, and take BC from the former line of equal parts = 56; laſtly, join the points A and C with the ſtraight line AC, ſo the triangle is conſtructed. AC may be found by taking it in your dividers and applying it to the line of equal parts; and the angles may be meaſured by the 6th Geometrical Problem.



By Calculation. 1*ſt*, For the angle A; ſuppoſing the baſe AB the radius, then the hypothenuſe becomes ſecant of the angle A, and the perpendicular BC, the tangent of the angle A: And as an angle is required, we muſt begin the analogy with a natural ſide.

As AB	94	1,97313
Is to BC	56	1,74819
So is tangent's radius	45°,00	10,
To the tangent of A	30°,47′	9,77506

The perpendicular might have been made radius, and then the proportion would have been, As BC : AB : : tang. rad. : tang. of C.

Now,

Now, as we have found the angle A, and as the angles A and C, taken together, are equal to 90°, therefore from 90°,00′

Take the angle A = 30°,47

And we have the angle C = 59°,13′

2d. *For the Hypothenuſe.* The baſe ſtill being radius, we have this analogy for finding the hypothenuſe : As T. R : Sec. A : : AB : AC. But this may be done without the help of ſecants : For, having found the angles, we may now make the hypothenuſe radius ; and as a natural ſide is required, we muſt begin the proportion with a tabular ſide ; therefore,

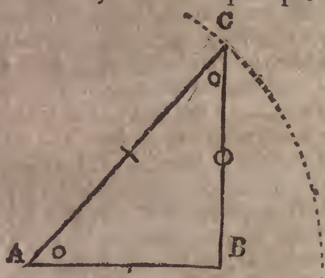
As the ſine of C	59°,13′	9.93405
Is to radius	90 ,00	10,
So is AB	94	1,97313
To AC	109,4	2,03908

Or the analogy might have been, As S. C : R : : BC : AC.

PROBLEMS V. *and* VI. *The Hypothenuſe and one of the Legs given, to find the Angles, and other Leg.*

EXAMP. In the triangle ABC, ſuppoſe the leg AB = 83, and the hypothenuſe AC = 126 ; Required the angles A & C, & the leg BC?

Geometrically. Draw AB = 83 from any line of equal parts ; and from the point B, raiſe the perpendicular BC of any length, then, take the length of AC 126 from the ſame line of equal parts, and ſetting one foot of the dividers in A, with the other croſs the perpendicular BC in C ; laſtly, join AC, ſo the triangle will be conſtructed, and the angles may be meaſured as directed in Problem 3d and 4th.



By Calculation. Firſt, for the angle C ; and as an angle is required, we muſt begin with a ſide, making the hypothenuſe radius.

As AC	126	2,10037
Is to AB	83	1,91908
So is radius	90°,00	10,
To ſine of C	41 12	9,81871

From 90°,00′

Take the angle at C = 41 ,12

And we have the angle A = 48°,48′

For the ſide BC. As a ſide is now required, we muſt begin with an angle ; therefore,

As radius	90°,00′	10,
Is to the ſine of A	48°,48′	9,87646
So is AC	126	2,10037
To BC	94.8	1,97683

SECTION

SECT. 2. *Of oblique angular Trigonometry.*

In any triangle, the ſides are proportional to the ſines of the oppoſite angles.

When two angles of any triangle are given, their ſum, being ſubtracted from 180°, leaves the third angle; and when one angle is given, that being ſubtracted from 180°, leaves the ſum of the two unknown angles.

When an angle exceeds 90°, ſubtract it from 180° and work with the remainder.

When the given and required parts, viz. ſides and angles, are oppoſite.

RULE 1.—As in right angled triangles.

As the ſine of any angle is to the ſine of any other angle: So is the ſide oppoſite to the firſt angle, to the ſide oppoſite to the other angle.

Or, As any one ſide is to any other ſide: So is the ſine of the angle oppoſite to the firſt ſide, to the angle oppoſite to the other ſide.

When any two ſides with the angle included between them are given.

RULE 2.—As the ſum of any two ſides is to their difference; So is the tangent of the half ſum of the two oppoſite angles, to the tangent of half the difference of thoſe two angles; which half difference, being added to the half ſum, gives the greater of the two angles, and, being ſubtracted from the half ſum, leaves the leſs of the two unknown angles.

When the three ſides are given, to find the angles.

RULE 3.—As the baſe of any triangle (or ſum of the ſegments of the baſe) is to the ſum of the other two ſides: So is the difference of thoſe ſides, to the difference of the two ſegments of the baſe, made by letting fall a perpendicular to the baſe from the angle oppoſite to it: Half of which difference, being added to half the ſum of the two ſegments, gives the longeſt, and, being ſubtracted, leaves the ſhorteſt.

The learner being now ſomewhat acquainted with the common method of working by logarithms, it will be proper to ſhew how to perform thoſe proportions without ſubtracting the firſt number from the ſum of the ſecond and third; which is done by ſetting down the arithmetical complement of the firſt term inſtead of the logarithm. This may be readily done thus; ſubtract the firſt figure of the logarithm from 10, and ſet down the remainder: Then ſubtract each of the other figures, index and all, from 9, ſetting down the remainders, and place a dot before the index, as in the caſe of the logarithm. Thus the arithmetical complement (uſually marked Co. Ar.) of the logarithm 9,66004 is 0,33996, and ſo of any other.

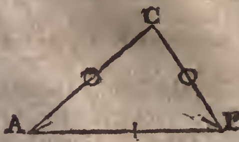
When the arithmetical complement of the firſt term is uſed inſtead of the logarithm, add all the three numbers together, and reject 10 out of the index of their ſum, as in thoſe caſes where the radius is the firſt term.

PROBLEM

PROBLEM I. *In the oblique angled triangle ABC, given two angles and a ſide oppoſite to one of them, to find the two other ſides.*

Suppoſe the angle at A 36°, 40′, the angle at B, 60°, 51′, and the baſe AB 85,6; Required AC and BC?

Geometrically. Draw the baſe AB, and from any ſcale of equal parts, lay thereon 85,6 from A to B; then, from the line of chords, lay off an angle of 36°, 40′ at A, and an angle of 60°, 51′ at B, and the meeting of theſe two lines in C completes the triangle, and AB and BC may be meaſured by the ſame line of equal parts.



From the ſum of all the angles	180°,00′
Take the ſum of the angles A and B, viz.	97°,31′
And we have the angle C equal to	82°,29′

Here we have the angle at C oppoſite to the given baſe, and the angles at A and B oppoſite the two required ſides, which may be found by the firſt rule, as follows:

By Calculation. For the ſide BC. Having to find a ſide, we begin with an angle.

As the ſine of the angle at C,	82°,29′	Co. Ar.	0,00375
Is to the ſine of the angle A,	36°,40′		9,77609
So is the baſe AB	85,6		1,93247
To ſide BC	51,55		1,71231

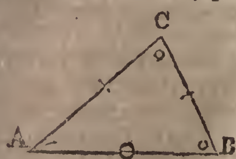
For the ſide AB.

As the ſine of C	82°,29′	Co. Ar.	0,00375
Is to the ſide of B	60°,51′		9,94118
So is AB	85,6		1,93247
To AC	75,4		1,87740

PROBLEM II. and III. *Two ſides, and an angle oppoſite to one of them, given, to find the two other angles and remaining ſide.*

In the oblique angled triangle ABC, given the ſide AC 75.4, the ſide BC 51,56, and the angle at A 36°, 40′, to find the baſe AB, and the angles at B and C.

Geometrically. Draw the baſe AB at pleaſure, and on any point aſſumed, as A, make an angle of 36°,40′; take 75,4 from the ſcale of equal parts and ſet it from A to C; then take 51,56 from the ſame ſcale; ſet one foot of the dividers in C, and with the other interſect the baſe in B; laſtly, draw BC, and the triangle is completed, and the baſe may be meaſured by the ſame ſcale of equal parts.



By

By Calculation. Here we have the ſide BC oppoſite the known angle at A, and the ſide AC oppoſite the unknown angle at B, which may be found by Rule 1ſt.

To find the angle at B. Having to find an angle, we begin with a ſide.

As BC	51,55	Co. Ar.	8,28778
Is to AC	75,4		1,87737
So is the ſine of the angle A	36°,40′		9,77609
To the ſine of the angle B	60°,51′		9,94124

From the ſum of all the angles	180°,00′
Take the ſum of the angles A and B	97°,01′
And we have the angle C equal to	82°,59′

For the baſe AB. Having to find a ſide, we begin with an angle.

As the ſine of A	36°,40′	Co. Ar.	0,22392
Is to the ſine of C	82°,29′		9,99625
So is BC	51,55		1,71223
To AB	85,6		1,93240

PROBLEM IV. *and* V. *Two ſides, and the angle included between them at* A, *given, to find the two other angles and the other ſide.*

In the oblique angled triangle ABC, given the ſide AC 75,4, the baſe AB 85,6, and the included angle at A 36°,40′, to find the angles B and C, and the ſide BC.

Geometrically. Draw the baſe AB, and, from any ſcale of equal parts, ſet off 85,6 from A to B; make an angle at A of 36°,40′, and draw AC, and from the ſame ſcale of equal parts, ſet 75,4 from A to C; laſtly, draw the line BC, and the triangle is completed: BC may be meaſured by the ſame ſcale of equal parts, and the angles B and C, on the line of chords.

By Calculation. Here we have given the two ſides AB and AC, with the angle included between them; and therefore theſe caſes muſt be ſolved by Rules 2d and 1ſt. Now, as the three angles of every triangle are equal to 180°, the angle at A 36°,40′ being ſubtracted from 180° leaves 143°,20′, the ſum of the two unknown angles B and C, half of which is 71°,40′; and half their difference may be found by the following proportion, according to Rule 2.

As the ſum of the two ſides AB and AC	161	Co. Ar.	7,79318
Is to their difference	10,2		1,00860
So is the tangent of half the ſum of the unknown angles B and C	71°,40′		10,47969
To the tangent of half their difference	10°,49′		9,28147

To

To the half ſum	71°,40′	From the half ſum	71°,40′
Add the half difference	10 ,49	Take the half diff.	10°,49′
The ſum is the greater ang. C	82 ,29	The remaind. is the leſs angle B	60°,51′

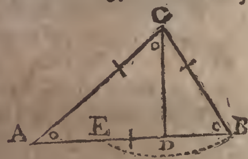
Having found the angles B and C, the ſide BC may be found by Rule 1.

As the ſine of C	82°,29′	Co. Ar.	0,00375
Is to the ſine of A	36°,40′		9,94118
So is AB	85,6		1,03247
To BC	51,56		1,87740

PROBLEM VI. *The three ſides given, to find the angles.*

In the oblique angled triangle ABC, given the baſe AB 85,6, the ſide AC 75,4, and the ſide BC 51,56; Required the angles?

Geometrically. Draw the baſe AB, and ſet off 85,6 from any ſcale of equal parts from A to B; take 75,4 from the ſame ſcale, and ſetting one foot in A, deſcribe an arch; then from the ſcale take 51,56, and, ſetting one foot in B, interſect the former arch in C; from C draw lines to A and B, and the triangle is completed. The angles may all be meaſured upon the line of chords.



By Calculation. Here being no angle given, theſe caſes muſt be ſolved by Rule 3d, in the following manner: Place one foot of the dividers in C, and extend the other ſo as to take in the ſhorteſt ſide BC, and deſcribe the arch BE; then, from C let fall a perpendicular on the baſe AB, which will divide it into two ſegments AD the greater, and DB the leſs, whoſe difference is AE: Then,

As the baſe AB	85,6	—Co. Ar.	8,06753
Is to the ſum of the two ſides AC and BC	126,96	——	2,10366
So is the difference of the ſides AC & BC	23,84	——	1,37730
To the difference of the ſegments of the baſe, or AE — —	35,36	—	1,54849
Half the difference of the ſegments is	17,68		

To half the baſe	42,8	From half the baſe	42,8
Add half the difference	17,68	Take the half difference	17,68
And the ſum is the greater ſegment AD	60,48	And the remainder is the leſs ſegment DB	25,12

Thus is the oblique angled triangle ABC divided into two right angled triangles ADC and BDC, both right angled at D, in each of which are given the baſe and hypothenuſe, to find the other parts.

Firſt, For the angle at C in the right angled triangle ADC, making the hypothenuſe radius.

As

As AC	75,4	Co. Ar. 8,12263
Is to AD	60,48	1,78161
So is radius	90°,00′	10,
To the sine of C	53°,20′	9,90424

The angle A, being the complement of the angle C, is 36°,40′. Then for the angle C in the right angled triangle BDC.

As BC	51,56	Co. Ar. 8,28778
Is to BD	25,12	1,40002
So is radius	90°,00′	10,
To the sine of C	29°,09′	9,68780

Whence the angle A is 60°,51′, being the complement of 29°,09′; and the angle at C, in one triangle, being added to the angle C in the other, is 82°,29′; thus the solution of the problem is finished.

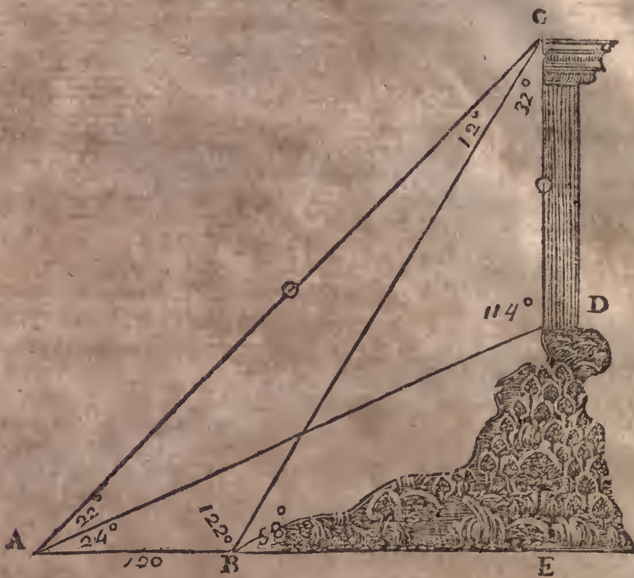
Trigonometry is easily applied to Navigation, and the Mensuration of Heights and Distances. With respect to the former; suppose in the first Problem of right angled Trigonometry, the angle at A is the ship's course, the base to be the true, (or meridional,) difference of latitude, the perpendicular to be the departure, or difference of longitude, and the hypothenuse to be the distance the ship is to run; then we have the course and true (or meridional) difference of latitude given, to find the distance, and departure from the meridian, (or difference of longitude.)

In Problem 2d, we have the course and distance given, to find the true (or meridional) difference of latitude, and the departure, (or difference of longitude.)

With respect to heights and distances; If we suppose, in the first Problem before mentioned, the angle at A to be the angle which the top of any distant object makes with the surface of the earth, where we stand—the base to be the distance of the object, (on level ground,) and the perpendicular, the object's height; then we have the angle A, and the distance AB, to find the height BC; but this will serve only on level ground, and where the object is accessible.

The distance of any *inaccessible* object may be found by Problem 1st, of oblique Trigonometry: For, if we suppose the object at C, then, at *two* stations, as at A and B, take the bearing of the place; also measure the stationary distance AB, and you will then have *two* angles and a side opposite to one of them, to find either of the other sides.

To take the height of an Object ſtanding on a hill, which is inacceſſible.



At two ſtations, as at A and B, take the angles, viz. CAE and CBE, which the top of the object makes with an horizontal line, and that, which the bottom of the object makes with the firſt ſtation, at A, viz. DAE, then take DAE from CAE, and the remainder is CAD.

Note, When an angle is expreſſed by three letters, the middle one ſhews the angle. Now, ſuppoſe the ſtationary diſtance AB 120, the angle ACB 12°, and angle CBA 122°, then by Problem 1ſt, of oblique Trigonometry, we have two angles and a ſide oppoſite to one of them given, to find the ſide AC. Therefore,

		Co. Ar.
As S. of ACB	12°,00′ —	0,68213
Is to S. of CBA	122,00′ —	9,92842
So is ſtationary diſtance	120 —	2,07918
To ſide AC	489,5 —	2,68973

Note, I ſubtracted 122° from 180°, and worked with the remainder, and in the following, 114° from 180°. Now, having found AC 489,5, ſuppoſe the angle CDA 114°, and the angle CAD 22°, and we have two angles and a ſide oppoſite to one of them, as before, to find the perpendicular height of the object CD. Therefore,

As

	deg. min.		Co. Ar.		deg.	Co. Ar.
As S. of CDA	114,00	Co. Ar. 0,03927		As S. of CDA	114	— 0,03927
Is to S. of CAD	22,00	9,57358		Is to S. of ACD	44	— 9,84177
So is side AC	489,5	2,68973		So is side AC	489,5	— 2,68973
To perpend. hht. CD	200,7	2,30258		To side AD	372,2	— 2,57077

To find the height of the mountain and object together; we have the right angled triangle ACE, in which are given the hypothenuse AC 489,5, angle CAE 46°, and the angle ACE 44°, whence, by Problem 2d, of right angled Trigonometry, we have these proportions.

As radius	90°	10,00000	As radius	90°	—10,00000
Is to S. of CAE	46°	9,85693	Is to S. of ACE	44°	— 9,84177
So is hypoth. AC	489,5	2,68973	So is AC	489,5	— 2,68973
To perp. hht. CE	252,1	2,54666	To AE	340	— 2,53150

If you subtract CD from CE, you will have the height of the hill 151,4.

Any figure in Navigation, or Mensuration of Heights and Distances, may be measured Geometrically, as directed in the aforegoing Problems of Trigonometry.

MENSURATION *of* SUPERFICIES *and* SOLIDS.

SECTION I. *Of* SUPERFICIES.

Superficies, or surfaces, are measured by the superficial inch, foot, yard, &c. according to the measures peculiar to different artists.

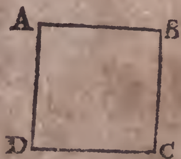
The superficial inch, foot, &c. is one inch, foot, &c. in length and breadth; and, because 12 inches make 1 foot of Long Measure, therefore, $12 \times 12 = 144$ inches make 1 superficial foot, $3 \times 3 = 9$ feet, a yard, &c.

The superficial content of every surface is found by the proper rule of its figure, whether square, triangle, polygon, or circle.

ARTICLE 1. *To measure a Square, having equal sides.*

RULE.—Multiply the side of the square into itself, and the product will be the area or superficial content, of the same name with the denomination taken, either in inches, feet, or yards, respectively.

Let ABCD represent a square, whose side is 12 feet. Multiply the side 12 by itself, thus,



12 inches.	12 feet.
12 inches.	12 feet.
Area = 144 inches.	144 feet.

By

By the Sliding Rule.

Set 1 to the length on B, then, find the breadth on A, and opposite to this on B, you will have the content.

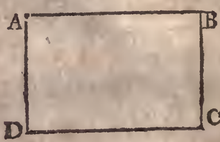
By Gunter's Scale.

Extend the dividers from 1, on the line of numbers, to the length ; that distance, laid the same way from the breadth, will point out the answer.

ART. 2. *To measure a Parallelogram, or long Square.*

RULE.—Multiply the length by the breadth, and the product will be the area, or superficial content.

Let ABCD represent a parallelogram, whose length is 16 feet, and breadth, 12 feet. Multiply 16 by 12.



Length 16
Breadth 12

192 area.

The content of this figure is found on the sliding rule and scale, as the former.

ART. 3. *When the breadth of a Superficies is given, to find how much in length will make a square foot, yard, &c.*

RULE.—As the breadth is to a foot, yard, &c. so is a foot, yard, &c. to the length required to make a foot, yard, &c. Or, divide 144 by the breadth, and the quotient will be the length required.

How much, in length, of a board $2\frac{1}{2}$ feet wide, will make a square foot ?

In. br. In. leng. In. br. In. leng.
As 30 : 12 :: 12 : 4,8
12

30)144(4,8 inches, length required.
120

240
240

In.
Breadth = 30)144(4,8 inches, *Ans.*

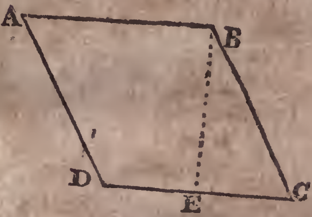
ART. 4. *To measure a Rhombus.*

Definition. A rhombus is a figure with four equal sides, in the form of a diamond on cards, having two angles greater, and two less, than the angles of a square : The former are called *obtuse* angles, and the latter, *acute*, or sharp, angles.

RULE.

RULE.—Multiply the ſide by the length of a perpendicular let fall from one of the obtuſe angles to the ſide oppoſite ſuch angle.

Let ABCD repreſent a rhombus, each of whoſe ſides is 15 feet: A perpendicular let fall from the obtuſe angle, at B, on the ſide DC, will interſect it in the point E, ſo will BE be 12 feet; and this being multiplied into the given ſide, the product will be the area of the rhombus.



Side = 16
Perp. = 12

192 area.

By the Sliding Rule.

Set 1 on A to the length on B; find the perpendicular height on A, againſt which on B is the content.

By Gunter.

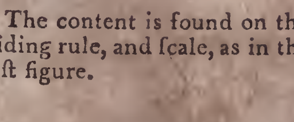
The extent from 1 to the perpendicular height will reach from the length to the content.

ART. 5. *To find the Area of a Rhomboides.*

Definition. A rhomboides is a figure whoſe oppoſite ſides and oppoſite angles are equal.

RULE.—Multiply one of the longeſt ſides by the perpendicular let fall from one of the obtuſe angles on one of the longeſt ſides.

Let ABCD repreſent a rhomboides; the longeſt ſides AB and CD being 16,5 feet, and the perpendicular AE, 9,7 feet.



Side = 16,5
Perp. 9,7

1155
1485

Anſ. 160,05 feet.

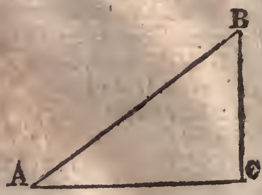
The content is found on the ſliding rule, and ſcale, as in the laſt figure.

ART. 6. *To meaſure a Triangle.*

RULE.—If it be a right angled triangle, multiply the baſe by half the perpendicular, or half the baſe by the perpendicular, and the product will be the area: But, if it be an oblique angled triangle (whether obtuſe, or acute) multiply half the baſe by the length of the perpendicular let fall on the baſe from the angle oppoſite to it, and the product will be the area. The longeſt ſide of a triangle is uſually called the baſe, except in a right angled

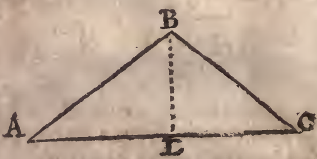
gled triangle, where the longeſt of the two legs, which include the right angle, is called the baſe.

In the right angled triangle ABC right angled at C ; the baſe AC is 18,8 feet, and the perpendicular BC = 12,6.



Baſe = 18,8	*Or*, Perp. = 12,6
Perp. = 6,3	½ Baſe = 9,4
564	504
1128	1134
118,44 area.	118,44 area.

The oblique angled triangle ABC being given, let fall a perpendicular from the angle at B on the baſe AC, and that perpendicular is the height of the triangle. The baſe AC being 15,6, and the perpendicular BD = 9, to find the area.



7,8 = half the baſe.
9 = height of the triangle.

70,2 = area.

By the Sliding Rule.

Set 1 on A to the length of the baſe on B, and oppoſite to half the length of the perpendicular, on A, you will have the content on B.

By Gunter.

The extent from 1 to half the length of the perpendicular will reach from the length of the baſe to the content.

In this place it may be proper to inſtruct the learner in one of the properties of a right angled triangle : viz. That the ſquare of the longeſt ſide of a right angled triangle, uſually called the hypothenuſe, is equal to the ſum of the ſquares of the two other ſides, uſually called the legs, which is of great uſe, for, by this mean, any two ſides of a right angled triangle being given, the other may be found by common Arithmetic. Thus, in the right angled triangle ABC, the baſe AC and perpendicular BC being given, the hypothenuſe AB may be found by extracting the ſquare root of the ſum of the ſquares of the baſe and perpendicular.

Baſe

Baſe 18,8	Perp. 12,6	353,44 = ſquare of the baſe.
18,8	12,6	158,76 = ſquare of the perp.
1504	756	. . .
1504	252	512,20(22,63 hypothenuſe.
188	126	4
353,44	158,76	42)112
		84
		446)2820
		2676
		4523)14400
		13569
		831

And, if the hypothenuſe and one of the legs be given, the other may be found by ſubtracting the ſquare of the given leg from the ſquare of the hypothenuſe.

There are ſome numbers, the ſum of whoſe ſquares make a perfect ſquare, of which ſort are 3 and 4, whoſe ſquares, being added together, make 25, which is the ſquare of 5 : Therefore, if the baſe of a triangle be 4, and the perpendicular 3, the hypothenuſe will be 5 ; and if any of theſe numbers be multiplied by any other number, thoſe products will be the ſides of right angled triangles, as 6, 8, 10, and 15, 20, 25, &c. Thus, artificers, when they ſet off the corner of a building, uſually meaſure 6 feet on one ſide, and 8 feet on the other, then laying a ten feet pole acroſs, it makes the corner a true right angle.

ART. 7. *There is another method of finding the area of triangles, the three ſides being given.*

RULE.—Add the three ſides together, then take the half of that ſum, and out of it ſubtract each ſide ſeverally ; multiply the half of the ſum and theſe remainders continually, and the ſquare root of this product will be the area of the triangle.

In the oblique triangle ABC, the baſe AC is given 15,6, the ſide AB is 10,4, and the ſide BC is 9,2, to find the area.

15,6	17,6	17,6	17,6
10,4	—15,6	—10,4	— 9,2
9,2	2	7,2	8,4
35,2 ſum.			
17,6 = half the ſum.			

17,6

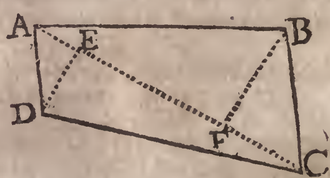
```
   17,6                    2128,8960(46,139 = area.
      2                    16
  -----                    ----
   35,2                 86)528
    7,2                    516
  -----                   -----
    704                921)1289
   2464                     921
  -----                    -----
 253,44               9223)36860
    8,4                     27669
  -----                    ------
 101376              92269)919100
 202752                    830421
 ------                    ------
2128,896                    88679
```

Art. 8. *To measure a Trapezium.*

Definition. A trapezium is an irregular figure of four unequal sides, and unequal angles.

Rule.—Draw a diagonal line from one of the angles to the opposite angle, as AC, and then will the trapezium be divided into two triangles, of which the diagonal is the common base: Then letting fall perpendiculars from the other opposite angles on the diagonal, add those perpendiculars together, and multiply half that sum into the diagonal, or half of the diagonal into the sum of the perpendiculars, and that product will be the area of the trapezium.

In the trapezium ABCD, the diagonal AC is 24, the perpendicular DE 6, and the perpendicular BF 10. The sum of the perpendiculars is 16, whose half is 8, which being multiplied into 24, will give the area.



$$\begin{array}{r} 24 \\ 8 \\ \hline 192 = \text{area.} \end{array}$$

By the Sliding Rule.

Set 1 on A to ½ the sum of the perpendiculars on B, and opposite the length of the diagonal on A, you will have the area on B.

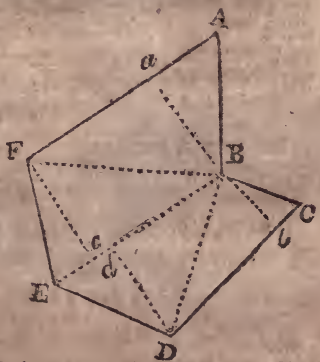
By Gunter.

The extent from 1 to ½ the sum of the perpendiculars will reach from the length of the diagonal to the area.

Art.

ART. 9. *To meaſure any irregular Figure.*

RULE.—Divide the figure into triangles, by drawing diagonals from one angle to another; then meaſure all the triangles by either of the rules, already taught, at Article 6 or 7, and the ſum of the ſeveral areas of all the triangles will be the area of the given figure.



The irregular figure ABCDEF being given, divide it into triangles by the diagonals FB, EB, and DB: Then may the triangles be meaſured by letting fall perpendiculars on their reſpective baſes, as B*a*, B*b*, D*c*, F*d*, and multiplying thoſe perpendiculars by half their reſpective baſes.

In the triangle AFB the baſe FA is 100, and the perpendicular B*a* 49; in the triangle FBE the baſe BE is 92, and the perpendicular F*d* 52; in the triangle EBD, the baſe BE is the ſame as before, and the perpendicular D*c* 44; and in the triangle DCB, the baſe DC is 80, and the perpendicular B*b* 38; by which the area of each may be found by Art. 6, as follows.

50 = half AF.
49 = perp. *a*B.
———
2450 = area of AFB.

46 = half BE.
44 = perp. D*c*.
———
184
184
———
2024 = area of EBD.

46 = half BE.
52 = perp. F*d*.
———
92
230
———
2392 = area of FBE.

38 = perp. B*b*.
40 = half DC.
———
1520 = area of DCB.

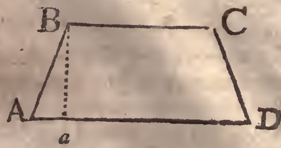
2450
2024
2392
1520
———
8386 = area of the figure ABCDEF.

In dividing any irregular figure into triangles, the triangles will be leſs, by two, and the diagonals leſs by three, than the number of the ſides of the figure.

ART. 10. *To meaſure a Trapezoid.*

Definition. A trapezoid is the ſegment of a triangle, cut by a line parallel to the baſe.

RULE.—Add the parallel ſides together, and multiply half that ſum by the perpendicular breadth.



In the trapezoid ABCD, the ſide AD is 24, the ſide BC is 16, and the perpendic. breadth B*a* is 10, to find the area by adding the ſides BC and AD and multiplying half their ſum by the perpendicular breadth B*a*.

24 = AD
16 = BC
——
40 = ſum.
——
20 = ½ ſum.
10 = B*a*.
——
200 = area.

By the Sliding Rule.

Set 1 on A to the equated length on B, and againſt the breadth on A you will have the area on B.

By Gunter.

The extent from 1 to the breadth will reach from the equated length to the area.

Art. 11. *To meaſure any regular Polygon.*

Definition. A regular polygon is a figure whoſe ſides and angles are all equal; they are uſually denominated from the number of their ſides:

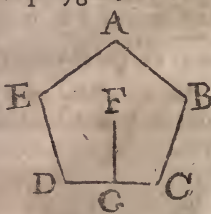
Thus, A figure having { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 } equal ſides and angles is a { Trigon. Tetragon. Pentagon. Hexagon. Heptagon. Octagon. Enneagon. Decagon. Endecagon. Dodecagon. }

3	Trigon.
4	Tetragon.
5	Pentagon.
6	Hexagon.
7	Heptagon.
8	Octagon.
9	Enneagon.
10	Decagon.
11	Endecagon.
12	Dodecagon.

Rule.—Multiply the length of one of the ſides by the number of ſides; then this product by the half of a perpendicular let fall from the centre of the figure to the middle of one of the ſides, and the product will be the area of the polygon.

In the pentagon ABCDE, each ſide is 95, and the perpendicular FG 65,36, to find the area.

95 = length of a ſide.
5 = number of ſides.
——
475 = ſum of the ſides.
32,68 = half of the perpendic.
——
3800
2850
950
1425
——
15523,00 = area of the pentagon.



By

By the Sliding Rule.

Set 1 on A to $\frac{1}{2}$ the perpendicular on B, and againſt the ſum of the ſides on A you will have the area on B.

By Gunter.

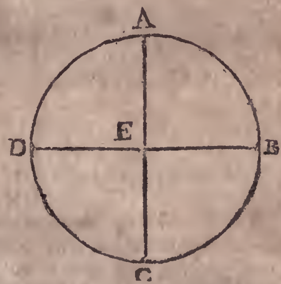
The extent, from 1 to $\frac{1}{2}$ the length of the perpendicular, will reach from the ſum of the ſides to the content.

But, for the more ready meaſuring regular polygons, the following Table, containing multipliers for all regular figures from the Triangle to the Dodecagon, will be of uſe to the learner.

Number of ſides.	*Names.*	*Multipliers.*	*Number of ſides.*	*Names.*	*Multipliers.*
3	Trigon.	,433013	8	Octagon.	4,828427
4	Tetragon.	1,	9	Enneagon.	6,181827
5	Pentagon.	1,720477	10	Decagon.	7,694209
6	Hexagon.	2,589076	11	Endecagon.	8,51425
7	Heptagon.	3,633959	12	Dodecagon.	9,330125

If the ſquare of the ſide of a polygon be multiplied by the multiplier of the like figure, the product will be the area of the figure ſought.

To meaſure a Circle and its Parts.



In the annexed circle ABCD, the arch line ABCD is called the *periphery*, the length of which is called the *circumference:* Any line, as DB or AC, paſſing through the centre E, cuts the circle into two equal parts, called *ſemicircles*, or half circles; and ſuch lines are called diameters of the circle: If two diameters be drawn through a circle, at right angles to each other, then, the four equal diviſions of the circle are called *quadrants:* Half the diameter, as EB, is called the *radius*, or ſemidiameter.

ART. 12. *The Diameter of a Circle being given, to find the Circumference.**

RULE.—This may be done by either of the following proportions in whole numbers, as 7 is to 22, or more exactly, as 113 is to

* *Note* 1. If the diameter of any circle be {multiplied / divided} by {3,14159 / ,31831} {the product / the quotient} is the circumference.

2. If the diameter of any circle be {multiplied / divided} by {,886227 / 1,128379} {the product / the quotient} is the ſide of an equal ſquare.

3. If

to 355; or in decimals, as 1 is to 3,14159; ſo is the diameter of a circle to the circumference.—Examp. A circle whoſe diameter is 12, to find the circumference.

As

3. If the diameter of any circle

be	multiplied	by	,866024	the product	is the ſide of the equilateral
	divided		,1547	the quotient	triangle inſcribed.

4. If the diameter of any circle

be	multiplied	by	,707016	the product	is the ſide of the ſquare
	divided		1,414213	the quotient	inſcribed.

5. If the ſquare of the diameter of any circle

be	multiplied	by	,785398	the product	is the area.
	divided		1,273241	the quotient	

6. If the circumference of any circle

be	multiplied	by	,31831	the product	is the diameter.
	divided		3,14159	the quotient	

7. If the circumference of any circle

be	multiplied	by	,282094	the product	is the ſide of the
	divided		3,544907	the quotient	ſquare equal.

8. If the circumference of any circle

be	multiplied	by	,2756646	the product	is the ſide of the equilateral
	divided		3,6275939	the quotient	triangle inſcribed.

9. If the circumference of any circle

be	multiplied	by	,225079	the product	is the ſide of the
	divided		4,442877	the quotient	ſquare inſcribed.

10. If the ſquare of the circumference of any circle

be	multiplied	by	,079577525	the product	is the area.
	divided		12,56636217	the quotient	

11. If the area of any circle

be	multiplied	by	1,273241	the product	is the ſquare of
	divided		,785398	the quotient	the diameter.

12. If the area of any circle

be	multiplied	by	12,56636217	the product	is the ſquare of the
	divided		,079577525	the quotient	circumference.

13. When the diameter of one circle is 1, and the diameter of another is 2, the circumference of the firſt is equal to the area of the ſecond, = 3,141592.

14. If the circumference be 4, the diameter and area are equal, = 1,273241.

15. If the diameter be 4, the circumference and area are equal, = 12,566368.

Hence, becauſe circles are the moſt capacious of all figures, if the *fourth* part of a circle be *ſquared*, it will not be equal to the *area* of that circle (as is commonly ſuppoſed) although the *four* ſides added together are equal to the *circumference* of that circle.

In a circle, whoſe diameter is 24, circumference 75,4, and area 452,4, the *fourth* part of the circumference is 18,85, the *ſquare* of which is only 355,3225, that is 97,0775 leſs than the truth; and the larger the circle is, the greater will the error be.

For further proof of this matter; If a cylindrical pint, beer meaſure, whoſe content is 35,25 cubic inches, be beaten into a perfectly *ſquare form*, it will contain only 28,902 cubic inches, which is leſs than the truth by 6,3484+; the area of the circle is 8,7615859288, and the area of the ſquare only 6,8813320653076624.

Hence appears the reaſon, why taking the *fourth part* of the girth in meaſuring a cylinder (or a round ſtick of timber) is falſe.

16. If the diameter of one circle be double to *that* of another, the *area* of the firſt circle will be *four times* the area of the ſecond.

As 7 : 22 :: 12
12
———
7)264(37,71 = circumference.
21
——
54
49
——
50
49
——
10
7
——
3

As 113 : 355 :: 12
12
———
113)4260(37,699 cir.
339
———
870
791
———
790
678
———
1120
1017
———
103

As 1 : 3,14159 :: 12
12
———
37,69908 cir.

Note, 3,14159 may be contracted to 3,1416 without any sensible difference.

ART. 13. *The Circumference of a Circle being given, to find the Diameter.*

RULE.—As 22 is to 7 ; or 355 to 113 ; or as 1 to ,31831, So is the circumference of a circle to the diameter.

EXAMP. The circumference of a circle being 326, to find the diameter.

As 22 : 7 :: 326
7
———
22)2282(103,72 diameter.
22
——
82
66
——
160
154
——
60
44
——
16

355 : 113 :: 326
326
———
678
226
339
———
355)36838(103,76 diameter.
355
———
1338
1065
———
2730
2485
———
245

1 : ,31831 :: 326
326
———
190986
63662
95493
———
103,76906 = diam.
This proportion is the most accurate.

ART. 14. *To find the Area of a Circle.*

RULE.—Multiply half the diameter by half the circumference, and the product is the area.

If the diameter be given, find the circumference by Art. 12.

If the circumference be given, find the diameter by Art. 13.

EXAMP. A circle, whose diameter is 12, and circumference is 37,7, given, to find the area ?

18,85 = half the circumference.
6 = half the diameter.
———
113,1 = area of the given circle.

ART.

ART. 15. *The Diameter being given, to find the Area of a Circle without finding the Circumference.*

RULE.—Multiply the ſquare of the diameter by ,7854, and the product will be the area of the circle, whoſe diameter was given.

EXAMP. The diameter of a circle being 12, to find the area?

```
             ,7854
12×12 =       144
           -------
             31416
            31416
            7854
          --------
          113,0976 = area.
```

By the Sliding Rule.

Set 1 on A to the diameter on B, then find ,7854 (which expreſſes the area of a circle whoſe diameter is 1) on A, againſt which on B is a 4th number, then find this 4th number on A, againſt which on B is the area.

By Gunter.

The extent from 1 to the length of the diameter reaches from ,7854 to a 4th number, and from that 4th number to the area.

ART. 16. *The Circumference of a Circle being given, to find the Area; without finding the Diameter.*

RULE.—Multiply the ſquare of the circumference by ,07958, and the product will be the area of the circle.

EXAMP. The circumference of a circle being 37,7, to find the area?

```
     37,7                  1421,29
     37,7                  ,079 58
    -----                 --------
     2639                  1137032
    2639                   710645
   1131                   1279161
   -------                994903
   1421,29 = ſquare.    -----------
                       113,1062582 = area of the circle.
```

ART. 17. *The Dimenſions of any of the parts of a Circle being given, to find the ſide of a Square equal to the Circle.*

RULE.—If the area of the circle be given, extract the ſquare root of the area, which will be the ſide of a ſquare equal to the circle: If the diameter or circumference be given, find the area by Art. 15 or 16, and then extract the ſquare root, as before. And this is a *general rule* to find the ſide of a ſquare equal to any ſuperficial figure, regular or irregular; For the ſquare root of the

area

area of any figure whatever, is the ſide of a ſquare equal to the given figure. But, with regard to circles, if the diameter be given; multiply it by ,886, and the product will be the ſide of an equal ſquare: Or, As 13,545 is to 12, or 1354 to 1200: So is the diameter of a circle to the ſide of a ſquare equal to the given circle. And, if the circumference be given, multiply it by ,282 for the ſide of an equal ſquare. Or, divide it by 3,545, and the quotient will be the ſide of an equal ſquare.

EXAMP. 1.

Let the diameter of a circle be 12, to find the ſide of a ſquare equal to the circle? ,886 × 12 = 10,632 = ſide of the ſquare. *Or,*
As 13,545 : 12 :: 12 : 10,631 = the ſide.

EXAMP. 2.

The circumference being 37,7 to find the ſide of an equal ſquare? 37,7 × ,282 = 10,631 = ſide of the ſquare.

Or, 37,7 ÷ 3,545 = 10,634

ART. 18. *The Area of a Circle being given, to find the Diameter.*

RULE.—Multiply the given area by 1,2732, and the product will be the ſquare of the diameter; then, extracting the ſquare root of the product, you will have the diameter.

EXAMP. The area of a circle being 113,09, to find the diameter.

```
  1,2732         143,986188(11,999 = 12 = diameter.
  113,09         1
 -------         --
  114588       21)43
 381960           21
 12732            ----
12732        229)2298
---------         2061
143,986188        -----
            2389)23761
                  21501
                  ------
          23989)226088
                 215901
                 ------
                  10187 remainder.
```

ART. 19. *The Area of a Circle being given, to find the Circumference.*

RULE.—Multiply the given area by 12,566, and extract the ſquare root of the product, which root will be the circumference required.

EXAMP. The area of a circle being 113,03 to find the circumference.

12,566

12,566	1420,3349(37,68 = circumference.
113,03	9
37698	67)520
376980	469
12566	746)5133
12566	4476
1420,33498	7528)65749
	60224
	5525 remainder.

ART. 20. *The Side of a Square being given, to find the Diameter of a Circle equal to the Square, whose side is given.*

RULE.—Multiply the given side by 1,128, and the product will be the diameter of a circle, whose area is equal to the area of the given square. Or, If the side of the square be divided by ,886, the quotient will be the diameter. Or, As 12 to 13,54, So is the side of any square to the diameter of an equal circle.

EXAMP. The side of a square being 10,635, to find the diameter of a circle equal to that square?

10,635 × 1,128 = 12 nearly. *Or*, 10,635 ÷ ,886 = 12 = diameter.
Or, As 12 : 13,54 :: 10,635 : 12 nearly.

ART. 21. *The Side of a Square being given, to find the Circumference of a Circle equal to the given Square.*

RULE.—Multiply the given side by 3,545 and the product will be the circumference required. Or, divide it by ,282, and the quotient will be the circumference.

EXAMP. The side of a square being 10,631, to find the circumference of a circle equal to that square. *Or*,

10,631 × 3,545 = 37,686 = circum. ,282)10,631(37,698 circum.

ART. 22. *To find the Area of a Semicircle, the Diameter being given.*

RULE.—Find the area of the circle by Art. 15, and take the half of it.

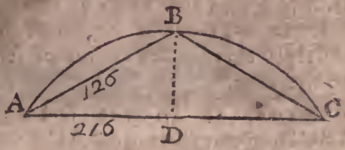
In the same manner may the area of a quadrant, or a quarter of a circle, be found, by taking a fourth part of the area of the whole circle.

But with regard to measuring a sector, or a segment of a circle, it will be necessary first to shew how to find the length of the arch line of a sector, and the diameter of the circle to a given segment.

ART. 23. *A Segment of a Circle being given, to find the Length of the Arch Line.*

RULE.—Divide the segment into two equal parts; then measure the chord of the half arch, from the double of which subtract the

the chord of the whole segment ; and one third of that difference, being added to the double of the chord of the half arch, will give the length of the arch line.



EXAMP. In the segment ABCD, the whole chord ADC is 216, and the chord AB or BC 126, to find the arch line ABC.

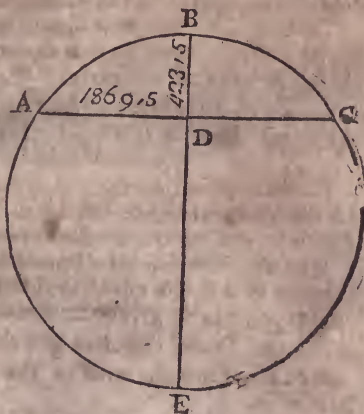
126 = chord AB or BC.
2

252 = double.
216 = ADC, to be subtracted.
3)36 = difference.
12 = $\frac{1}{3}$ difference.

252 = double of AB.
12 = $\frac{1}{3}$ difference, added.
264 = length of the arch ABC.

ART. 24. *The Chord and versed Sine of a Segment being given, to find the Diameter of a Circle.*

RULE.—Multiply half the chord by itself, and divide the product by the versed sine ; then add the quotient to the versed sine, and the sum will be the diameter of the circle.



EXAMP. In the segment ABCD, the chord AC is 1869,5, and the versed sine BD 423,5, to find the diameter.

934,75 { half the chord AC
934,75

467375
654325
373900
280425
841275

423,5)873757,5625(2063,1 = DE.
8470
26757
25410
13475
12705
7706
4235
3471

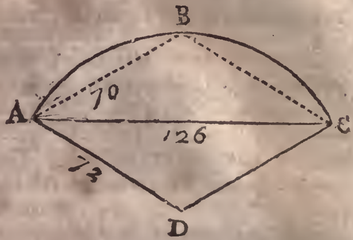
423,5 = BD, add.
2486,6 = diameter BDE.

ART. 25. *To meaſure a Sector.*

Definition. A ſector is a part of a circle, contained between an arch line and two radii or ſemidiameters of the circle.

RULE.—Find the length of half the arch by Art. 23: Then multiply this by the radius or ſemidiameter, and the product will be the area.

EXAMP. 1. In the ſector ABCD, given the radius AD or DC 72 feet, the chord AC = 126 feet, and the chord AB or BC = 70, to find the area of the ſector.



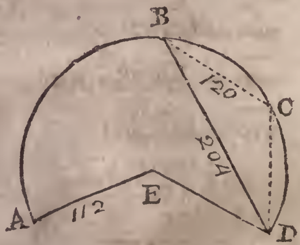
First.

70 = chord AB or BC.
2
——
140
126 = AC, ſubtract.
——
3)14
——
4,66
140
——
144,66 = length of the arch ABC, (by Art. 23.
——
72,33

Secondly.

72,33 = half the arch.
72 = radius.
——
14466
50631
——
5207,76 = area.

EXAMP. 2. In the ſector ABCD, greater than a ſemicircle, given the radius AE or ED = 112, the chord BD (of half the arch ABD) = 204, and the chord BC (of half the arch BCD) = 120, to find the area of the ſector.



120 = BC.
2
——
240
204 ſubtract.
——
3)36
——
12
240 Add.
——
252 = { Length of the arch BCD, by Art. 23.

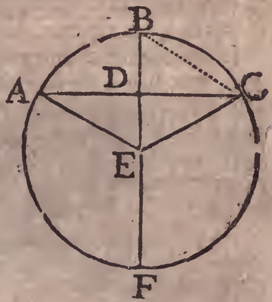
252 = half of the arch ABD.
112 = radius.
——
504
252
252
——
28224 = area of the ſector.

ART.

ART. 26. *To find the Area of a Segment of a Circle.*

Definition. A ſegment of a circle is any part of a circle cut off by a right line drawn acroſs the circle, which does not paſs through the centre, and is always greater or leſs than a ſemicircle.

EXAMP. 1. To find the area of the ſegment ABC, whoſe chord AC is 172, the chord of half the arch ABC, viz. BC = 104, and the verſed ſine BD = 58,48.



RULE.—By Art. 23, find the length of the arch line ABC, and by Art. 24, the diameter BF; then multiply half the chord of the arch ABC by half the diameter, and the product will be the area of the ſector ABCE: Then find the area of the triangle AEC, whoſe baſe AC is 172, and perpendicular height 34, found by ſubtracting the verſed ſine BD from half the diameter; and the area of the triangle AEC, being ſubtracted from the area of the ſector ABCE, will leave the area of the ſegment ABC.

104 = BC.
2
———
208
172 = AC, ſubtract.
———
3)36
———
12
208 add.
———
220 = arch line ABC.
———
110 = half arch

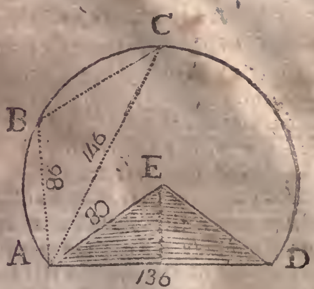
92,475 = radius.
110
———
924750
92475
———
10172,25 = area of the ſector.
86 = half the baſe = AD.
34 = perpendicular DE.
———
344
258
———
2924 = area of the triangle.

86 = half ADC.
86
———
516
688
———
58,48)7396,00(126,47 = DEF.
5848
———
15480
11696
———
37840
35088
———
27520
23392
———
41280
40936
———
344

58,48 = BD, add.
———
184,95 = diameter BF.
———
92,475 = radius or ſemidiameter.

10172,25 = area of the ſector.
2924 = area of triangle.
———
7248,25 = area of the ſeg.

EXAMP. 2. In the ſegment, ABCD greater than a ſemicircle, given the chord of the whole ſegment AD = 136, the chord AC of half the arch ACD = 146, the chord AB or BC of one fourth of the arch ACD = 86, and the radius AE or ED = 80, to find the area of the ſegment ABCD.



Firſt find the area of the ſector ABCDE, by Art. 25, at the ſecond Example; then find the area of the triangle AED, by Art. 6, and, adding the area of the triangle to the area of the ſector, you will have the area of the ſegment.

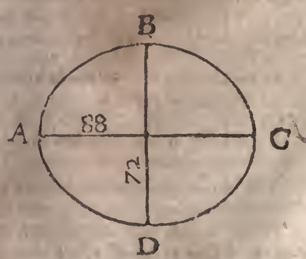
86 = chord AB.
2
———
172
146 = chord AC, ſubtract.
———
3)26
———
8,666
172 = double of AB, add.
———
180,666 = arch line ABC.
80 = radius.
———
14453,280 = area of the ſector.

68 = half the baſe AD.
42 = perpendicular E 136.
———
136
272
———
2856 = area of the triangle AED.
14453,28 = area of the ſector, add.
———
17309,28 = area of the ſegment.

ART. 27. *To find the Area of an Ellipſis.*

Definition. An ellipſis, or oval is a curve which returns into itſelf like a circle, but has two diameters, one longer than the other, the longeſt of which is called the tranſverſe, and the ſhorteſt, the conjugate diameter.

RULE.—Multiply the two diameters of the ellipſis together; then, multiplying the product by ,7854, this laſt product will be the area of the ellipſis.



EXAMP. In the ellipſis ABCD, the tranſverſe diameter AC is 88, and the conjugate diameter BD is 72, to find the area.

$$
\begin{array}{r}
88 \\
72 \\
\hline
176 \\
616 \\
\hline
6336 \\
,7854 \\
\hline
25344 \\
31680 \\
50688 \\
44352 \\
\hline
4976,2944
\end{array}
$$

4976,2944 = area.

The content is found by the ſliding rule and Gunter, in the ſame way as the circle, only uſing the product of the two diameters as the diameter of a circle.

Menſuration of Superficies is eaſily applied to *Surveying :* Thus, take the angles of the plot with a good compaſs, then meaſure the ſides with Gunter's chain, which note down in links (or chains and links, which is done by ſeparating the two right hand figures of your links by a comma, your chain being 100 links) then caſt up the contents, according to the rule of the figure, cutting off the five right hand figures of the product, and thoſe at the left hand, if any, are acres ; then, multiply the five figures, cut off, by 4, by 40, and by $272\frac{1}{4}$, cutting off as before, and thoſe at the left hand will be roods, poles and feet, reſpectively.

SECTION II. *Of* SOLIDS.

Solids are meaſured by the ſolid inch, foot or yard, &c. 1728 of theſe inches, that is $12 \times 12 \times 12$, make 1 cubic or ſolid foot.

The ſolid content of every body is found by rules adapted to their particular figures.

ART. 28. *To meaſure a Cube.**

Definition. A cube is a ſolid of ſix equal ſides, each of which is an exact ſquare.

The

* Here follows a Table of the Proportions, which the following *Solids* have to the *Cube* and *Cylinder*, having the ſame *Baſe* and *Altitude.*

	Solid Inches.
1. A *Cube*, whoſe ſide is 12 inches, contains - - - -	1728
2. A *Priſm*, having an equilateral triangle, whoſe ſide is 12 inches for its *Baſe*, and its *Altitude* 12 inches, contains - - -	784,24
3. A *Square Pyramid*, whoſe height and the ſide of its baſe, are each 12 inches, is $\frac{1}{3}$ of the above cube, and therefore contains -	576
4. A *Triangular Pyramid*, whoſe height and ſide of its triangular baſe are each 12 inches, is near $\frac{1}{7}$ of the cube, and contains -	249,413
5. A *Cylinder*, whoſe diameter and height are each 12 inches, is $\frac{11}{14}$ of the above cube, and contains - - - - - -	1357,17
6. A *Sphere* or *Globe*, whoſe axis or diameter is 12 inches, equal to the ſide of the cube, is $\frac{11}{21}$ of it, and contains - - -	904,78
7. A *Cone*, whoſe baſe and altitude are each 12 inches, equal to the ſide of the cube, is $\frac{5}{19}$ of it, and contains - - - -	452,38829

A 8.

The ſolid foot is compoſed of 1728 inches: For a ſolid, that is 1 foot, or 12 inches every way, that is $12 \times 12 \times 12$, contains 1728 inches.

RULE.

	Solid Inch.
8. A *Parabolic Conoid*, whoſe diameter at the baſe, and height are each 12 inches, being $\frac{1}{2}$ its circumſcribing cylinder, contains - -	678,583
9. A *Hyperbolic Conoid*, whoſe height, and diameter at the baſe, are each 12 inches, is $\frac{5}{12}$ of its circumſcribing cylinder, and contains	565,49
10. A *Parabolic Spindle*, whoſe height and middle diameter are each 12 inches, is $\frac{8}{15}$ of its circumſcribing cylinder, and contains -	723,824

Hence ariſes a different method of finding their contents.

General Rule. If the baſe of the ſolid, whoſe content you would find, be rectilinear, conſider it as a *Parallelopipedon*; if curved, as a *Cylinder*, and find the content accordingly: Then take ſuch a part of the content, thus found, as is ſpecified in the preceding Table, which, if the parts be taken in inches, will be the ſolid content of the given figure, in inches, which, divided by 1728, will give the cubic feet.

EXAMP. 1. There is a triangular priſm, the ſide of whoſe baſe is 48 inches, and whoſe perpendicular height is 108 inches; What is its ſolid content?

The baſe being right lined, I conſider it as a *parallelopipedon*, the ſide of whoſe baſe is 48 inches, and whoſe length is 108 inches, and as 784,24 is contained 2,20340712 times in a cubic foot; 2,20340712 is a diviſor, to divide the content of the parallelopipedon by; therefore $48 \times 48 \times 108 \div 2{,}20340712 = 112930{,}56$ ſolid inches $= 65{,}353$ ſolid feet.

Had the dimenſions been given in feet, it would have been $4 \times 4 \times 9 \div 2{,}20340712 = 65{,}353$ feet.

EXAMP. 2. There is a ſquare pyramid, whoſe height is 12 feet, and the ſide of whoſe baſe is 3,5 feet; What is its content?

$3{,}5 \times 3{,}5 \times 12 \div 3 = 49$ *feet, Anſ.*

EXAMP. 3. There is a triangular pyramid, whoſe height is 15 feet, and the ſide of whoſe baſe is 5 feet; What is its content?

$5 \times 5 \times 15 \div 7 = 53{,}57$ *feet, Anſ.*

EXAMP. 4. There is a cylinder, whoſe diameter is 2,5 feet, and whoſe length is 24 feet; What is its content?

Here, the diameter is to be conſidered as the ſide of the baſe of a parallelopipedon. Therefore, $2{,}5 \times 2{,}5 \times 24 \times 11 \div 14 = 117{,}857$ *feet, Anſ.*

EXAMP. 5. There is a ſpherical balloon, whoſe diameter is 50 feet; How many cubic feet of air does it contain?

Here, the diameter is to be conſidered as the ſide of a cube. Therefore,

$50 \times 50 \times 50 \times 11 \div 21 = 65476{,}19$ *feet, Anſ.*

EXAMP. 6. There is a cone whoſe height is 15 feet, and the diameter of whoſe baſe is 5 feet; What is its content?

Here, the diameter of the baſe is to be conſidered as the ſide of the baſe of a parallelopipedon, and its height, as the length. Therefore,

$5 \times 5 \times 15 \times 5 \div 19 = 98{,}684$ *feet, Anſ.*

EXAMP. 7. There is a parabolic conoid, whoſe diameter at the baſe, is 2,9 feet, and whoſe height is 6 feet; What is the content?

This ſolid, being $\frac{1}{2}$ of a cylinder; we muſt firſt find the content as of that of a cylinder, and then halve it. Therefore,

$2{,}9 \times 2{,}9 \times 6 \times 11 \div 14 = 39{,}647$, and $39{,}647 \div 2 = 19{,}823$, *Anſ.*

EXAMP. 8. There is a hyperbolic conoid, whoſe diameter at the baſe is 2,9 feet and whoſe height is 6 feet; What is the content?

Firſt, find the content of a cylinder.

$2{,}9 \times 2{,}9 \times 6 \times 11 \div 14 = 39{,}647$, and $39{,}647 \times \frac{5}{12} = 16{,}519$ *feet, Anſ.*

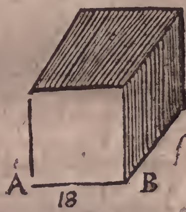
EXAMP. 9. There is a parabolic ſpindle, whoſe middle diameter is 2,9 feet, and whoſe length is 6 feet; Required the content?

Firſt, find the content of a cylinder.

$2{,}9 \times 2{,}9 \times 6 \times 11 \div 14 = 39{,}647$, and $39{,}647 \times \frac{8}{15} = 21{,}145$ *feet, Anſ.*

Rule.—Multiply the ſide by itſelf, and that product by the ſame ſide, and this laſt product will be the ſolid content of the cube.

Examp. The ſide of a cube AB, being 18 inches, or 1 foot and 6 inches, to find the content ?



1 foot, 6 inches = 1,5 foot.	18 inches.
1,5	18
75	144
15	18
2,25	324
1,5	18
1125	2592
225	324
3,375	1728)5832(3,375
	5184
	6480
	5184
	12960
	12096
	8640
	8640

In this operation, the inches are changed into the decimal parts of a foot.

I have done this two different ways, that the learner may ſee they come out the ſame. The content in inches is 5832, which being divided by 1728, the inches in a ſolid foot, and the diviſion continued by annexing cyphers, it comes out the ſame as the decimal operation.

Note, The area of the ſurface, or ſuperficial content of the cube and parallelopipedon is found by adding the areas of the ſeveral quadrilateral figures which compoſe them.

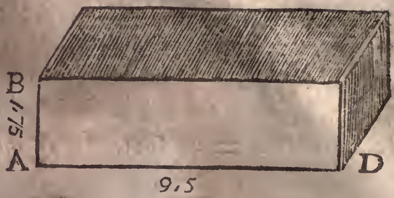
Art. 29. *To meaſure a Parallelopipedon.*

Definition. A parallelopipedon is a ſolid of three dimenſions, length, breadth, and thickneſs ; as a piece of timber exactly ſquared, whoſe length is more than the breadth and thickneſs. The ends are called baſes, which are equal.

Rule.—Find the area of the baſe, then multiply that by the length, and it will give the ſolid content.

Examp. 1. The ſide AB is 1,75 foot, and the length AD 9,5 feet, to find the ſolid content ?

1,75 =



1,75 = 1 foot, 9 inc.
1,75

875
1225
175

3,0625 = area of base.
9,5

153125
275625

29,09375 = solid content.

EXAMP. 2. A vessel 3,5 feet each side within, and 5 feet deep, to find the content?

3,5
3,5

175
105

12,25
5

61,25 = the content.

If a piece of timber, or any other thing, be of an equal bigness through its whole length, though there be a difference between the breadth and thickness, if the breadth and thickness are multiplied together, and that product multiplied by the length, this last product will be the solid content.

EXAMP. 3. A piece of timber being 1 foot and 6 inches, or 18 inches broad, 9 inches thick, and 9 feet 6 inches, or 114 inches long, to find the content?

1 foot, 6 inches = 1,5 foot.
9 inches = ,75 foot.

75
105

1,125
9 Ft. 6 inches = 9,5

5625
10125

10,6875 = content.

In this operation the inches are changed into the decimal fractions of a foot.

Breadth = 18 inches.
Depth = 9 inches.

162
Length = 114 inches.

648
162
162

1728)18468(10,6875 = content, as before.
1728

11880
10368

15120
13824

12960
12096

8640
8640

Note,

Note, When the end is given in inches and the length in feet, find the area at the end in inches, multiply that by the length in feet, and divide this product by 144 (the ſquare inches in a foot) and the quotient will be feet.

Take the laſt example.

Foot.

1,5 = 18 inches.
,75 = 9 inches.

162 area in inches.
9,5 feet = length.

810
1458

144)1539(10,6875 = content.
144

990
864

1260
1152

1080
1008

720
720

By the Sliding Rule.

Set 12 inches on the girt line D to the ſide of the ſquare end on C, then, againſt the length on D, you will have the anſwer on C.

By Gunter.

Extend the compaſſes from 12 inches to the length of the ſide of the ſquare end; that diſtance, twice turned over from the length, will reach to the content.

When the ſide of a ſquare ſolid is given, in inches, to find how much in length will make a foot ſolid.

Rule.—As the given ſide is to 12, ſo is 12 to a fourth number, and ſo is that fourth number to the required length.—Or divide 1728 by the area at the end, and the quotient will be the length making a ſolid foot.

If the given ſide is in foot meaſure, then,

Rule.—As the given ſide is to 1; ſo is 1 to a fourth number, and ſo is that fourth number to the required length.

When two ſides of an unequal ſquare ſolid (that is, of unequal breadth) are given, to find what length will make any number of ſolid feet:

Rule.—Multiply the propoſed number of feet by 144: Divide that product by the product of the breadth and depth, and the quotient will be the length required.

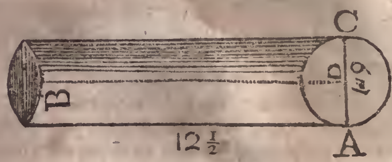
L l l   Art.

ART. 30. *To meaſure a Cylinder.*

Definition. A cylinder is a round body, whoſe baſes are circles, like a round column, or a rolling ſtone of a garden.

RULE.—The diameter of the baſe being given, find the area of the end by Art. 15, then, multiplying the area of the baſe by the length, that product will be the content of the cylinder.

EXAM. The diameter of the baſe AC being 1 foot and 9 inches, and the length BD 12 feet and 6 inches, to find the content.




```
   1,75 = diam. of the baſe.
   1,75
   ----
    875
  1225
  175
  ------
  3,0625
   ,7854
  ------
   122500
  153125
 245000
214375
---------
2,40528750 = area of the baſe.
```

```
  2,405 = area of the baſe.
   12,5 = length.
  -----
  12025
  4810
 2405
-------
30,0625 = content.
```

If the ſquare of the diameter of a cylinder be multiplied by ,7854, and the ſolidity divided by that product, the quotient will be the length.

The learner may, for his practice, reduce all the dimenſions to inches, and find the ſolid content in inches, which being divided by 1728, the quotient will be the ſolid content in feet: Or, if he finds the area at the end in inches, and multiplies that by the length in feet, and divides by 144, the quotient will be feet.

This is a general rule for finding the content of any ſtraight ſolid body, of equal bigneſs from end to end, of whatever form the baſes are: For, if the area of the baſe be multiplied by the length, the product will be the ſolid content.

By the Sliding Rule.

Set 13,5, the ſquare root of 183,34 (which is a gauge point ariſing from the diviſion of 144 by ,7854) found on D, to the diameter

ameter found on C, and oppoſite to the length, on D, you will find the content on C.

Or, As 42,54 is to the circumference ; So is the length in feet to a fourth number, and ſo is that fourth number, to the anſwer.

Note, The ſuperficial content of a cylinder is found by multiplying the circumference of one of the baſes into the length, and to the product adding the areas of the two baſes, or ends.

When the diameter is given in inches, to find what length will make a ſolid foot.

Rule.—As the given diameter is to 13,531 : So is 12 to a 4th number, and ſo is that 4th number to the required length. If the diameter be given in foot meaſure : *Rule*, As the given diameter is to 1,128 : So is 1 to a 4th number, and ſo is that 4th number to the required length. Or, divide 1728 by the area at the end in inches, and the quotient will be the required length.

To find how much a Cylindric or round Tree, that is equally thick from end to end, will hew to, when made ſquare.

Rule.—Multiply twice the ſquare of its ſemidiameter by the length, then divide the product by 144, and the quotient will be the anſwer.

If the diameter of a round ſtick of timber be 24 inches from end to end, and its length 20 feet ; How many ſolid feet will it contain, when hewn ſquare ; and what will be the content of the ſlabs which reduce it to a ſquare ?

$$\frac{12\times12\times2\times20}{144}=40 \text{ feet, the ſolidity when hewn ſquare.}$$

$$\frac{24\times24\times,7854\times20}{144}=62,8 \text{ feet, or } 2\times2\times,7854\times20=62,8$$

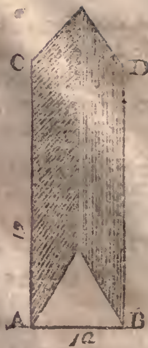
the total ſolidity, whence 62,8—40 = 22,8 feet, the ſolidity of the ſlabs.

Art. 31. *To meaſure a Priſm.*

Definition. A priſm is a body with two equal or parallel ends, either ſquare, triangular, or polygonal, and three or more ſides, which meet in parallel lines, running from the ſeveral angles at one end, to thoſe of the other.

Rule.—Priſms of all kinds, whether ſquare, triangular or polygonal, are meaſured by one general rule, viz. Find the ſuperficial content, or area at the baſe (or end) by the proper rule of Sect. 1. and this multiplied by the length, or height of the priſm, will give the ſolid content.

Examp-



EXAMP. The ſide of a ſtick of timber, AB, hewn three ſquare, is 10 inches, and the length, AC, is 12 feet, to find the content?

Side = 10 inches.
$\frac{1}{2}$ Perpend. = 4,33 inches.

43,3 = area at the end.
12 feet = length.

144)519,6(3,6 feet, content.
432

876
864

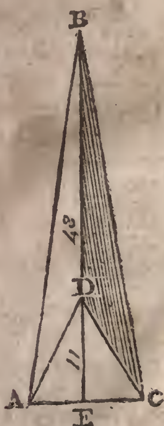
12

Note, The ſuperficial content is found by adding the areas of the ſeveral quadrilateral and triangular figures, which compoſe it.

ART. 32. *To meaſure a Pyramid.*

Definition. Solids, which decreaſe gradually from the baſe till they come to a point, are generally called pyramids, and are of different kinds, according to the figure of their baſes; thus, if it has a ſquare baſe, it is called a *ſquare pyramid:* If a triangular baſe, a *triangular pyramid:* If the baſe be a circle, a *circular pyramid,* or ſimply a *cone.* The point, in which the top of a pyramid ends, is called a *Vertex*, and a line drawn from the vertex, perpendicular to the baſe, is called the height of the pyramid.

RULE.—Find the area of the baſe, whether triangular, ſquare, polygonal or circular, by the rules in ſuperficial meaſure; then, multiply this area by one third of the height, and the product will be the ſolid content of the pyramid.



EXAMP. 1. In a triangular pyramid, the height BE, being 48, and each ſide of the baſe 13: The baſe being a triangle, let the perpendicular height DE be 11; to find the content.

5,5 = half ED.
13 = baſe AC.

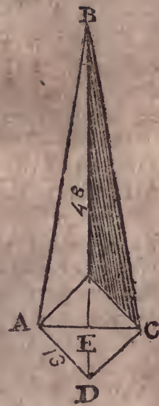
165
55

71,5 = area of the baſe.
16 = $\frac{1}{3}$ of the height EB.

4290
715

1144,0 = content.

EXAMP.



EXAMP. 2. In a quadrangular pyramid, the height BE being 48, and each ſide of the baſe 13, to find the content.

```
    13
    13
    --
    39
   13
   ---
   169 = area of the baſe.
    16 = 1/3 of the height EB.
   ----
  1014
  169
  ----
  2704 = content.
```

EXAMP. 3. *To meaſure a Cone.*—The diameter AC being 13, and the height BD 48, to find the content.

```
        13
        13
        --
        39
       13
       ---
       169
     ,7854
     -----
       676
      845
    1352
   1183
   --------
   132,7326 = area of the baſe.
         16 = 1/3 of the height.
   --------
   7963956
   1327326
   ---------
  2123,7216 = content.
```

Note, The ſuperficial content of all pyramids is found by taking the ſum of the ſeveral areas, which compoſe them. That of a cone, by multiplying the circumference of the baſe into half the line joining the vertex and any point in that circumference, and adding the area of the baſe to the product.

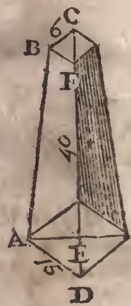
ART.

ART. 33. *To meaſure the Fruſtrum of a Pyramid.*

Definition. The fruſtrum of a pyramid is what remains after the top is cut off by a plane parallel to the baſe, and is in the form of a log greater at one end than the other, whether round, or hewn three or four ſquare, &c.

RULE.—If it be the fruſtrum of a ſquare pyramid, multiply the ſide of the greater baſe by the ſide of the leſs ; to this product add one third of the ſquare of the difference of the ſides, and the *ſum* will be the mean area between the baſes ; but, if the baſe be any other regular figure, multiply this *ſum* by the proper multiplier of its figure in the Table, Art. 11. and the product will be the mean area between the baſes : Laſtly, multiply this by the height, and it will give the content of the fruſtrum.

EXAMP. 1. In the fruſtrum of a ſquare pyramid the ſide of the greater baſe AD = 15, the ſide of the leſs, BC = 6, and the height EF = 40, to find the content.



15 = AD.	15
6 = BC.	6
Prod. = 90	9 = difference.
Add 27	9
117	3)81 = ſquare of the difference.
× 40	27 = $\frac{1}{3}$ of the ſquare.
4680 = content.	

Or, If it be a tapering ſquare ſtick of timber, take the girth of it in the middle ; ſquare $\frac{1}{4}$ of the girth (or multiply it by itſelf in inches) then ſay, as 144 (inches) to that product ; ſo is the length, taken in feet, to the content in feet.

EXAMP. 2. What is the content of a tapering ſquare ſtick of timber, whoſe ſide of the largeſt end is 12 inches, of the leaſt end, 8 inches, and whoſe length is thirty feet ?

One fourth of the girth in the middle = 10, and 10 × 10 = 100, the area in the middle; then, As 144 : 100 : : 30 feet : 20,83 feet, the content.

By the Sliding Rule.

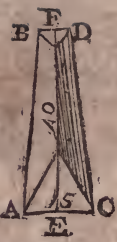
Set 12 on D to $\frac{1}{4}$ of the circumference on C, and againſt the length on D is the anſwer on C.

By Gunter.

The extent from 12 to $\frac{1}{4}$ of the circumference doubled, or twice turned over, will reach from the length to the content.

EXAMP.

EXAMP. 3. In the fruſtrum of a triangular pyramid, the ſide of the greater baſe AC = 15, as before, the fide of the leſs BD = 6, and the height EF = 40, to find the content.



15 = AC.
6 = BD.
—
9 = difference of the ſides.
9
—
3)81 = ſquare of the difference.
—
27 = $\frac{1}{3}$ of the ſquare.

15
6
—
90
Add 27
—
117
,433 multiplier.
—
351
351
468
—
50,661 = mean area.
40 = height.
—
2026,440 = content.

Or, If it be a tapering three ſquare ſtick of timber, you may find the area midway from end to end, then, As 144 is to that area ; ſo is the length, taken in feet, to the content in feet.

EXAMP. 4. *To meaſure the Fruſtrum of a Cone.*

RULE.—Multiply the diameters of the two baſes together, and to the product add one third of the ſquare of the difference of the diameters : Then multiplying this ſum by ,7854, it will be the mean area between the two baſes, which being multiplied by the length of the fruſtrum, will give the ſolid content.

Or, To the areas of the top and bottom add the ſquare root of the product of thoſe areas, and the ſum, multiplied by *one third* of the height of the fruſtrum, will give the ſolidity.

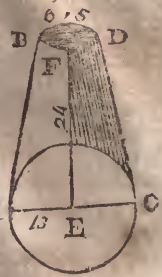
When figures run uniformly taper ; but not to a point (they being conſidered as portions of the cone or pyramid) we may find the ſolidity by ſupplying what is wanting to complete the figure, and then deducting the part cut off.

A general rule for completing every ſtraight ſided ſolid, whoſe ends are parallel and ſimilar.

As the difference of the top and bottom diameters is to the perpendicular height, (or depth, which is the ſame :) So is the longeſt diameter to the altitude of the whole cone or pyramid,

The

The former cone in Art. 32, Examp. 3, being cut off in the middle, the greater diameter AC is 13, the lefs BD $6\frac{1}{2}$, and height EF 24, to find the content of the fruftrum.



AC = 13 inches.
BD = 6,5 inches.

65
78

84,5
Add 14,083

98,583
,7854

394332
492915
788664
690081

77,427|0882 = mean area.
24 feet = length.

309708
154854

1858,248 = content.

13
6,5

6,5 = difference.
6,5

325
390

3)42,25 = { fquare of the differ.

14,083 = $\frac{1}{3}$ of the fquare.

144)1858,248(12,973 feet, content.
144

418
288

1302
1296

1064
1008

568
432

136

ART. 34. *To meafure a Sphere or Globe.*

Definition. A fphere or globe is a round folid body, in the middle of which is a point, from which all lines drawn from the furface are equal.

RULE.—Multiply the cube of the diameter by ,5236, and the product will be the folid content.

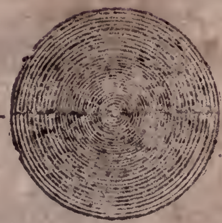
Or, Multiply the circumference by the diameter, which will give the fuperficial content; then multiply the furface by *one fixth* of the diameter, and it will give the folidity.

Or, Multiply the cube of the diameter by 11, and the product divided by 21, will give the folidity.

EXAMP. The diameter, AB, of a globe, is 4,5 feet; to find the folid content.

4,5

4,5
4,5
———
225
180
———
20,25
4,5
———
10125
8100
———
91,125
,5236
———
546750
273375
182250
455625
———
47,7130500



Note. If the circumference, or greateſt circle of the ſphere, be given, multiply the cube of it by ,016887 for the content.

The ſurface of the globe may be found by multiplying the ſquare of the diameter by 3,1416 ; or by multiplying the area of its greateſt circle by 4, or the ſquare of the circumference by ,3183.

When the ſolidity of a globe is given, the diameter may be found by dividing the ſolidity by ,5236, and extracting the cube root of the quotient.

Or, If the circumference be required, divide the ſolidity by ,016887, and the cube root of the quotient will give it.

ART. 35. *To meaſure the Solidity of a Fruſtrum or Segment of a Globe.*

Definition. The fruſtrum of a globe is any part cut off by a plane.

RULE.—To three times the ſquare of the ſemidiameter of the baſe, add the ſquare of the height ; then multiplying that ſum by the height, and the product by ,5236, you will have the ſolid content.

EXAMP. The height BD being 9 inches, and the diameter of the baſe AC 24 inches : To find the content.

12 = ſemidiam.
12
———
144 = ſquare.
× 3
———
432
Add 9×9 = 81 = { ſquare of the hht.
———
513
× 9 = height.
———
4617

4617
,5236
———
27702
13851
9234
23085
———
2417,4612 = ſolid content

Te

To meaſure the Surface of a Fruſtrum or Segment of a Globe.

RULE.—Find the diameter of the globe by Art. 24, and the ſurface of the whole globe, by Art. 34; then, As the diameter of the globe is to the height of the fruſtrum; So is the ſurface of the globe, to the ſurface of the fruſtrum; then, by Art. 15, find the area of the baſe; add theſe two together, and the ſum will be the whole ſurface of the fruſtrum.

ART. 36. *To meaſure the middle Zone of a Globe.*

Definition. This part of a globe is ſomewhat like a caſk, two equal ſegments being wanting, one on each ſide of the axis.

RULE.—To twice the ſquare of the middle diameter, add the ſquare of the end diameter; multiply that ſum by ,7854, and that product, multiplied by *one third* of the length, will give the ſolidity.

Or, To four times the ſquare of the middle diameter add twice the ſquare of the end diameter, that ſum multiplied by ,7854, and that product by *one ſixth* of the length, will give the ſolidity.

Note. This rule is applicable to the fruſtrum of a cone or pyramid.

If the middle diameter of a zone be 20 inches, the end diameters each 16 inches, and length 12 inches; Required its ſolidity?

$$\overline{\overline{20\times 20\times 2}+\overline{16\times 16}}\times ,7854\times 4 = 3317,5296 \text{ Anſ.}$$

ART. 37. *To meaſure a Spheroid.*

Definition. A ſpheroid is a ſolid body like an egg, only both its ends are the ſame.

RULE.—Multiply the ſquare of the diameter of the greateſt circle, viz. the diameter of the middle (DB in the figure) by the length AC, and that product by ,5236, and you will have the ſolidity.

EXAMP. The diameter BD being 20, and the length AC 30, to find the content.

$$20\times 20\times 30\times ,5236 = 6283,2 \text{ Anſ.}$$

ART. 38. *To meaſure the middle Fruſtrum of the Spheroid.*

Definition. This is a caſklike ſolid, wanting two equal ſegments to complete the ſpheroid.

RULE.—The ſame as in Article 36.

If the middle and end diameters of the middle fruſtrum of a ſpheroid be 40 and 30 inches, and its length 50; What is its ſolidity?

$$50\div 3=16,6, \text{ then } \overline{\overline{40\times 40\times 2}+\overline{30\times 30}}\times ,7854\times 16,6=53669 \text{ Anſ.}$$

ART. 39. *To meaſure a Segment, or Fruſtrum of a Spheroid.*

Definition. This is a part of a ſpheroid made by a plane, parallel to its greateſt circular diameter.

RULE.

Rule.—To four times the ſquare of the middle diameter add the ſquare of the baſe diameter, then multiply that ſum by ,7854, and the product by *one ſixth* of the altitude, and it will give the ſolidity.

If the baſe diameter of the end fruſtrum of a ſpheroid be 36, diameter at the middle of the height 30, and the height 20 inches; Required its ſolidity?

$\overline{30\times 30\times 4}+\overline{36\times 36}\times ,7854\times 3,3 = 12817,728$ *Anſ.*

N. B. $20\div 6 = 3,3$.

Art. 40. *To meaſure a Parabolic Conoid.*

Definition. This ſolid may be generated by turning a ſemiparabola about its abſciſſa or altitude.

Rule.—As a parabolic conoid is half of its circumſcribing cylinder, of the ſame baſe and altitude; multiply the area of the baſe by half the height, for the ſolidity.

If the diameter of the baſe of a parabolic conoid be 40 inches, and its height 42; What is the ſolidity?

$40\times 40\times ,7854\times 21 = 26389,44$ *Anſ.*

Art. 41. *To meaſure the lower Fruſtrum of a Parabolic Conoid.*

Definition. This ſolid is made by a plane paſſing through the conoid, parallel to its baſe.

Rule.—Multiply the ſum of the ſquares of the diameters of the baſes by ,7854, and that product by half the height, for the ſolidity.

If the diameters of a fruſtrum of a parabolic conoid be 40 and 30 inches, and its height 20 inches; Required its ſolidity?

$\overline{\overline{40\times 40}+\overline{30\times 30}}\times ,7854\times 10 = 19635$ *Anſ.*

Art. 42. *To meaſure a Parabolic Spindle.*

Definition. This ſolid is formed by an obtuſe parabola, turned about its greateſt ordinate.

Rule.—This ſolid being eight fifteenths of its leaſt circumſcribing cylinder, multiply the area of its middle or greateſt diameter by eight fifteenths of its perpendicular length, and it will give its ſolidity.

If the diameter at the middle of a parabolic ſpindle be 20 inches, and its length 60; Required its ſolidity?

$20\times 20\times ,7854\times 32\ (= 60\times 8\div 15) = 10053,12$ *Anſ.*

Art. 43. *To meaſure the middle Zone, or middle Fruſtrum of a Parabolic Spindle.*

Definition. This is a caſklike ſolid, wanting two equal ends of ſaid ſpindle.

Rule.—To the ſum and half ſum of the ſquares of the two diameters add three tenths of the difference of their ſquares, which multiply by a third of the length, and the product will be the ſolidity.

If the middle and end diameters of the middle fruſtrum of a parabolic ſpindle be 40 and 30 inches, and its length 60 ; What is its ſolidity ?

$40\times40=1600$ $\quad 1600-900=700$ the diff. of the ſquares.
$30\times30=\ \ 900$ $\quad 700\times,3=210=$ three tenths of do. then,
$\overline{2500+1250+210}\times20(=\frac{1}{3}\text{ of }60)=79200$ *Anſ.*
Sum $=2500$
Half ſum $=1250$

Art. 44. *To meaſure a Cylinderoid, or Priſmoid.*

Definition. A cylinderoid is a ſolid ſomewhat like the fruſtrum of a cone, one baſe may be an ellipſis, and the other a diſproportional ellipſis or circle.

A priſmoid is a ſolid ſomewhat like the fruſtrum of a pyramid, but its baſes are diſproportional.

Rule.—The ſame as for the fruſtrum of a cone or pyramid : Or, to the areas of both baſes add a mean area, that is, the ſquare root of the product of the two baſes, then multiply that ſum by a third of the height or length, and it will give the ſolidity.

If the diameters of the greater baſe of a cylinderoid be 30 and 20 inches, the diameter of the leſs baſe 12, and length 60 inches ; What is the ſolidity ?

$30\times20=600$
$12\times12=144$
$\sqrt{144\times600}=293,9$
$1037,9$

$1037,9\times,7854\times20(=60\div3)=16303,33$ *Anſ.*

If the diameters of the greater baſe of a priſmoid be 30 and 20 inches, the leſs baſe 20 by 10 inches, and length 30 inches ; What is its ſolidity ?

$30\times20=600$
$20\times10=200$
$\sqrt{600\times200}=346,4$
$1146,4$

$1146,4\times10\ (=30\div3)=11464$ ſolidity in inches.

Art. 45. *To meaſure a Solid Ring.*

Rule.—Meaſure the internal diameter of the ring, and its girth, or circumference ; then multiply the girth by ,31831, and the product will be the diameter of the wire, which add to the internal diameter ; multiply this ſum by 3,1416, and the product will be the length of a cylinder equal to the ring of the ſame baſe. Then the area of a ſection of the ring multiplied by the length of the ſaid cylinder will give the ſolidity of the ring.

If

If an iron ring be 12 inches in girth, and its internal diameter be 20 inches; What is its solidity?

$,31831 \times 12 = 3,8 =$ ring's diameter. $\overline{20+3,8} \times 3,1416 = 74,77$ the length of a cylinder equal to the ring: And

$$3,8 \times 3,8 \times ,7854 \times 74,77 = 847,97 = \text{solidity.}$$

ART. 46. *To measure the Solidity of any irregular Body, whose Dimensions cannot be taken.*

Take any regular vessel, either square or round, and put the irregular body into it; pour so much water into the vessel as will exactly cover the body, and measure the dry part from the top of the vessel to the water; then take out the body, and measure again from the top of the vessel to the water, and subtract the first measure from the second, and the difference is the fall of the water: Then, if the vessel be square, multiply the side by itself, and that product by the fall of the water, and you will have the content of the body; but if it be a long square, multiply the length by the breadth, and that product by the fall of the water; or, lastly, if it be a round vessel, multiply the square of the diameter by ,7854, and that product by the fall of the water, and you will have the content.

EXAM. 1. A body being put into a vessel 18 inches square, on taking out the body, the water sunk 9 inches; Required the content of the body?

18 inch. = 1,5 foot.
9 inch. = ,75 foot.
$1,5 \times 1,5 \times ,75 =$ 1,6875 foot, content.

EXAM. 2. A body put into a cistern 4 feet by 3, on taking it out, the water fell 6 inches; Required the content of the body?

$4 \times 3 \times ,5 = 6$ feet, content.

EXAM. 3. A body being put into a round tub, whose diameter was 1,5 foot, on taking out the body, the water fell 1,5 foot; What was the content of the body?

$1,5 \times 1,5 \times ,7854 \times 1,5 = 2,65$ feet, content.

Of the five Regular Bodies.

There are five solids contained under equal regular sides, which, by way of distinction, are called *the five regular bodies.*

These are the *Tetraedron*, the *Hexaedron* or *Cube*, the *Octaedron*, the *Dodecaedron*, and the *Eicosiedron*. The measuring of the cube was shewn at Art. 28. I shall now shew how to measure the other four, by the following Table, which is the shortest method.

A Table of the solid and superficial content of each of the five bodies, the sides being unity, or 1.

Names of the Bodies.	*Solidity.*	*Superficies.*
Tetraedron.	0.11785	1.73205
Hexaedron.	1.	6.
Octaedron.	0.4714	3.464
Eicosiedron.	2.181695	8.66025
Dodecaedron.	7.663119	20.6457

All

All like ſolid bodies being in proportion to one another as the cubes of their like ſides, the ſolid content of any of theſe bodies may be found by multiplying the cubes of their ſides by the numbers in the ſecond column under *Solidity*; and their ſuperficies, by multiplying the ſquares of their ſides into the numbers in the third column, under *Superficies*.

Of the TETRAEDRON.

This ſolid is contained under four equal and equilateral triangles, that is, it is a triangular pyramid of four equal faces, the ſide of whoſe baſe is equal to the ſlant height of the pyramid, from the angles to the vertex.

ART. 47. *The ſide of a Tetraedron being* 3, *to find the ſolid and ſuperficial content.*

Cube $= 3\times3\times3 = 27$, and $27\times,11785 = 3,18195 =$ ſolidity.
Square $= 3\times3 = 9$, and $9\times1,73205 = 15,58845 =$ ſuperficies.

Of the OCTAEDRON.

This ſolid is contained under eight equal and equilateral triangles, which may be conceived to conſiſt of two quadrangular pyramids of equal baſes joined together, the ſides of whoſe baſes are equal to the given ſides of the triangles, under which it is contained.

ART. 48. *The ſide of an Octaedron being* 3, *to find the ſolid and ſuperficial content.*

Cube $= 3\times3\times3 = 27$, and $27\times,4714 = 12,7278 =$ ſolidity.
Square $= 3\times3 = 9$, and $9\times3,464 = 31,176 =$ ſuperficies.

Of the DODECAEDRON.

This ſolid is contained under 12 equilateral pentagons, and may be conceived to conſiſt of twelve pentagonal pyramids, of equal baſes and altitude, whoſe vertices meet in the centre of the dodecaedron.

ART. 49. *The ſide of a Dodecaedron being* 3, *to find the ſolid and ſuperficial content.*

Cube $= 3\times3\times3 = 27$, and $27\times7,663119 = 206,904$.
Square $= 3\times3 = 9$, and $9\times20,6457 = 185,8113$.

Of the EICOSIEDRON.

This ſolid is contained under twenty equal and equilateral triangles, and may be conceived to conſiſt of twenty equal triangular pyramids, whoſe vertices all meet in the centre.

ART. 50. *The ſide of an Eicoſiedron being* 3, *to find the ſolid and ſuperficial content.*

Cube $= 3\times3\times3 = 27$, and $27\times2,18169 = 58,90563 =$ ſolidity.
Square $= 3\times3 = 9$, and $9\times8,66025 = 77,94225 =$ ſuperficies.

As the figures of ſome of theſe bodies would give but a confuſed idea of them, I have omitted them; but the following figures

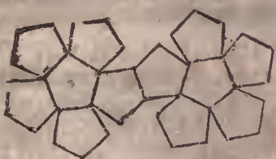
ures, cut out in paſteboard, and the lines cut half through, will fold up into the ſeveral bodies.

Tetraedron.

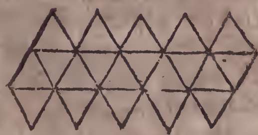
Hexaedron.

Octaedron.

Dodecaedron.



Eicoſiedron.



Of CASK GAUGING.

Among the many different canons, drawn from Stereometry, for gauging *caſks*, the following is as exact as any.

Take the dimenſions of the caſk in inches, viz. the diameter at the bung and head, and length of the caſk : Subtract the head diameter from the bung diameter, and note the difference.

If the ſtaves of the caſk be much curved or bulging *between the bung and the head*, multiply the difference by ,7 ; if not quite ſo curve, by ,65 ; if they bulge yet leſs, by ,6 ; and if they are almoſt or quite ſtraight, by ,55, and add the product to the head diameter ; the ſum will be a mean diameter, by which the caſk is reduced to a cylinder.

Square the mean diameter, thus found, then multiply it by the length ; divide the product by 359 for ale or beer gallons, and by 294 for wine gallons.

Note 1. The length is moſt conveniently taken by callipers, allowing, for the thickneſs of both heads, 1 inch, 1½ inch, or 2 inches, according to the ſize of the caſk : But if you have no callipers, do thus ; meaſure the length of the ſtave ; then take the depth of the chimes, which, with the thickneſs of the head, being ſubtracted from the length of the ſtave, leaves the length within.

Note 2. You muſt take the head diameter, cloſe to its outſide, and, for ſmall caſks, add 3 tenths of an inch ; for caſks of 30, 40, or 50 gallons, 4 tenths, and for larger caſks, 5 or 6 tenths, a

the ſum will be very nearly the head diameter within. In taking the bung diameter, obſerve, by moving the rod backward and forward, whether the ſtave, oppoſite the bung, be thicker or thinner than the reſt, and if it be, make allowance accordingly.

By the Sliding Rule.

On D is 18,94, the gauge point for ale or beer gallons, marked AG, and 17,14, the gauge point for wine gallons, marked WG: Set the guage point to the length of the caſk on C, and againſt the mean diameter, on D, you will have the anſwer in ale or wine gallons accordingly as which gauge point you make uſe of.

By the Scale.

Take the extent from the gauge point to the mean diameter, ſet one foot of the dividers in the length, and, turning them twice over, they will point out the content.

ART. 51. Required the content, in ale and wine gallons, of a caſk, whoſe bung diameter is 35 inches, head diameter 27 inches, and length 45 inches?

```
   Bung diameter = 35
   Head diameter = 27
                   --
      Difference =  8
                   ,7
                   --
                  5,6
Add the head dia. = 27
                  ----
  Mean diameter = 32,6
                  32,6
                  ----
                  1956
                  652
                 978
                -------
Squared     1062,76
```

```
Square of the diam. = 1062,76
           Length =      45
                   -------
                    531380
                   425104
                   --------
           359)47824,20(133,21
                          [ale gall.
  294)47824,2(162,66 wine gall.
```

ART. 52. A round maſh tub is 42 inches diameter at the top, within, and 36 inches at the bottom, and the perpendicular height 48 inches; Required the content in beer and wine gallons?

This being the lower fruſtrum of a cone, to the product of the diameters add $\frac{1}{3}$ of the ſquare of their difference; multiply this ſum by the length, and it will give the ſolidity in ſuch parts as the dimenſions are taken in. If they be taken in inches, divide by 359 for beer, and 294 for wine gallons.

$$\overline{42\times36+\frac{\overline{42-36}\times\overline{42-36}}{3}}\times48\div\begin{cases}359=203\frac{1}{2}\text{ ale gallons.}\\294=248\frac{3}{4}\text{ wine gallons.}\end{cases}$$

ART.

ART. 53. Let the difference of diameters of this tub be 6 inches, the height 48 inches, and the content $203\frac{3}{4}$ gallons, to find the diameters?

Multiply the content, if beer meaſure, by 359; if wine meaſure, by 294, and divide the product by the length: From the quotient ſubtract $\frac{1}{3}$ of the ſquare of the difference of the diameters; to this remainder add the ſquare of $\frac{1}{2}$ the difference of the diameters, and extract the ſquare root of the ſum; from the ſquare root ſubtract $\frac{1}{2}$ the difference of the diameters, and it will give the leaſt diameter to great exactneſs, to which add the difference of the diameters, and the ſum is the greateſt diameter.

$$\sqrt{\frac{203{,}75\times359}{48}-\frac{6\times6}{3}+3\times3}-3=36, \text{ and } 36+6=42.$$

The diameters are 36 and 42.

The content of any veſſel, in gallons, &c. may be thus found: Meaſure the inſide of the veſſel, according to the rule of the figure, and find the content in cubic inches; then,

Divide by	1728	and the quotient will be the content in	cubic feet.
	282		ale or beer gallons.
	231		wine gallons.
	2150,425		buſhels.

ART. 54. *To ullage a Caſk, lying on one ſide, by the Gauging Rod, when the Bung Diameter, and the Content, one, or both, are greater or leſs than the Table on the Rod is made for.*

RULE.—As the bung diameter of the caſk to be meaſured, is to the bung diameter that the table is made for; ſo are the dry inches of the caſk, to a *fourth* number, which find in the table on the rod, and note the number of gallons anſwering to it. Then, as the content of the caſk that the table is made for, is to the content of the caſk to be meaſured; ſo is the number of gallons anſwering to the aforeſaid *fourth* number, to the number of gallons your caſk wants of being full.

ART. 55. *To find a Ship's Burthen, or to Gauge a Ship.*

There is ſuch diverſity in the forms of ſhips, that no general rule can be applied to anſwer all varieties; however, the following rules are practiſed.

RULE 1.—Multiply the breadth at the main beam, half the breadth, and length together: Divide the product by 94, and the quotient is the tons.

RULE 2.—Divide the continued product of the length, breadth and depth, in feet, by 100, for ſhips of war, and 95 for merchant ſhips, in which nothing is allowed for guns, &c. and the quotient is the tons.

RULE 3.—Take the length, from the stern post to the upper part of the stem; subtract two thirds of her breadth from that length: Multiply the remainder by the whole breadth, and that product by half the breadth, in feet, and divide by 100 for war, and 94 for merchant tonnage.

RULE 4.—The weight of a ship's burthen is half the weight of water she can hold.

What is the tonnage of a ship, whose length is 97 feet, breadth 31 feet, and depth $15\frac{1}{2}$ feet?

By Rule 1st.

$\frac{1}{2}$ breadth 15,5
Breadth 31
155
465
480,5
Length 97
33635
43245
94)46608,5(495,83 tons.
376
900
846
548
470
785
752
330
282
48

100)46608,5(466 tons.
400
660
600
608
600
85

By Rule 2d.

Length 97
Breadth 31
97
291
3007
Depth = 15,5
15035
15035
3007
95)46608,5(490,61 tons.
380
860
855
585
570
150
95
55

By

By Rule 3.

$$
\begin{array}{lr}
\text{Length} = & 97 \\
\text{Subtract } \tfrac{2}{3} \text{ of breadth} = & 20{,}66 \\
\hline
 & 76{,}33 \\
\text{Multiply by the breadth} & 31 \\
\hline
 & 7633 \\
 & 22899 \\
\hline
 & 2366{,}23 \\
\text{Multiply by } \tfrac{1}{2} \text{ breadth} & 15{,}5 \\
\hline
 & 1183115 \\
 & 1183115 \\
 & 236623 \\
\hline
\end{array}
$$

94)36676,565(390,176 tons.

Allowing the Cubit, as it is found by modern travellers, to be 22 inches, the content of Noah's Ark is as follows, viz.

	Feet.	
Length of the keel,	300	Its burthen as a man of war 27729 tons.
Breadth by the mid ship beam	50	
Depth in the hold	30	As a merchant ship, 29188,6 tons.

QUESTIONS *in* MENSURATION.

1. The largest of the Egyptian pyramids is square at the base, and measures 693 feet on a side; How much ground does it cover?

$\frac{693 \times 693}{272{,}25} = 1764$ poles, and $\frac{1764}{160} = 11$ acres and 4 poles, *Ans.*

2. What difference is there between a floor 20 feet square, and two others, each 10 feet square?

$\overline{20 \times 20} - \overline{\overline{10 \times 10} + \overline{10 \times 10}} = 200$ feet, *Ans.*

3. There is a square of 2500 yards in area; What is each side of the square, and the breadth of a walk along one side and one end, which may take up just one half of the square?

$\sqrt{2500} = 50$ yards, each side. $\sqrt{\frac{2500}{2}} = 35{,}35$, and $50 - 35{,}35 = 14{,}65$ yards, breadth of the walk, *Ans.*

4. A pine plank is 16 feet and 5 inches long, and I would have just a square yard slit off; At what distance from the edge must the line be drawn?

A square yard $= 1296$ inches, and 16 feet 5 inches $= 197$ inches. Therefore, $\frac{1296}{197} = 6\frac{114}{197}$ inches, *Ans.*

5. If

5. If the area of a triangle be 900 yards, and the perpendicular 40 yards; Required the length of the base?

$$\frac{900\times 2}{40} = 45 \text{ yards, } Ans.$$

6. If the three sides of a plain triangle be 24, 16 and 12 perches; Required its area?

$\frac{24+16+12}{2} = 26$; $26-24=2$; $26-16=10$; $26-12=14$, and $\sqrt{26\times 14\times 10\times 2} = 85{,}32$ perches, = area. Again, As 24 : 16+12 :: 16−12 : 4,6+, the difference of the segments of the base; then, $12-\frac{4{,}6+}{2}=9{,}6$, and $\sqrt{12\times 12-9{,}6\times 9{,}6} = 7{,}11$ the perpendicular on the longest side; whence $24\div 2\times 7{,}11 = 85{,}32$, area, as above.

7. Required the area of a circular garden, whose diameter is 12 rods? $12\times 12\times{,}7854 = 113{,}0976$ poles, *Ans.*

8. The wheel of a perambulator turns just once and an half in a rod; What is its diameter?

$16{,}5\times\frac{2}{3} = 11$, circumf. and $11\times{,}31831 = 3\frac{1}{2}$ feet, *Ans.*

9. Agreed for a platform to the curb of a round well, at $7\frac{1}{2}d.$ per square foot; the inward part, round the mouth of the well, is 36 inches diameter, and the breadth of the platform was to be $15\frac{1}{2}$ inches; What will it come to?

$36+15{,}5\times 2 = 67$ the greatest diameter; $\overline{67\times 67\times{,}7854}-\overline{36\times 36\times{,}7854} = \frac{2507{,}8722}{144} = 17{,}4157$ square feet, at $7\frac{1}{2}d.$ per foot, = 10*s.* $10\frac{6}{10}d.$ *Ans.*

10. Required the difference between the area of a circle, whose radius (or semidiameter) is 50 yards, and its greatest inscribed square?

$50\times 2 = 100$ the diameter, and $100\times 100\times{,}7854 = 7854$ the area of the circle; then, $50\times 50\times 2 = 5000$ the area of the greatest inscribed square, and $7854-5000 = 2854$ *Ans.*

11. There is a section of a tree 25 inches over; I demand the difference of the areas of the inscribed and circumscribed squares, and how far they differ from the area of the section?

$\overline{25\times 25}-\overline{12{,}5\times 12{,}5\times 2} = 312{,}5$ the difference of the squares.
$25\times 25-25\times 25\times{,}7854 = 134{,}125$ the circumscribed square, more than the section, and $\overline{25\times 25\times{,}7854}-\overline{12{,}5\times 12{,}5\times 2} = 178{,}375$ inscribed square, less than the area of the section.

12. Four men bought a grindstone of 60 inches diameter; How much of its diameter must each grind off, to have an equal share of the stone, if one first grind his share, and then another, till the stone is ground away, making no allowance for the eye?

RULE.

RULE.—Divide the ſquare of the diameter by the number of men, ſubtract the quotient from the ſquare, and extract the ſquare root of the remainder, which is the length of the diameter after the firſt man has ground his ſhare; this work being repeated by ſubtracting the ſame quotient from the remainder, for every man, to the laſt; extract the ſquare root of the remainders, and ſubtract thoſe roots from the diameters, one after another; the ſeveral remainders will be the anſwers.

60
60
4)3600
Quot. = 900

From 3600
Take 900
$\sqrt{2700} = 51,9615$, to be taken from 60.
Subt. 900
$\sqrt{1800} = 42,4264$, from 51,9615
Subt. 900
$\sqrt{900} = 30$, from 42,4264

From 60
Take 51,9615
Remains 8,0385 = 1ſt ſhare.

From 51,9615
Take 42,4264
Rem. 9,5351 = 2d ſhare.

From 42,4264
Take 30
Rem. 12,4264 = 3d ſhare.

And 30 inch. = 4th ſhare.

13. If a cubic foot of iron were hammered, or drawn, into a ſquare bar, an inch about, that is, $\frac{1}{4}$ of an inch ſquare; Required its length, ſuppoſing there is no waſte of metal?

$$\frac{12 \times 12 \times 12}{,25 \times ,25 \times 4} = 6912 \text{ inches}, = 576 \text{ feet, } Anſ.$$

14. Required the axis of a globe, whoſe ſolidity may be juſt equal to the area of its ſurface?

$$\frac{,7854 \times 4}{,5236} = 6 \text{ inches, } Anſ.$$

15. A joiſt is $7\frac{1}{2}$ inches wide, and $2\frac{1}{4}$ thick; but I want one juſt twice as large, which ſhall be $3\frac{3}{4}$ inches thick; What will be the breadth?

$$\frac{7.5 \times 2,25 \times 2}{3,75} = 9 \text{ inches, } Anſ.$$

16. I have a ſquare ſtick of timber 18 inches by 14; but one of a third part of the timber in it, provided it be 8 inches deep, will ſerve; How wide will it be?

$$\frac{18 \times 14}{3} \div 8 = 10\tfrac{1}{2} \text{ inches, } Anſ.$$

17. A had a beam of oak timber 18 inches ſquare throughout, and 25 feet long, which he bartered with B, for an equilateral triangular

triangular beam of the ſame length, each ſide 24 inches; Required the balance, at 1*s.* 4*d.* per foot?

$$\frac{18\times18\times25}{144}=56,25 \text{ ſolidity of the ſquare beam.}$$

The perpendicular let fall on one of the ſides of the triangular beam is 21 inches, and the half perp.$=10,5$; then, $\frac{10.5\times24}{144}=1,75$ foot, area at the end, and $1,75\times25=43,75$ feet, ſolidity of the triangular beam; therefore $56,25-43,75=12,5$ feet, at 1*s.* 4*d.* per foot $=$ 16*s.* 8*d.* balance due to A, *Anſ.*

18. What is the difference between a ſolid half foot, and half a foot ſolid?

$$\frac{12\times12\times6}{6\times6\times6}=4, \text{therefore, one is but } \tfrac{1}{4} \text{ of the other.}$$

19. A lent B a ſolid ſtack of hay, meaſuring 20 feet every way; ſometime afterward, B returned a quantity, meaſuring every way 10 feet; What proportion of the hay remains due?

$$20\times20\times20-10\times10\times10=7000 \text{ feet}=\tfrac{7}{8}\ Anſ.$$

20. A ſhip's hold is $75\frac{1}{2}$ feet long, $18\frac{1}{2}$ wide, and $7\frac{1}{4}$ deep; How many bales of goods $3\frac{1}{2}$ feet long, $2\frac{1}{4}$ deep, and $2\frac{3}{4}$ wide, may be ſtowed therein, leaving a gang way the whole length, of $3\frac{1}{4}$ feet wide?

$$\frac{\overline{75,5\times18,5\times7,25}-\overline{75,5\times7,25\times3,25}}{3,5\times2,25\times2,75}=385,44 \text{ bales, } Anſ.$$

21. If a ſtick of timber be $20\frac{1}{2}$ feet long, 16 inches broad, and 8 inches thick, and $3\frac{1}{2}$ ſolid feet be ſawed off one end; How long will the ſtick then be?

$$20\tfrac{1}{2}-\frac{1728\times3,5}{16\times8}=16 \text{ feet, } 6\tfrac{3}{4} \text{ inches, } Anſ.$$

22. The ſolid content of a ſquare ſtone is found to be $136\frac{1}{2}$ feet; its length is $9\frac{1}{2}$ feet; What is the area of one end? and if the breadth be 3 feet 11 inches; What is the depth?

$\frac{136,5\times1728}{9.5\times12}=$ area 2069,0526 inches, and $\frac{2069.0526}{47}=44,022$ inches, *Anſ.*

23. I would have a cubic box made capable of receiving juſt 50 buſhels, the buſhel containing 2150,425 ſolid inches; What will be the length of the ſide?

$$\sqrt[3]{2150.4\times50}=47,55 \text{ inches.}$$

24. A ſtatute buſhel is to be made 8 inches high, and $18\frac{1}{2}$ inches diameter, to contain 2176 cubic inches; (though the content of the dimenſions is but 2150,425 inches) I demand what the diameter of the buſhel muſt be, the height being 8 inches; and what

what the height, the diameter being $18\frac{1}{2}$ inches, to contain 2176 cubic inches ?

Solidity.

Height $= 8)2176$ and $\sqrt{272 \times 1{,}273} = 18{,}6$ diameter. $18{,}5 \times 18{,}5 \times {,}7854 = 268{,}80315 =$ area, and the

Area $= 272$ solidity $2176 \div 268{,}8 = 8{,}0956$ inches, hht.

25. There is a garden rolling ſtone 66 inches in circumference, and $3\frac{1}{2}$ cubic feet are to be cut off from one end, perpendicular to the axis : Where muſt the ſection be made ?

Area $= \dfrac{1728 \times 3{\cdot}5}{412{,}5} = 14{,}65$ inches from one end, *Anſ.*

26. I would have a ſyringe of $1\frac{1}{2}$ inch diameter in the bore to hold a quart, wine meaſure ; What muſt be the length of the piſton, ſufficient to make an injection with ?

$1{,}5 \times 1{,}5 \times {,}7854 = 1{,}76715$, and $231 \div 4 = 57{,}75$ the cubic inches in a quart, then, $\dfrac{57{,}75}{1{,}76715} = 32{,}679$ inches, *Anſ.*

27. If a round pillar, 9 inches diameter, contain 5 feet ; Of what diameter is that column, of equal length, which meaſures 10 times as much ?

As $5 : 9 \times 9 :: 5 \times 10 : 810$, and $\sqrt{810} = 28{,}46$ inches, *Anſ.*

28. There is a ſquare pyramid, each ſide of whoſe baſe is 30 inches, and whoſe perpendicular height is 120 inches, to be divided by ſections parallel to its baſe into 3 equal parts : Required the perpendicular height of each part ?

$30 \times 30 \times 40 = 36000$ the ſolidity in inches, now $\frac{2}{3}$ thereof is 24000, and $\frac{1}{3}$ is 12000. Therefore,

As $36000 : 120 \times 120 \times 120 :: \left\{ \begin{matrix} 24000 \\ 12000 \end{matrix} \right\} \begin{matrix} : 1152000 \\ : 576000 \end{matrix} \Big\{$ Then,

$\sqrt[3]{1152000} = 104{,}8$. Alſo, $\sqrt[3]{576000} = 83{,}2$. Then, $120 - 104{,}8 = 15{,}2$ length of the thickeſt part, and $104{,}8 - 83{,}2 = 21{,}6$ length of the middle part, conſequently $83{,}2$ is the length of the top part.

29. Suppoſe the diameter of the baſe of a conical ingot of gold to be 3 inches, and its height 9 inches ; What length of wire may be expected from it, without loſs of metal, the diameter of the wire being one hundredth part of an inch ?

$3 \times 3 \times {,}7854 \times 3 = 21{,}2058$ the ſolidity of the cone.

$\dfrac{21{,}2058}{{,}01 \times {,}01 \times {,}7854} = 270000$ inch. $= 4$ miles, and 460 yds. *Anſ.*

30. Suppoſe a pole to ſtand on an horizontal plane 75 feet in height above the ſurface : At what height from the ground muſt it be cut off, ſo as that the top of it may fall on a point 55 feet from the bottom of the pole, the end, where it was cut off, reſting on the ſtump, or upright part ?

As

As the whole length of the pole is equal to the ſum of the hypothenuſe and perpendicular of a triangle, (the 55 feet on the ground being the baſe) this, as well as the following queſtion, may be ſolved by this

RULE.—From the ſquare of the length of the pole (that is, of the ſum of the hypothenuſe and perpendicular) take the ſquare of the baſe; divide the remainder by twice the length of the pole, and the quotient will be the perpendicular, or height at which it muſt be cut off.

$$\frac{\overline{75\times75}-\overline{55\times55}}{75\times2}=17\tfrac{1}{3}\text{ feet, } Anſ.$$

31. Suppoſe a ſhip ſails from lat. 43°, north, between north and eaſt, till her departure from the meridian be 45 leagues, and the ſum of her diſtance and difference of latitude to be 135 leagues; I demand her diſtance ſailed, and latitude come to?

$$\frac{\overline{135\times135}-\overline{45\times45}}{135\times2}=60\text{ leagues, and } 60\times3=180\text{ miles}=3$$

degrees the difference of latitude, 135—60 = 75 leagues the diſtance. Now, as the veſſel is ſailing from the equator, and conſequently the latitude is increaſing: Therefore,

To the latitude ſailed from	43°,00′*N.*
Add the difference of latitude	3 ,00
And the ſum is the lat. come to =	46 ,00*N.*

AN

INTRODUCTION TO ALGEBRA,

DESIGNED FOR THE

USE OF ACADEMIES.

DEFINITIONS.

ALGEBRA is the art of computing by ſymbols.

1. *Like quantities* are thoſe which conſiſt of the ſame letters.
2. *Unlike quantities* are thoſe which conſiſt of different letters.
3. *Given quantities* are thoſe whoſe values are known.
4. *Unknown quantities* are thoſe whoſe values are unknown.
5. *Simple quantities* are thoſe which conſiſt of one term only.
6. *Compound quantities* are thoſe which conſiſt of ſeveral terms.
7. *Poſitive* or *affirmative quantities* are thoſe to be added.
8. *Negative quantities* are thoſe to be ſubtracted.
9. *Like ſigns* are all + or all —.
10. *Unlike ſigns* are + and —.
11. *The coefficient* of any quantity is the number prefixed to it.
12. *A binomial quantity* is one conſiſting of two terms; a *trinomial*, of three terms; and a *quadrinomial*, of four terms, &c.
13. *A reſidual quantity* is a binomial, where one of the terms is a negative.

In the computation of problems, the firſt letters of the alphabet are put for known quantities, and letters of the latter part of the alphabet for thoſe which are unknown.

AXIOMS.

1. If equal quantities be added to, ſubtracted from, multiplied or divided by, equal quantities, the wholes, remainders, products and quotients will be reſpectively equal.
2. The equal powers or roots of equal quantities are equal.
3. Two quantities reſpectively equal to a third, are equal to each other.
4. The whole is equal to all its parts taken together.

ADDITION.

CASE I. *To add quantities which are alike, and have like ſigns.**

RULE.—Add all the coefficients together, and to their ſum adjoin the letters common to each term, prefixing the common ſign.

5a

* When a leading quantity has no ſign before it, + is always underſtood; and a quantity without any coefficient prefixed to it, is ſuppoſed to have unity, or 1.

Ooo

$$\begin{array}{ccccc}
5a & -6br & 8bru & 5r^2+ru & 7ar-u \\
7a & -3br & 7bru & 3r^2+2ru & 8ar-3u \\
8a & -2br & 3bru & r^2+3ru & 6ar-2u \\
10a & -7br & 4bru & 7r^2+8ru & 4ar-3u \\
2a & -br & 5bru & r^2+ru & ar-u \\
a & -5br & bru & 2r^2+3ru & 3ar-2u \\
\hline
33a & -24br & 28bru & 19r^2+18ru & 29ar-12u
\end{array}$$

CASE II. *To add quantities which are like, but have unlike ſigns.*

RULE 1. Add all the affirmative coefficients into one ſum, and all the negative ones into another.

2. Subtract the leaſt ſum from the greateſt, and to the difference prefix the ſign of the greateſt, with the common quantity.

$$\begin{array}{ccccc}
-3a & +8ar^2 & +6r^{\frac{1}{3}}+8u & -2ru+8 & +8r^2-u+3\sqrt{r} \\
+7a & +7ar^2 & -3r^{\frac{1}{3}}+7u & -3ru+7 & -10r^2-3u+2\sqrt{r} \\
+8a & -3ar^2 & -13r^{\frac{1}{3}}+8u & +ru-10 & -4r^2-2u+\sqrt{r} \\
-a & -4ar^2 & +2r^{\frac{1}{3}}-3u & +5ru-7 & +9r^2+6u-10\sqrt{r} \\
-2a & +4ar^2 & +r^{\frac{1}{3}}-u & -ru+2 & +r^2 * -\sqrt{r} \\
\hline
+9a & +12ar^2 & -7r^{\frac{1}{3}}+19u & * & +4r^2 * -5\sqrt{r}
\end{array}$$

CASE III. *To add quantities which are unlike, and have unlike ſigns.*

RULE.—Collect the like quantities together by the laſt rule, and ſet down thoſe which are unlike, one after another, with their proper ſigns.

$$\begin{array}{ccc}
2r & 2r-r^2 & \\
3u & 3a-r & 12ar-r^2-6+\sqrt{ar-r^2} \\
-a & 2ar+6r^2 & -6ar+r^2-r+10 \\
r^2 & 3\sqrt{r}-2ar & 3u-ar-4-2\sqrt{ar-r^2} \\
\hline
2r+3u-a+r^2 & r+5r^2+3a+3\sqrt{r} & 5ar-\sqrt{ar-r^2}--r+3u
\end{array}$$

SUBTRACTION.

RULE.—Change the ſigns of all the quantities to be ſubtracted, and then add them together, as in Addition.

$$\begin{array}{cccc}
3a^2-2b & 6r^2-8u+2 & 35ru-2+8r-u^{\frac{1}{2}} & 8ar-2\sqrt{ru}-10 \\
2a^2-3b & r^2+9u-20 & 24ru-8-8r-3u & 10r-6\sqrt{ru}-ar \\
\hline
a^2+b & 5r^2-17u+22 & 11ru+6+16r-u^{\frac{1}{2}}+3u & 9ar+4\sqrt{ru}-10-10r
\end{array}$$

MULTIPLICATION.

MULTIPLICATION.

CASE I. *To multiply ſimple quantities.*

RULE.—Multiply the coefficients of the two terms together, and to the product annex all the letters in thoſe terms.

Note. Like ſigns produce +, and unlike ſigns —.

$2a$	$-2a$	$5a$	$-9r$	$6a^2r$	$-r^2u$	$-7ru$
$3b$	$+4b$	$-6r$	$-5b$	$5r$	ru^2	$-ru$
$6ab$	$-8ab$	$-30ar$	$+45br$	$30a^2r^2$	$-r^3u^3$	$+7r^2u^2$

CASE II. *When one of the factors is a compound quantity.*

RULE.—Find the products of the multiplier and every particular term of the multiplicand ſeparately, and place them one after another with their proper ſigns.

$4a-2b$	$6ru-8$	$8a^2-2r+6$	$3u-8+2ru$
$3a$	$2r$	$3ru$	ru
$12a^2-6ab$	$12r^2u-16r$	$24a^2ru-6r^2u+18ru$	$3ru^2-8ru+2r^2u^2$

CASE III. *When both the factors are compound quantities.*

RULE.—Multiply every particular term of the multiplier into every term of the multiplicand reſpectively, and ſet down the products one after another with their proper ſigns, and their ſum will be the whole product.

$$\begin{array}{l} r+u \\ r+u \\ \hline r^2+ru \\ \quad +ru+u^2 \\ \hline r^2+2ru+u^2 \end{array} \qquad \begin{array}{l} r-u \\ r-u \\ \hline r^2-ru \\ \quad -ru+u^2 \\ \hline r^2-2ru+u^2 \end{array} \qquad \begin{array}{l} 3r^2-2ru+5 \\ r^2+2ru-3 \\ \hline 3r^4-2r^3u+5r^2 \\ \quad +6r^3u-4r^2u^2+10ru \\ \quad\quad -9r^2+6ru-15 \\ \hline 3r^4+4r^3u-4r^2-4r^2u^2+16ru-15 \end{array}$$

When two ſurd numbers are to be multiplied together, multiply them without any regard to the radical ſign, and prefix the radical ſign to the product. Thus,

$$\sqrt{3}\times\sqrt{2}=\sqrt{6};\ \sqrt{a}\times\sqrt{b}=\sqrt{ab},\ \&c.$$

DIVISION.

CASE I. *When the diviſor is a ſimple quantity.*

RULE 1.—Place the dividend above a line, and the diviſor under it, like a vulgar fraction.

2. Expunge

2. Expunge thoſe letters which are common to both the factors, and divide the coefficients of all the terms by any number which will divide them without a remainder.

Note. Like ſigns make $+$, and unlike ſigns $-$, as in Multiplication.

$$\frac{a}{a}=1;\ \frac{8bc}{2b}=4c;\ \frac{abc}{bcd}=\frac{a}{d};\ \frac{10ab+15ac}{20ad}=\frac{2b+3c}{4d};\ \frac{ab+b^2}{2b}=\frac{a+b}{2};\ \frac{12ru}{6r^2}=\frac{2u}{r};\ \frac{30ar-54au}{12ab}=\frac{5r-9u}{2b};\ \frac{10r^2u-15u^2-5u}{5u}=2r^2-3u-1;\ \frac{3a^2-15+6a+3b}{3a}=a-\frac{5}{a}+2+\frac{b}{a}.$$

Case II. *When the diviſor and dividend are both compound quantities.*

Rule 1.—Range the terms of both the quantities according to the dimenſions of ſome letter in them, ſo that the firſt term may have the higheſt power of that letter, and the ſecond term the next higheſt power ; and ſo on.

2. Divide the firſt term of the dividend by the firſt term of the diviſor, and place the reſult in the quotient.

3. Multiply the whole diviſor by the quotient term laſt found, and ſubtract the reſult from the dividend.

4. To this remainder bring down the next term of the dividend and divide as before, and ſo on, as in common arithmetic.

$$\begin{array}{l}
a+r)\,a^3+5a^2r+5ar^2+r^3\,(a^2+4ar+r^2 \\
\qquad a^3+a^2r \\
\hline
\qquad\quad 4a^2r+5ar^2 \\
\qquad\quad 4a^2r+4ar^2 \\
\hline
\qquad\qquad\quad ar^2+r^3 \\
\qquad\qquad\quad ar^2+r^3 \\
\hline
\qquad\qquad\qquad *
\end{array}$$

$$\begin{array}{l}
r-3)\,r^3-9r^2+27r-27\,(r^2-6r+9 \\
\qquad r^3-3r^2 \\
\hline
\qquad -6r^2+27r \\
\qquad -6r^2+18r \\
\hline
\qquad\qquad\quad 9r-27 \\
\qquad\qquad\quad 9r-27 \\
\hline
\qquad\qquad\qquad *
\end{array}$$

$$\begin{array}{l}
a-r)\,a^3-r^3\,(a^2+ar+r^2 \\
\qquad a^3-a^2r \\
\hline
\qquad\quad a^2r-r^3 \\
\qquad\quad a^2r-ar^2 \\
\hline
\qquad\qquad ar^2-r^3 \\
\qquad\qquad ar^2-r^3 \\
\hline
\qquad\qquad\quad *
\end{array}$$

ALGEBRAIC

ALGEBRAIC FRACTIONS.

PROBLEM I. *To reduce a mixed quantity to an improper fraction.*

RULE.—Multiply the integer by the denominator of the fraction, and to the product add the numerator, and the denominator being placed under this ſum will give the improper fraction required.

$$r+\frac{r^2}{a}=\frac{ar+r^2}{a};\ a-\frac{b}{c}=\frac{ac-b}{c};\ 1-\frac{2r}{a}=\frac{a-2r}{a};$$
$$a-r+\frac{a^2-ar}{r}=\frac{a^2-r^2}{r}.$$

PROB. II. *To reduce an improper fraction to a whole, or mixed quantity.*

RULE.—Divide the numerator by the denominator, for the integral part, and place the remainder over the denominator, for the fractional part.

$$\frac{ar+a^2}{r}=a+\frac{a^2}{r};\ \frac{au+2u^2}{a+u}=u+\frac{u^2}{a+u};\ \frac{ab-a^2}{b}=a-\frac{a^2}{b};$$
$$\frac{a^2+2r^2}{a-r}=a+r+\frac{r^2}{a-r}.$$

PROB. III. *To reduce fractions of different denominators, to thoſe of the ſame value, which ſhall have a common denominator.*

RULE.—Multiply every numerator ſeparately into all the denominators but its own, for new numerators, and all the denominators together, for a common denominator.

1. Reduce $\frac{a}{b}$ and $\frac{b}{c}$ to fractions of equal values, having a common denominator.

$a\times c=ac$ new numer. $\quad \frac{ac}{bc}$ and $\frac{b^2}{bc}=$ fractions required.
$b\times b=b^2$ new numer.

$b\times c=bc$ common denominator.

2. Reduce $\frac{a}{b}$, $\frac{b}{c}$ & $\frac{c}{d}$ to equivalent fractions, having a common denominator.

$a\times c\times d=acd$
$b\times b\times d=b^2d$ $\quad \frac{acd}{bcd}$, $\frac{b^2d}{bcd}$ and $\frac{c^2b}{bcd}$, *Anſ.*
$c\times b\times c=c^2b$

$b\times c\times d=bcd$.

PROB. IV. *To find the greateſt common meaſure of a fraction.*

RULE 1. Range the quantities according to the dimenſions of ſome letter, as was ſhewn in diviſion.

2. Divide

2. Divide the greater term by the leſs, and the laſt diviſor by the laſt remainder, and ſo on, till nothing remain, and the diviſor laſt uſed, will be the common meaſure required.

Note. All the letters or figures, which are common to the diviſor, and dividend, muſt be cancelled in the diviſor before they be uſed in the operation.

To find the greateſt common meaſure of $\frac{cr+r^2}{ca^2+a^2r}$

$$\begin{array}{l} {}^*\,cr+r^2)ca^2+a^2r \\ \text{Or, } c+r)ca^2+a^2r(a^2 \\ \qquad\quad ca^2+a^2r \\ \qquad\quad \overline{\qquad\qquad} \\ \qquad\qquad * \end{array}$$

Therefore the greateſt common meaſure is $c+r$.

2. To find the greateſt common meaſure of $\frac{b^3-b^2r}{r^2+2br+b^2}$

$$\begin{array}{l} r^2+2br+b^2)r^3-b^2r(r \\ \qquad\qquad r^3+2br^2+b^2r \\ \qquad\qquad \overline{\qquad\qquad\qquad} \\ \dagger\ -2br^2-2b^2r)r^2+2br+b^2 \\ \quad \text{Or, } r+b\ )r^2+2br+b^2(r+b \\ \qquad\qquad\qquad r^2+br \\ \qquad\qquad\qquad \overline{\qquad\qquad} \\ \qquad\qquad\qquad br+b^2 \\ \qquad\qquad\qquad br+b^2 \\ \qquad\qquad\qquad \overline{\qquad\qquad} \\ \qquad\qquad\qquad\quad * \end{array}$$

Therefore $r+b$ is the greateſt common diviſor.

PROB. V. *To reduce a fraction to its loweſt terms.*

RULE 1.—Find the greateſt common meaſure, as in the laſt problem.

2. Divide both the terms of the fraction by the common meaſure thus found.

1. Reduce

* Here I find that r is common to both diviſor and dividend, I therefore cancel r in the diviſor, that is, I divide $cr+r^2$ by r, and $c+r$ is the quotient: Thus,

$$\begin{array}{l} r)cr+r^2(c+r, \text{ for the diviſor.} \\ \quad cr \\ \quad \overline{\qquad} \\ \quad +r^2 \\ \quad +r^2 \\ \quad \overline{\qquad} \\ \qquad * \end{array}$$

† Here $-2br$ is common to the diviſor and dividend; I therefore firſt divide $-2br^2-2b^2r$ by r, and the quotient is $-2br-2b^2$, thus,

$$\begin{array}{l} r)-2br^2-2b^2r(-2br-2b^2 \\ \quad -2br^2 \\ \quad \overline{\qquad\qquad} \\ \qquad -2b^2r \\ \qquad -2b^2r \\ \qquad \overline{\qquad} \\ \qquad\quad * \end{array}$$

I then divide $-2br-2b^2$ by $-2b$, and the quotient is $r+b$, thus,

$$\begin{array}{l} -2b)-2br-2b^2(r+b, \text{ for the diviſor.} \\ \qquad -2br \\ \qquad \overline{\qquad\qquad} \\ \qquad\quad -2b^2 \\ \qquad\quad -2b^2 \\ \qquad\quad \overline{\qquad} \\ \qquad\qquad * \end{array}$$

1. Reduce $\frac{cr+r^2}{ca^2+a^2r}$ to its lowest terms.

$$cr+r^2)ca^2+a^2r$$

$$\text{Or, } c+r\ )ca^2+a^2r(a^2$$

$$ca^2+a^2r$$

$$*$$

Therefore, $c+r$ is the greatest common measure,

and $c+r\Big)\frac{cr+r^2}{ca^2+a^2r}\Big(\frac{r}{a^2}=$ fraction required.

2. Reduce $\frac{r^3-b^2r}{r^2+2br+b^2}$ to its lowest terms.

$$r^2+2br+b^2)r^3-b^2r(r$$

$$r^3+2br^2+b^2r$$

$$-2br^2-2b^2r)r^2+2br+b^2$$

$$\text{Or, } r+b\ )r^2+2br+b^2(r+b$$

$$r^2+\ br$$

$$br+b^2$$

$$br+b^2$$

$$*$$

Therefore $r+b$ is the greatest common measure, and

$r+b\Big)\frac{r^3-b^2r}{r^2+2br+b^2}\Big(\frac{r^2-br}{r+b}=$ fraction required.

Prob. VI. *To add fractional quantities together.*

Rule 1. Reduce the fractions to a common denominator.

2. Add all the numerators together, and under their sum write the common denominator.

1. Add $\frac{r}{2}$ and $\frac{r}{3}$.

$$r\times 3=3r$$

$$r\times 2=2r$$

$$2\times 3=6$$

$$\frac{3r}{6}+\frac{2r}{6}=\frac{5r}{6}=\text{sum.}$$

2. Add $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$.

$$a\times d\times f=adf \quad \frac{adf}{bdf}+\frac{cbf}{bdf}+\frac{ebd}{bdf}=\frac{adf+cbf+ebd}{bdf}=\text{sum.}$$

$$c\times b\times f=cbf$$

$$e\times b\times d=ebd$$

$$b\times d\times f=bdf.$$

3. Add

3. Add $a - \frac{3r^2}{b}$ and $b + \frac{r-2b}{c}$.

$$3r^2 \times c = 3cr^2$$
$$\overline{r-2b} \times b = br - 2b^2$$
$$b \times c = bc$$

$$a - \frac{3cr^2}{bc}$$
$$b + \frac{br-2b^2}{bc}$$
$$a + b + \frac{br - 3cr^2 - 2b^2}{bc} = \text{sum.}$$

PROB. VII. *To subtract one fractional quantity from another.*

RULE 1.—Reduce the fractions to a common denominator.

2. Subtract the numerators, and under their difference write the common denominator.

1. Required the difference of $\frac{r}{3}$ and $\frac{2r}{11}$?

$$r \times 11 = 11r$$
$$2r \times 3 = 6r$$
$$3 \times 11 = 33$$

$$\frac{11r}{33} - \frac{6r}{33} = \frac{5r}{33} = \text{difference.}$$

2. What is the difference of $\frac{r-a}{3b}$ and $\frac{2a-4r}{5c}$?

$$\overline{r-a} \times 5c = 5cr - 5ac$$
$$\overline{2a-4r} \times 3b = 6ab - 12br$$
$$3b \times 5c = 15bc$$

$$\frac{5cr-5ac}{15bc} - \frac{6ab-12br}{15bc} = \frac{5cr - 5ac - 6ab + 12br}{15bc} = \text{difference.}$$

PROB. VIII. *To multiply fractional quantities.*

RULE.—Multiply the numerators together for a new numerator, and the denominators, for a new denominator.

1. Multiply $\frac{r}{6}$ and $\frac{2r}{9}$ together.

$$\left.\begin{matrix} r \times 2r \\ 6 \times 9 \end{matrix}\right\} = \frac{2r^2}{54} = \frac{r^2}{27} = \text{product.}$$

2. Find the product of $\frac{r}{2}$, $\frac{4r}{5}$ and $\frac{10r}{21}$.

$$\left.\begin{matrix} r \times 4r \times 10r \\ 2 \times 5 \times 21 \end{matrix}\right\} = \frac{40r^3}{210} = \frac{4r^3}{21} = \text{product.}$$

3. Find

3. Find the product of $\frac{r}{a}$ and $\frac{r+a}{a+c}$.

$$\left.\begin{matrix} r\times\overline{r+a} \\ a\times\overline{a+c}\end{matrix}\right\} = \frac{r^2+ar}{a^2+ac} = \text{product.}$$

PROB. IX. *To divide one fractional quantity by another.*

RULE.—Invert the divisor, and proceed as in multiplication.

1. Divide $\frac{r}{3}$ by $\frac{2r}{9}$. $\frac{r}{3}\times\frac{9}{2r}=\frac{9r}{6r}=\frac{3}{2}=1\frac{1}{2}=$ quotient.

2. Divide $\frac{2a}{b}$ by $\frac{4c}{d}$. $\frac{2a}{b}\times\frac{d}{4c}=\frac{2ad}{4bc}=\frac{ad}{2bc}=$ quotient.

3. Divide $\frac{r+a}{2r-2b}$ by $\frac{r+b}{5r+a}$.

$$\frac{r+a}{2r-2b}\times\frac{5r+a}{r+b}=\frac{5r^2+6ar+a^2}{2r^2-2b^2}=\text{quotient.}$$

INVOLUTION.

Involution is the raising of powers from any proposed root; or the method of finding the square, cube, biquadrate, &c. of any given quantity.

RULE.—Multiply the quantity into itself as often as is denoted by the index, and the last product will be the power required.

Or, Multiply the index of the quantity by the index of the power, and the result will be the same as before.

Note. When the sign of the root is $+$ all the powers of it will be $+$; and when the sign is $-$, all the odd powers will be $-$, and all the even powers $+$.

$$\text{Root} = a\begin{cases} a^2 = \text{square.} \\ a^3 = \text{cube.} \\ a^4 = \text{4th power.} \\ a^5 = \text{5th power.}\end{cases} \qquad \text{Root} = a^2\begin{cases} a^4 = \text{square.} \\ a^6 = \text{cube.} \\ a^8 = \text{4th pow.} \\ a^{10} = \text{5th pow.}\end{cases}$$

$$\text{Root} = -3a\begin{cases} +\ 9a^2 = \text{square.} \\ -\ 27a^3 = \text{cube.} \\ +\ 81a^4 = \text{4th power.} \\ -\ 243a^5 = \text{5th power.}\end{cases}$$

$$\text{Root} = -2ar^2\begin{cases} +\ 4a^2r^4 = \text{square.} \\ -\ 8a^3r^6 = \text{cube.} \\ +16a^4r^8 = \text{4th pow.} \\ -32a^5r^{10} = \text{5th pow.}\end{cases} \qquad \text{Root} = \frac{r}{a}\begin{cases} \frac{r^2}{a^2} = \text{squa.} \\ \frac{r^3}{a^3} = \text{cube.} \\ \frac{r^4}{a^4} = \text{biqd.}\end{cases}$$

Root =

$$\text{Root} = -\frac{2ar^2}{3b}\left\{\begin{array}{l} +\dfrac{4a^2r^4}{9b^2} = \text{ſquare.} \\ -\dfrac{8a^3r^6}{27b^3} = \text{cube.} \\ +\dfrac{16a^4r^8}{81b^4} = \text{biquadrate.}\end{array}\right.$$

$$\begin{array}{l}
r+a \\
r+a \\
\hline
r^2+ar \\
\quad +ar+a^2 \\
\hline
r^2+2ar+a^2 = \text{ſquare.} \\
r+a \\
\hline
r^3+2ar^2+a^2r \\
\quad + ar^2+2a^2r+a^3 \\
\hline
r^3+3ar^2+3a^2r+a^3 = \text{cube.} \\
r+a \\
\hline
r^4+3ar^3+3a^2r^2+a^3r \\
\quad + ar^3+3a^2r^2+3a^3r+a^4 \\
\hline
r^4+4ar^3+6a^2r^2+4a^3r+a^4 = \text{biquadrate.} \\
\hline
\end{array}$$

Of the Compoſition and Reſolution of a Square raiſed from a Binomial Root.

A binomial is a quantity conſiſting of two parts or members, connected together by the ſign + or —, as $r+a$, $r-a$, $r+\frac{b}{2}$, $r-\frac{b}{2}$, and a ſquare raiſed from a binomial root is nothing elſe but the ſquare of ſuch a quantity: Thus, the ſquare of $r+\frac{b}{2}$ is $r^2+br+\frac{b^2}{4}$, and that of $r-\frac{b}{2}$ is $r^2-br+\frac{b^2}{4}$.

$r+\frac{b}{2}$

$$\begin{array}{ll} r+\frac{b}{2} & r-\frac{b}{2} \\ r+\frac{b}{2} & r-\frac{b}{2} \\ \hline r^2+\frac{br}{2}+\frac{b^2}{4} & r^2-\frac{br}{2}+\frac{b^2}{4} \\ \quad +\frac{br}{2} & \quad -\frac{br}{2} \\ \hline r^2+\frac{2br}{2}+\frac{b^2}{4}=r^2+br+\frac{b^2}{4} & r^2-\frac{2br}{2}+\frac{b^2}{4}=r^2-br+\frac{b^2}{4} \end{array}$$

The difference between theſe two ſquares ariſes from the different ſign of b, and that only affects the ſecond member; for the third member $\frac{bb}{4}$ will be the ſame, whether the quantity b be affirmative or negative; therefore, if thoſe caſes be thrown into one, it will ſtand thus: The ſquare of $r\pm\frac{b}{2}$; viz. $+ br$ when the root is $r+\frac{b}{2}$, and $-br$ when the root is $r-\frac{b}{2}$. Now, of the three members, which compoſe this ſquare, the firſt, r^2 is the ſquare of r, the ſecond, $\pm br$ is the root of that ſquare multiplied into the coefficient $\pm b$, and the third member, $\frac{b^2}{4}$ is the ſquare of $\pm\frac{b}{2}$, that is, the ſquare of half the coefficient of the ſecond member; whence may be deduced the following obſervations.

1. Any quantity conſiſting of two members, as $r^2\pm br$, whereof one, as r^2 is a ſquare, and the other $\pm br$ is the root of that ſquare multiplied into ſome given coefficient $\pm b$, it may be conſidered as an imperfect ſquare raiſed from a binomial root, and may eaſily be completed by adding $\frac{b^2}{4}$, that is, by adding the ſquare of half the coefficient of r in the ſecond term; thus r^2+6r, when completed, is r^2+6r+9; r^2+3r becomes $r^2+3r+\frac{9}{4}$, becauſe half the coefficient 3 is $\frac{3}{2}$. Again, $r^2+\frac{2r}{3}$ becomes $r^2+\frac{2r}{3}+\frac{1}{9}$, becauſe half the coefficient is $\frac{1}{3}$, the ſquare of which is $\frac{1}{9}$: Laſtly, $r^2-\frac{br}{a}$ becomes $r^2-\frac{br}{a}+\frac{b^2}{4a^2}$: For the coefficient is $-\frac{b}{a}$, its half $-\frac{b}{2a}$, and the ſquare $\frac{b^2}{4a^2}$.

2. The

2. The root of ſuch a ſquare, when completed, that is, the root of $r^2 \pm br + \frac{b^2}{4}$ will always be $r \pm \frac{b}{2}$, that is, it will always be the ſquare root of the firſt, together with half the coefficient of the ſecond: Thus the ſquare root of r^2+6r+9 will be $r+3$, that of $r^2+3r+\frac{9}{4}$ will be $r+\frac{3}{2}$, that of $r^2+\frac{2r}{3}+\frac{1}{9}$ will be $r+\frac{1}{3}$, and laſtly, that of $r^2-\frac{br}{a}+\frac{b^2}{4a^2}$ will be $r-\frac{b}{2a}$.

SIR ISAAC NEWTON's RULE *for raiſing a binomial or reſidual quantity to any power whatever.*

1. *To find the terms without the coefficients.*

The index of the firſt, or leading quantity, begins with that of the given power, and decreaſes continually by 1, in every term to the laſt; and in the following quantity the indices of the terms are 0, 1, 2, 3, 4, &c.

2. *To find the unciæ or coefficients.*

The firſt is always 1, and the ſecond is the index of the power: And, in general, if the coefficient of any term be multiplied by the index of the leading quantity, and the product be divided by the number of terms to that place, it will give the coefficient of the term next following.

Note. The whole number of terms will be one more than the index of the given power; and when both terms of the root are $+$, all the terms of the power will be $+$; but if the ſecond term be $-$, then all the odd terms will be $+$, and the even terms $-$.

1. Let $a+r$ be involved to the fifth power.

The terms without the coefficients will be $a^5, a^4r, a^3r^2, a^2r^3, ar^4, r^5$, and the coefficients will be $1, 5, \frac{5\times4}{2}, \frac{10\times3}{3}, \frac{10\times2}{4}, \frac{5\times1}{5}$, or 1, 5, 10, 10, 5, 1, and therefore the 5th power is $a^5+5a^4r+10a^3r^2+10a^2r^3+5ar^4+r^5$.

2. Let $r-a$ be involved to the 6th power.

The terms without the coefficients will be $r^6, r^5a, r^4a^2, r^3a^3, r^2a^4, ra^5, a^6$, and the coefficients will be $1, 6, \frac{6\times5}{2}, \frac{15\times4}{3}, \frac{20\times3}{4}, \frac{15\times2}{5}, \frac{6\times1}{6}$ or 1, 6, 15, 20, 15, 6, 1; and therefore the 6th power of $r-a$ is $r^6-6r^5a+15r^4a^2-20r^3a^3+15r^2a^4-6ra^5+a^6$.

E V O L U T I O N.

Evolution is the reverſe of Involution, and teaches to find the roots of any given powers.

CASE

CASE I. *To find the roots of simple quantities.*

RULE.—Extract the root of the coefficient, for the numerical part, and divide the index of the letters by the index of the power, and it will give the root required.

1. The square root of $9r^2 = 3r^{\frac{2}{2}} = 3r$.

2. The cube root of $8r^3 = 2r^{\frac{3}{3}} = 2r$.

3. The square root of $3a^2r^6 = a^{\frac{2}{2}}r^{\frac{6}{2}}\sqrt{3} = ar^3\sqrt{3}$.

4. The cube root of $-125a^3r^6 = -5a^{\frac{3}{3}}r^{\frac{6}{3}} = -5ar^2$.

5. The biquadrate root of $16a^4r^8 = 2a^{\frac{4}{4}}r^{\frac{8}{4}} = 2ar^2$.

CASE II. *To find the square root of a compound quantity.*

RULE 1.—Range the quantities according to the dimensions of some letter, and set the root of the first term in the quotient.

2. Subtract the square of the root, thus found, from the first term, and bring down the two next terms to the remainder, for a dividend.

3. Divide the dividend by double the root, and set the result in the quotient.

4. Multiply the divisor and quotient by the term last put in the quotient, aud subtract the product from the dividend, and so on, as in common Arithmetic.

1. Extract the square root of $4a^4+12a^3r+13a^2r^2+6ar^3+r^4$.

$$
\begin{array}{r}
4a^4+12a^3r+13a^2r^2+6ar^3+r^4(2a^2+3ar+r^2 \\
4a^4 \qquad\qquad\qquad\qquad\qquad\qquad \\
\hline
4a^2+3ar)12a^3r+13a^2r^2 \qquad\qquad \\
12a^3r+\ 9a^2r^2 \qquad\qquad \\
\hline
4a^2+6ar+r^2)4a^2r^2+6ar^3+r^4 \\
4a^2r^2+6ar^3+r^4 \\
\hline
*
\end{array}
$$

2. Extract the square root of $r^4-4r^3+6r^2-4r+1$.

$$
\begin{array}{r}
r^4-4r^3+6r^2-4r+1(r^2-2r+1 \\
r^4 \qquad\qquad\qquad\qquad\qquad \\
\hline
2r^2-2r)-4r^3+6r^2 \qquad\qquad \\
-4r^3+4r^2 \qquad\qquad \\
\hline
2r^2-4r+1)2r^2-4r+1 \\
2r^2-4r+1 \\
\hline
*
\end{array}
$$

CASE III. *To find the roots of powers in general.*

RULE 1.—Find the root of the firſt term, and place it in the quotient.

2. Subtract the power, and bring down the ſecond term for a dividend.

3. Involve the root, laſt found, to the next loweſt power, and multiply it by the index of the given power, for a diviſor.

4. Divide the dividend by the diviſor, and the quotient will be the next term of the root.

5. Involve the whole root, and ſubtract and divide as before; and ſo on till the whole be finiſhed.

1. Required the ſquare root of $a^4-2a^3r+3a^2r^2-2ar^3+r^4$.

$$a^4-2a^3r+3a^2r^2-2ar^3+r^4)a^2-ar+r^2$$
$$a^4$$
$$2a^2)-2a^3r$$
$$a^4-2a^3r+a^2r^2$$
$$2a^2)2a^2r^2$$
$$a^4-2a^3r+3a^2r^2-2ar^3+r^4$$

*

2. Extract the cube root of $r^6+6r^5-40r^3+96r-64$

$$r^6+6r^5-40r^3+96r-64(r^2+2r-4$$
$$r^6$$
$$3r^4)6r^5$$
$$r^6+6r^5+12r^4+8r^3$$
$$3r^4)-12r^4$$
$$r^6+6r^5-40r^3+96r-64$$

*

INFINITE SERIES.

An *Infinite Series* is formed from a vulgar fraction, having a compound denominator, or by extracting the root of a ſurd quantity; and is ſuch, as, being continued, would run on *ad infinitum*, in the manner of a decimal fraction. And, by obtaining a few of the firſt terms, the law of the progreſſion will be manifeſt, ſo that the ſeries may be continued, without actually performing the whole operation.

PROBLEM

PROBLEM I. *To reduce fractional quantities into infinite series.*

RULE.—Divide the numerator by the denominator, and the operation continued, as far as shall be thought necessary, will give the operation required.

1. Let $\frac{1}{1+r}$ be thrown into an infinite series.

$$1+r)1 \ldots (1-r+r^2-r^3+r^4, \&c.$$
$$1+r$$
$$\overline{\quad -r \quad}$$
$$-r-r^2$$
$$\overline{\quad +r^2 \quad}$$
$$+r^2+r^3$$
$$\overline{\quad -r^3 \quad}$$
$$-r^3-r^4$$
$$\overline{\quad +r^4, \&c. \quad}$$

2. Let $\frac{1}{1-r}$ be thrown into an infinite series.

$$1-r)1 \ldots (1+r+r^2+r^3+r^4, \&c.$$
$$1-r$$
$$\overline{\quad +r \quad}$$
$$+r-r^2$$
$$\overline{\quad +r^2 \quad}$$
$$+r^2-r^3$$
$$\overline{\quad +r^3 \quad}$$
$$+r^3-r^4$$
$$\overline{\quad +r^4, \&c. \quad}$$

3. Let $\frac{ar}{a-r}$ be proposed.

$$a-r)ar \ldots (r+\frac{r^2}{a}+\frac{r^3}{a^2}+\frac{r^4}{a^3}, \&c.$$
$$ar-r^2$$
$$\overline{\quad r^2 \quad}$$
$$r^2-\frac{r^3}{a}$$
$$\overline{\quad \frac{r^3}{a} \quad}$$
$$\frac{r^3}{a}-\frac{r^4}{a^2}$$
$$\overline{\quad \frac{r^4}{a^2} \quad}$$
$$\frac{r^4}{a^2}-\frac{r^5}{a^3}$$
$$\overline{\quad \frac{r^5}{a^3}, \&c. \quad}$$

4. Let $\frac{a^2}{a^2+2ar+r^2}$ be proposed.

$$a^2+2ar+r^2)a^2 \ldots (1-\frac{2r}{a}+\frac{3r^2}{a^2}-\frac{4r^3}{a^3}$$
$$a^2+2ar+r^2$$
$$\overline{\quad -2ar-r^2 \quad}$$
$$-2ar-4r^2-\frac{2r^3}{a}$$
$$\overline{\quad 3r^2+\frac{2r^3}{a} \quad}$$
$$3r^2+\frac{6r^3}{a}+\frac{3r^4}{a^2}$$
$$\overline{\quad -\frac{4r^3}{a}-\frac{3r^4}{a^2} \quad}$$
$$-\frac{4r^3}{a}-\frac{8r^4}{a^2}-\frac{4r^5}{a^3}$$
$$\overline{\quad \frac{5r^4}{a^2}+\frac{4r^5}{a^3} \&c \quad}$$

PROBLEM

PROBLEM II. *To reduce a compound surd into an infinite series.*

RULE.—Extract the root to such degree of exactness as shall be thought necessary.

Extract the square root of a^2+r^2 in an infinite series.

$$a^2+r^2\left(a+\frac{r^2}{2a}-\frac{r^4}{8a^3}+\frac{r^6}{16a^5}-\frac{5r^8}{128a^7},\ \text{\&c.}\right.$$

$$a^2$$

$$2a+\frac{r^2}{2a}\Big)\,r^2$$

$$r^2+\frac{r^4}{4a^2}$$

$$2a+\frac{r^2}{a}-\frac{r^4}{8a^3}\Big)-\frac{r^4}{4a^2}$$

$$-\frac{r^4}{4a^2}-\frac{r^6}{8a^4}+\frac{r^8}{64a^6}$$

$$2a+\frac{r^2}{a}-\frac{r^4}{4a^3},\ \text{\&c.}\Big)\frac{r^6}{8a^4}-\frac{r^8}{64a^6}$$

$$\frac{r^6}{8a^4}+\frac{r^8}{16a^6},\ \text{\&c.}$$

$$\frac{5r^8}{64a^6},\ \text{\&c.}$$

ARITHMETICAL PROPORTION.

A Series in Arithmetical Proportion is thus expressed, $a, a+b, a+2b, a+3b, a+4b$, &c. Here the common difference is b. See page 218, &c.

Note. The most useful part of Arithmetical Proportion is contained in the 1st, 3d, and 4th, Theorems.

GEOMETRICAL PROPORTION.

A Series in Geometrical Proportion is thus expressed, a, ar, ar^2, ar^3, ar^4, &c. Here r is the ratio. See page 235, &c.

Note. The most useful part of Geometrical Proportion is contained in the 1st, 3d, 5th and 8th, Theorems.

SIMPLE EQUATIONS.

An *Equation* is the comparing of two equal quantities which are differently expressed, together, by means of the sign $=$ placed between them.

Thus $12-7=5$ is an equation, expressing the equality of the quantities $12-7$ and 5.

A

A Simple Equation is that which contains only one unknown quantity, without including its power. Thus $r-a+b=c$ is a ſimple equation, containing only the unknown quantity r.

Reduction of Equations is the method of finding the value of the unknown quantity; which is ſhewn in the following rules.

RULE 1.—Any quantity may be tranſpoſed from one ſide of the equation to the other, by changing its ſign.

Thus, if $r+3=7$, then will $r=7-3=4$. And, if $r-4+6=8$, then will $r=8+4-6=6$. Alſo, if $r-a+b=c-d$, then will $r=c-d+a-b$. And, if $4r-8=3r+20$, then will $4r-3r=20+8$, or $r=28$.

RULE 2.—If the unknown term be multiplied by any quantity, it may be taken away by dividing all the other terms of the equation by it.

Thus, if $ar=ab-a$, then will $r=b-1$. If $2r+4=16$, then will $r+2=8$. and $r=8-2=6$. Alſo, if $ar+2ba=3c^2$, then will $r+2b=\frac{3c^2}{a}$, and $r=\frac{3c^2}{a}-2b$.

RULE 3.—If the unknown term be divided by any quantity, it may be taken away by multiplying all the other terms of the equation by it.

Thus, if $\frac{r}{2}=5+3$, then will $r=10+6=16$. If $\frac{r}{a}=b+c-d$, then will $r=ab+ac-ad$. Alſo, if $\frac{2r}{3}-2=6+4$, then will $2r-6=18+12$, and $2r=18+12+6=36$, or $r=\frac{36}{2}=18$.

RULE 4.—The unknown quantity in any equation may be made free from ſurds, by tranſpoſing the reſt of the terms according to the rule, and then involving each ſide to ſuch a power as is denoted by the index of the ſaid ſurd.

Thus, if $\sqrt{r}-2=6$, then will $\sqrt{r}=6+2=8$, and $r=8^2=64$. If $\sqrt{4r+16}=12$, then will $4r+16=144$, and $4r=144-16=128$, or $r=\frac{128}{4}=32$. Alſo, if $\sqrt[3]{2r+3}+4=8$; then will $\sqrt[3]{2r+3}=8-4=4$, and $2r+3=4^3=64$, and $2r=64-3=61$, or $r=\frac{61}{2}=30\frac{1}{2}$.

RULE 5.—If that ſide of the equation, which contains the unknown quantity, be a complete power, it may be reduced by extracting the root of ſaid power from both ſides of the equation.

Thus, if $r^2+6r+9=25$, then will $r+3=\sqrt{25}=5$, or $r=5-3=2$. If $3r^2-9=21+3$, then will $3r^2=21+3+9=33$,

and $r^2 = \frac{33}{3} = 11$, or $r = \sqrt{11}$. Alſo, if $\frac{2r^2}{3} + 10 = 20$, then will $2r^2 + 30 = 60$, and $r^2 + 15 = 30$, or $r^2 = 30 - 15$, or $r = \sqrt{15}$.

Rule 6.—Any analogy, or proportion, may be converted into an equation, by making the product of the two mean terms equal to that of the two extremes.

Thus, if $3r : 16 :: 5 : 10$, then will $3r \times 10 = 16 \times 5$, and $30r = 80$, or $r = \frac{80}{30} = 2\frac{2}{3}$. If $\frac{2r}{3} : a :: b : c$, then will $\frac{2cr}{3} = ab$, and $2cr = 3ab$, or $r = \frac{3ab}{2c}$. Alſo, if $12 - r : \frac{r}{2} :: 4 : 1$, then will $12 - r = \frac{4r}{2} = 2r$, and $2r + r = 12$, or $r = \frac{12}{3} = 4$.

Rule 7.—If any quantity be found on both ſides of the equation with the ſame ſign, it may be taken away from them both; and if every term in an equation be multiplied or divided by the ſame quantity, it may be ſtruck out of them all.

Thus, if $4r + a = b + a$, then will $4r = b$, and $r = \frac{b}{4}$. If $3ar + 5ab = 8ac$, then will $3r + 5b = 8c$, and $r = \frac{8c - 5b}{3}$. Alſo, if $\frac{2r}{3} - \frac{8}{3} = \frac{16}{3} - \frac{8}{3}$, then will $2r = 16$, and $r = 8$.

Miscellaneous Examples.

1. Given $5r - 15 = 2r + 6$, to find the value of r.

Firſt, $5r - 2r = 6 + 15$, then $3r = 21$, and $r = \frac{21}{3} = 7$.

2. Given $40 - 6r - 16 = 120 - 14r$ to find r.

Firſt, $14r - 6r = 120 - 40 + 16$, then $8r = 96$, therefore $r = \frac{96}{8} = 12$.

3. Given $5ar - 3b = 2dr + c$, to find r.

Firſt, $5ar - 2dr = c + 3b$, or $\overline{5a - 2d} \times r = c + 3b$, therefore $r = \frac{c + 3b}{5a - 2d}$.

4. Given $3r^2 - 10r = 8a + r^2$, to find r.

Firſt, $3r - 10 = 8 + r$, then $3r - r = 8 + 10$, therefore $2r = 18$, and $r = \frac{18}{2} = 9$.

5. Given $6ar^3 - 12abr^2 = 3ar^3 + 6ar^2$, to find r.

Firſt, dividing the whole by $3ar^2$, we ſhall have $2r - 4b = r + 2$, then $2r - r = 2 + 4b$, whence $r = 2 + 4b$.

6. Given

6. Given $\frac{r}{2} - \frac{r}{3} + \frac{r}{4} = 10$, to find r.

First, $r - \frac{2r}{3} + \frac{2r}{4} = 20$, then $3r - 2r + \frac{6r}{4} = 60$, and $12r - 8r + 6r = 240$, therefore $10r = 240$, and $r = \frac{240}{10} = 24$.

7. Given $\frac{r-3}{2} + \frac{r}{3} = 20 - \frac{r+19}{2}$, to find r.

First, $r - 3 + \frac{2r}{3} = 40 - r - 19$, then $3r - 9 + 2r = 120 - 3r - 57$, therefore $3r + 2r + 3r = 120 - 57 + 9$, that is, $8r = 72$, or $r = \frac{72}{8} = 9$.

8. Given $\sqrt{\frac{2}{3}r + 5} = 7$, to find r.

First, $\sqrt{\frac{2}{3}r} = 7 - 5 = 2$, then $\frac{2}{3}r = 2^2 = 4$, and $2r = 12$, or $r = \frac{12}{2} = 6$.

9. Given $r + \sqrt{a^2 + r^2} = \frac{2a^2}{\sqrt{a^2 + r^2}}$, to find r.

First, $r\sqrt{a^2 + r^2} + a^2 + r^2 = 2a^2$, then $r\sqrt{a^2 + r^2} = a^2 - r^2$, and $r^2 \times \overline{a^2 + r^2} = \overline{a^2 - r^2}|^2 = a^4 - 2a^2r^2 + r^4$, or $a^2r^2 + r^4 = a^4 - 2a^2r^2 + r^4$, whence $a^2r^2 + 2a^2r^2 = a^4$, or $3a^2r^2 = a^4$, consequently $r^2 = \frac{a^4}{3a^2}$, and $r = \sqrt{\frac{a^4}{3a^2}} = a\sqrt{\frac{1}{3}}$.

PROBLEM I. *To exterminate two unknown quantities, or to reduce the two simple equations, containing them, to a single one.*

RULE 1st.—1. Observe which of the unknown quantities is the least involved, and find its value in each of the equations, by the methods already explained.

2. Let the two values thus found be made equal to each other, and there will arise a new equation with only one unknown quantity in it, whose value may be found as before.

1. Given $\left\{\begin{array}{l} 2r + 3u = 23 \\ 5r - 2u = 10 \end{array}\right\}$ to find r and u.

From the first equation, $r = \frac{23 - 3u}{2}$, and from the second, $r = \frac{10 + 2u}{5}$, and consequently $\frac{23 - 3u}{2} = \frac{10 + 2u}{5}$, or $115 - 15u = 20 + 4u$, or $19u = 115 - 20 = 95$, and $u = \frac{95}{19} = 5$, whence $r = \frac{23 - 15}{2} = 4$.

2. Given

2. Given $\left\{\begin{array}{l} r+u=a \\ r-u=b \end{array}\right\}$ to find r and u.

From the firſt equation, $r=a-u$, and from the ſecond, $r=b+u$, therefore $a-u=b+u$, or $2u=a-b$, conſequently $u=\frac{a-b}{2}$, and $r=a-u=a-\frac{a-b}{2}=\frac{a+b}{2}$.

3. Given $\left\{\begin{array}{l} \frac{r}{2}+\frac{u}{3}=7 \\ \frac{r}{3}+\frac{u}{2}=8 \end{array}\right\}$ to find r and u.

From the firſt equation, $r=14-\frac{2u}{3}$, and from the ſecond, $r=24-\frac{3u}{2}$, therefore $14-\frac{2u}{3}=24-\frac{3u}{2}$, and $42-2u=72-\frac{9u}{2}$, or $84-4u=144-9u$; whence $5u=144-84=60$, and $u=\frac{60}{5}=12$, and $r=14-\frac{2u}{3}=14-\frac{24}{3}=6$.

Rule 2d.—1. Conſider which of the unknown quantities you would firſt exterminate, and let its value be found in that equation where it is the leaſt involved.

2. Subſtitute the value, thus found, for its equal in the other equation, and there will ariſe a new equation, with only one unknown quantity, whoſe value may be found as before.

1. Given $\left\{\begin{array}{l} r+2u=17 \\ 3r-u=2 \end{array}\right\}$ to find r and u.

From the firſt equation, $r=17-2u$, and this value, ſubſtituted for r in the ſecond, gives $\overline{17-2u}\times 3-u=2$, or $51-6u-u=2$, or, $51-7u=2$; that is, $7u=51-2=49$; whence $u=\frac{49}{7}=7$, and $r=17-2u=17-14=3$.

2. Given $\left\{\begin{array}{l} a : b :: r : u \\ r^2+u^2=c \end{array}\right\}$ to find r and u.

The firſt analogy, turned into an equation, is $br=au$, or $r=\frac{au}{b}$, and this value of r, ſubſtituted in the ſecond, gives $\left.\frac{au}{b}\right|^2+u^2=c$, or $\frac{a^2u^2}{b^2}+u^2=c$, or $a^2u^2+b^2u^2=cb^2$, or $u^2=\frac{cb^2}{a^2+b^2}$, therefore $u=\left.\overline{\frac{cb^2}{a^2+b^2}}\right|^{\frac{1}{2}}$, and $r=\left.\overline{\frac{ca^2}{a^2+b^2}}\right|^{\frac{1}{2}}$.

Rule

RULE 3.—Let the given equations be multiplied or divided by ſuch numbers or quantities as will make the term, which contains one of the unknown quantities, to be the ſame in both equations, and then by adding or ſubtracting the equations, accordingly as is required, there will ariſe a new equation with only one unknown quantity, as before.

1. Given $\left\{\begin{array}{r} 3r+5u=40 \\ r+2u=14 \end{array}\right\}$ to find r and u.

Firſt, multiply the 2d equation by 3, and we ſhall have $3r+6u=42$, then from this laſt equation ſubtract the firſt, and it will give $6u-5u=42-40$, or $u=2$, therefore $r=14-2u=14-4=10$.

2. Given $\left\{\begin{array}{r} 5r-3u=9 \\ 2r+5u=16 \end{array}\right\}$ to find r and u.

Let the firſt equation be multiplied by 2, and the 2d by 5, and we ſhall have $\left\{\begin{array}{r} 10r-6u=18 \\ 10r+25u=80 \end{array}\right\}$ and if the former of theſe be ſubtracted from the latter, it will give $31u=62$, or $u=\frac{62}{31}=2$, conſequently, $r=\frac{9+6}{5}=\frac{15}{5}=3$.

Another Method.

Multiply the 1ſt equation by 5, and the 2d by 3, and we ſhall have $\left\{\begin{array}{r} 25r-15u=45 \\ 6r+15u=48 \end{array}\right\}$ Now, add theſe two equations, and it will give $31r=93$, or $r=\frac{93}{31}=3$, conſequently $u=\frac{16-6}{5}=\frac{10}{5}=2$, as before.

PROB. II. *To exterminate three unknown quantities, or to reduce the three ſimple equations, containing them, to a ſingle one.*

RULE 1. Let r, u, and z, be the three unknown quantities to be exterminated.

2. Find the value of r, from each of the three given equations.

3. Compare the firſt value of r with the ſecond, and an equation will ariſe, involving only u and z.

4. Compare the firſt value of r with the third, and another equation will ariſe, involving only u and z.

5. Find the value of u and z from theſe two equations, according to the former rules, and r, u, and z, will be exterminated as required.

Note. Any number of unknown quantities may be exterminated in nearly the ſame manner.

1. Given $\left\{\begin{array}{l} r+u+z=29 \\ r+2u+3z=62 \\ \frac{r}{2}+\frac{u}{3}+\frac{z}{4}=10 \end{array}\right\}$ to find r, u, and z.

From

From the first equation, $r = 29 - u - z$. From the 2d $r = 62 - 2u - 3z$. From the 3d $r = 20 - \frac{2u}{3} - \frac{z}{2}$, whence $29 - u - z = 62 - 2u - 3z$, and $29 - u - z = 20 - \frac{2u}{3} - \frac{z}{2}$; but from the first of these equations, $u = 62 - 29 - 2z = 33 - 2z$; and from the 2d $u = 27 - \frac{3z}{2}$, therefore $33 - 2z = 27 - \frac{3z}{2}$, or $z = 12$, and $u = 62 - 29 - 2z = 62 - 29 - 24 = 9$, and $r = 29 - u - z = 29 - 12 - 9 = 8$.

2. Given $\left\{\begin{array}{l} \frac{r}{2} + \frac{u}{3} + \frac{z}{4} = 62 \\ \frac{r}{3} + \frac{u}{4} + \frac{z}{5} = 47 \\ \frac{r}{4} + \frac{u}{5} + \frac{z}{6} = 38 \end{array}\right\}$ to find r, u, and z.

First, the given equations, cleared from fractions, become

$$12r + 8u + 6z = 1488$$
$$20u + 15u + 12z = 2820$$
$$30r + 24u + 20z = 4560$$

Then, if the 2d equation be subtracted from double the 1st, and three times the 3d from five times the 2d, we shall have

$$4r + u = 156$$
$$10r + 3u = 420.$$

And again, if the second of these be subtracted from three times the first, it will give $12r - 10r = 468 - 420$, or $r = \frac{48}{2} = 24$, therefore $u = 156 - 4r = 60$, and $z = \frac{1488 - 8u - 12r}{6} = 120$.

Questions producing simple Equations.

1. To find two such numbers, as that their sum shall be 40, and their difference 16.

Let r denote the least of the two numbers required, then will $r + 16 =$ the greater, $r + r + 16 = 40$ by the question, that is, $2r = 40 - 16 = 24$, or $r = \frac{24}{2} = 12 =$ least number, and $r + 16 = 12 + 16 = 28 =$ greater number required.

2. What

2. What number is that, whoſe $\frac{1}{3}$ part exceeds its $\frac{1}{4}$ part by 16?

Let $r=$ number required, then will its $\frac{1}{3}$ part be $\frac{r}{3}$, and its $\frac{1}{4}$ part $\frac{r}{4}$; therefore $\frac{r}{3}-\frac{r}{4}=16$, by the queſtion, that is $r-\frac{3r}{4}=48$, or $4r-3r=192$; whence $r=192$ the number required.

3. Divide £1000 between A, B and C, ſo that A ſhall have £72 more than B, and C, £100 more than A.

Let $r=$ B's ſhare of the given ſum, then will $r+72=$ A's ſhare, and $r+172=$ C's ſhare; and the ſum of all theſe ſhares $r+r+72+r+172$, or $3r+244=1000$, by the queſtion, that is, $3r=1000-244=756$, or $r=\frac{756}{3}=£252=$ B's ſhare, and $r+72=252+72=£324=$ A's ſhare, and $r+172=252+172=£424=$ C's ſhare.

Proof, $252+324+424=£1000$.

4. A prize of £1000 is to be divided between two perſons, whoſe ſhares therein are in the proportion of 7 to 9; Required the ſhare of each?

Let $r=$ the firſt perſon's ſhare, then will $£1000-r=$ 2d perſon's ſhare, and $r:1000-r::7:9$, by the queſtion, that is, $9r=\overline{1000-r}\times7=7000-7r$, or $16r=7000$, whence $r=\frac{7000}{16}=£437\ 10s.=$ 1ſt ſhare, and $1000-r=1000-£437\ 10s.=£562\ 10s.=$ 2d ſhare.

5. The paving of a ſquare at 2*s.* per yard, coſt as much as the incloſing of it, at 5*s.* per yard; Required the ſide of the ſquare?

Let $r=$ ſide of the ſquare ſought, then $4r=$ yards of incloſure, and $r^2=$ yards of pavement; whence $4r\times5=20r=$ price of incloſing, and $r^2\times2=2r^2=$ price of paving. But $2r^2=20r$, by the queſtion, therefore $r^2=10r$, and $r=10=$ length of the ſide required.

6. A labourer engaged to ſerve 40 days upon theſe conditions, that for every day he worked he ſhould receive 20*d.* but for every day he was abſent, he was to forfeit 8*d.* Now, at the end of the time, there was due to him £1 11*s.* 8*d.*; How many days did he work, and how many was he abſent?

Let r be the number of days he worked, then will $40-r$ be the number of days he was abſent; alſo, $r\times20=20r=$ ſum earned, and $\overline{40-r}\times8=320-8r=$ ſum forfeited; whence $20r-\overline{320-8r}=380d.$ ($=£1\ 11s.\ 8d.$) by the queſtion, that is, $20r-320+8r=380$, or $28r=380+320=700$, and $r=\frac{700}{28}=25=$ number of days he worked; and $40-r=40-25=15=$ number of days he was abſent.

7. Out

7. Out of a cask of wine, which had leaked away $\frac{1}{3}$, 21 gallons were drawn; and then, being gauged, it appeared to be half full; How much did it hold?

Let it be supposed to have holden r gallons, then it would have leaked $\frac{r}{3}$ gallons, and consequently there had been taken away $21+\frac{r}{3}$ gallons. But $21+\frac{r}{3}=\frac{r}{2}$ by the question, that is, $63+r=\frac{3r}{2}$, or $126+2r=3r$, hence $3r-2r=126$, or $r=126$, *Answer.*

8. What fraction is that, to the numerator of which if 1 be added, the value will be $\frac{1}{3}$; but if one be added to the denominator, its value will be $\frac{1}{4}$?

Let the fraction be represented by $\frac{r}{u}$ then will $\frac{r+1}{u}=\frac{1}{3}$ and $\frac{r}{u+1}=\frac{1}{4}$, or $3r+3=u$, and $4r=u+1$; hence $4r-3r-3=u+1-u$, that is, $r-3=1$, or $r=4$, and $u=3r+3=12+3=15$. So that $\frac{4}{15}=$ fraction required.

9. A market woman bought a certain number of eggs, at 2 for a cent, and as many, at 3 for a cent, and sold them all out again, at the rate of 5 for 2 cents, and, by so doing, lost 4 cents; What number of eggs had she?

Let $r=$ number of eggs of each sort, then will $\frac{r}{2}=$ price of the 1st sort, and $\frac{r}{3}=$ price of the 2d sort. But $5:2::2r$ (the whole number of eggs) $:\frac{4r}{5}$; therefore $\frac{4r}{5}=$ price of both sorts together, at 5 for 2 cents, and $\frac{r}{2}+\frac{r}{3}-\frac{4r}{5}=4$, by the question; that is, $r+\frac{2r}{3}-\frac{8r}{5}=8$; or $3r+2r-\frac{24r}{5}=24$; or $15r+10r-24r=120$; whence $r=120=$ number of eggs of each sort required.

10. A person in the afternoon being asked what o'clock it was, answered, that $\frac{3}{5}$ of the time from noon was equal to $\frac{5}{8}$ of the time to midnight; Required the time?

Let $r=$ the time sought from noon, then will $12-r=$ the time to midnight, $\frac{3}{5}$ of the time from noon $=\frac{3r}{5}$, and $\frac{5}{8}$ of the time to midnight $=\frac{60-5r}{8}$, therefore $\frac{3r}{5}=\frac{60-5r}{8}$ by the question; whence, $3r=\frac{300-25r}{8}$ and $24r=300-25r$, or $24r+25r=300$ or $r=\frac{300}{49}=6h.\ 7'\ 20''\frac{40}{49}$, *Answer.*

11. A

11. A merchant ſhips goods for Southcarolina to the amount of £700; What ſum, at 5 per cent. ſhould he get inſured, to cover his adventure?

Let $r =$ ſum to be inſured, then will $r - \frac{5r}{100} = 700$, whence $100r - 5r = 70000$, and $r = \frac{70000}{95} = £736$ 16s. $10\frac{10}{95}$d. *Anſ.*

12. A man lays out 30 cents for apples and pears, buying his apples, at 4, and his pears, at 5 for a cent, and afterwards ſold $\frac{1}{2}$ of his apples, and $\frac{1}{3}$ of his pears for 13 cents, which was the prime coſt; I demand the number he bought of each?

Let $r =$ the number of apples, and $z =$ the number of pears; then, if 4 apples coſt a cent, r will coſt $\frac{r}{4}$ cents, and for the ſame reaſon z will coſt $\frac{z}{5}$ cents, and we ſhall have $\frac{r}{4} + \frac{z}{5} = 30$, for one fundamental equation. Again, the price of $\frac{r}{2} = \frac{1}{2}$ of his apples will be $\frac{r}{8}$, and the price of $\frac{z}{3} = \frac{1}{3}$ of his pears will be $\frac{z}{15}$; hence $\frac{r}{8} + \frac{z}{15} = 13$, for another fundamental equation: Now, croſs multiplying $\frac{r}{4} + \frac{z}{5} = 30$, and then multiplying 30 by 4 and 5 we ſhall have the firſt equation $= 5r + 4z = 600$; and doing the ſame by $\frac{r}{8} + \frac{z}{15} = 13$, we have the 2d equation $= 15r + 8z = 1560$. Subtract the 2d equation from three times the 1ſt, and we ſhall have the 3d equation $= 4z = 240$. therefore 4th equation $= z = 60 =$ the number of pears: Now, ſubſtitute 60 for z, that is, 240 for $4z$, in the 1ſt equation. $5r + 4z = 600$, we ſhall have $5r + 240 = 600$, whence, equation 5th $r = 72 =$ the number of apples.

QUADRATIC EQUATIONS.

A Simple Quadratic Equation is that which involves the ſquare of the unknown quantity only.

An Adfected Quadratic Equation is that which involves the ſquare of the unknown quantity, together with the product, which ariſes from multiplying it by ſome known quantity.

Thus, $ar^2 = b$, is a ſimple quadratic equation, and $ar^2 + br = c$ is an adfected quadratic equation.

All adfected quadratic equations fall under the three following forms.

1ſt. $r^2 + ar = b$. 2d. $r^2 - ar = b$. 3d. $r^2 - ar = -b$. And the rule for finding the value of r, in each of theſe equations, is as follows:

RULE* 1.—Tranſpoſe all the terms, which involve the unknown quantity, to one ſide of the equation, and the known terms to the other ſide, and let them be ranged according to their dimenſions.

2. When

* *The ſquare root of any quantity may be either* $+$ *or* $-$, *and therefore all quadratic equations admit of two ſolutions. Thus the ſquare root of* $+n^2$ *is* $+n$, *or* $-n$, *for* $+n\times+n$, *or* $-n\times-n$, *are each equal to* $+n^2$. *So, in the firſt form, where* $r+\frac{a}{2}$ *is found* $=\sqrt{b+\frac{a^2}{4}}$, *the root may be either* $+\sqrt{b+\frac{a^2}{4}}$: *or* $-\sqrt{b+\frac{a^2}{4}}$, *ſince either of them being multiplied by itſelf will produce* $b+\frac{a^2}{4}$. *And this ambiguity is expreſſed by writing the uncertain ſign* $\pm$ *before* $\sqrt{b+\frac{a^2}{4}}$; *thus* $r=\pm\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$. *In the firſt form, where* $r=\pm\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$ *the firſt value of r, viz.* $r=+\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$ *is always affirmative.*

The ſecond value, viz. $r=-\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$, *will always be negative, becauſe it is compoſed of two negative terms; therefore, when* $r^2+ar=b$, *we ſhall have* $r=+\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$ *for the affirmative value of r, and* $r=-\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$ *for the negative value of r.*

In the ſecond form, where $r=\pm\sqrt{b+\frac{a^2}{4}}+\frac{a}{2}$, *the firſt value, viz.* $r=+\sqrt{b+\frac{a^2}{4}}+\frac{a}{2}$ *is always affirmative, ſince it is compoſed of two affirmative terms, and the ſecond value, viz.* $r=-\sqrt{b+\frac{a^2}{4}}+\frac{a}{2}$ *will always be negative; therefore when* $r^2-ar=b$, *we ſhall have* $r=+\sqrt{b+\frac{a^2}{4}}+\frac{a}{2}$, *for the affirmative value of r, and* $r=-\sqrt{b+\frac{a^2}{4}}+\frac{a}{2}$, *for the negative value of r.*

In

2. When the ſquare of the unknown quantity has any coefficient prefixed to it, let all the reſt of the terms be divided by that coefficient.

3. Add the ſquare of half the coefficient of the ſecond term to both ſides of the equation, and that ſide, which involves the unknown quantity, will be a complete ſquare.

4. Extract the ſquare root from both ſides of the equation, and the value of the unknown quantity will be determined.

Note, 1. The ſquare root of one ſide of the equation is always equal to the unknown quantity, with half the coefficient of the ſecond term ſubjoined to it.

2. All equations, wherein there are two terms involving the unknown quantity, and the index of one is juſt double that of the other, are ſolved like quadratics, by completing the ſquare.

Thus, $r^4+ar^2=b$, or $r^n+ar^{\frac{n}{2}}=b$, are the ſame as quadratics, and the value of the unknown quantity may be determined accordingly.

From this rule may be formed a general theorem with which all particular equations may be compared, and by means whereof they may be more readily reſolved.

Suppoſe $ar^2=br+c$ be the general quadratic equation propoſed to be reſolved; where a, b, and c denote known integral quantities, whether affirmative, or negative, and $r=$ the unknown quantity; to find the values of r in this equation.

Here, tranſpoſing br, we have $ar^2-br=c$, then dividing by a, in order to free r^2 the higheſt power of r from its coefficient, we have $r^2-\frac{br}{a}=\frac{c}{a}$; this being done, the firſt ſide, $r^2-\frac{br}{a}$ may be conſidered as an imperfect ſquare raiſed from a binomial root, and accordingly we may complete that ſquare by adding the ſquare of

In the third form, where $r=\sqrt{\frac{a^2}{4}-b}+\frac{a}{2}$, both the values of r will be poſitive, ſuppoſing $\frac{a^2}{4}$ to be greater than b. Therefore when $r^2-ar=-b$, we ſhall have $r=+\sqrt{\frac{a^2}{4}-b}+\frac{a}{2}$, and $-\sqrt{\frac{a^2}{4}-b}+\frac{a}{2}$, both, for the affirmative value of r.

But in this third form, if b be greater than $\frac{a^2}{4}$, the ſolution of the propoſed queſtion will be impoſſible. For ſince the ſquare of any quantity is always affirmative, the ſquare root of a negative quantity is impoſſible.

of half the coefficient of the ſecond term: But if $\frac{bb}{4a^2}$ muſt be added to the firſt ſide of the equation, to complete the ſquare, it muſt be alſo added to the other ſide, to preſerve the equality, otherwiſe by an unequal addition, the equation would be deſtroyed; this equal addition therefore being made, the equation will ſtand thus, $r^2 - \frac{br}{a} + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} + \frac{c}{a}$; but the two fractions $\frac{b^2}{4a^2}$ and $\frac{c}{a}$ when added, give $\frac{ab^2+4a^2c}{4a^3}$, which divided by a, gives $\frac{b^2+4ac}{4a^2}$; therefore $r^2 - \frac{br}{a} + \frac{b^2}{4a^2} = \frac{b^2+4ac}{4a^2}$; therefore the ſquare root of one ſide will be equal to the ſquare root of the other; but the ſquare root of $\frac{b^2+4ac}{4a^2}$, as it here ſtands in letters, cannot be extracted, becauſe, although the denominator $4a^2$ be a ſquare, yet there is no literal quantity whatever, which, being multiplied into itſelf, will produce b^2+4ac, therefore, to put this numerator into the form of a ſquare, let us ſuppoſe $b^2+4ac = ss$, and then the equation will be $r^2 - \frac{br}{a} + \frac{b^2}{4a^2} = \frac{ss}{4a^2}$; but the ſquare root of $r^2 - \frac{br}{a} + \frac{b^2}{4a^2}$ is $r - \frac{b}{2a}$, and that of $\frac{ss}{4a^2}$ is $\pm\frac{s}{2a}$, therefore this equation will now be reduced to a ſimple one, and will ſtand thus, $r - \frac{b}{2a} = \pm\frac{s}{2a}$, therefore $r = \frac{b \pm s}{2a}$, that is, $r = \frac{b+s}{2a}$ and $r = \frac{b-s}{2a}$.

Note. When the quantity c (and conſequently $4ac$) is negative, the quantity ss, or b^2+4ac muſt be conſidered as the ſum of the affirmative quantity b^2 and the negative one $4ac$, when added together according to the common rules of Addition.

Examples of the reſolution of Adfected Equations with and without the general Theorem.

1. Given $r^2 = 140 - 4r$, to find the values of r.

Firſt, tranſpoſing $-4r$, it is $r^2+4r = 140$, then, $r^2+4r+4 = 144$ by completing the ſquare; then $\sqrt{r^2+4r+4} = \sqrt{144}$, by extracting the root; or $r+2 = \pm 12$, that is, $r = -2 \pm 12 = +10$, or -14.

By the general Theorem. a, in the general theorem, anſwers to 1 in the particular one, that is, to the coefficient of r^2, b anſwers to 4, and c, to 140, that is, $a = 1$, $b = -4$, $c = 140$, and $4ac = 560$, therefore ss, or b^2+4ac will be the ſum of 16 and $560 = 576$, therefore

therefore, $s = 24$, $\frac{b+s}{2a} = \frac{-4+24}{2} = +10$, and $\frac{b-s}{2a} = \frac{-4-24}{2} = -14$; therefore the two roots of this equation are 10 and -14.

2. Given $r^2+8 = 6r+80$, to find r.

First, $r^2-6r = 72$, by transposition; then $r^2-6r+9 = 72+9 = 81$, by completing the square, and $r-3 = \sqrt{81} = \pm 9$, by extracting the root, therefore $r = +3 \pm 9 = +12$, or -6.

By the Theorem. $a = 1$, $b = 6$, $c = 72$, and $4ac = 288$, therefore $ss = 36+288 = 324$, therefore $s = 18$, $\frac{b+s}{2a} = 12$, and $\frac{b-s}{2a} = -6$.

3. Given $2r^2-20 = 70-8r$ to find r.

First, $2r^2+8r = 70+20 = 90$, by transposition, then $r^2+4r = 45$, by dividing by the coefficient 2, and $r^2+4r+4 = 45+4 = 49$, by completing the square; whence $r+2 = \sqrt{49} = \pm 7$, therefore, $r = -2 \pm 7 = 5$ or -9.

By the Theorem. $a = 2$, $b = -8$, $c = 90$, $4ac = 720$, $ss = 64+720 = 784$, therefore $s = 28$, $\frac{b+s}{2a} = +5$, $\frac{b-s}{2a} = -9$, so that $+5$, and -9 are the values of r.*

As the general Theorem is sufficiently exemplified in the preceding problems, the following equations will be solved by the Rule only.

4. Given $3r^2-3r+6 = 5\frac{1}{3}$, to find r.

Here $r^2-r+2 = 1\frac{7}{9}$ by dividing by 3, and $r^2-r = 1\frac{7}{9}-2$, by transposition; also $r^2-r+\frac{1}{4} = 1\frac{7}{9}-2+\frac{1}{4} = \frac{1}{36}$ by completing the square; and $r-\frac{1}{2} = \sqrt{\frac{1}{36}} = \pm\frac{1}{6}$, by evolution; therefore $r = +\frac{1}{2} \pm \frac{1}{6} = \frac{2}{3}$, or $\frac{1}{3}$.

5. Given $\frac{r^2}{2} - \frac{r}{3} + 20\frac{1}{2} = 42\frac{2}{3}$, to find r.

Here $\frac{r^2}{2} - \frac{r}{3} = 42\frac{2}{3} - 20\frac{1}{2} = 22\frac{1}{6}$, by transposition, and $r^2 - \frac{2r}{3} = 44\frac{1}{3}$, by dividing by $\frac{1}{2}$, whence $r^2 - \frac{2r}{3} + \frac{1}{9} = 44\frac{1}{3}+\frac{1}{9} = 44\frac{4}{9}$, by completing the square, and $r-\frac{1}{3} = \sqrt{44\frac{4}{9}} = \pm 6\frac{2}{3}$, therefore $r = +\frac{1}{3} \pm 6\frac{2}{3} = 7$, or $-6\frac{1}{3}$.

6. Given

* In a quadratic equation pf this form $ar^2 = br+c$, the sum of the roots will always be $\frac{b}{a}$, and the product of their multiplication $\frac{-c}{a}$; therefore, if $a = 1$, that is, if the equation be $r^2 = br+c$, the sum of the roots will be b, and their product $-c$, or the sum will be the coefficient of the unknown quantity on the second side of the equation, and their product, what is called the absolute term, with its sign changed.

6. Given $ar^2+br=c$, to find r.

First, $r^2+\frac{b}{a}r=\frac{c}{a}$, by division; then $r^2+\frac{b}{a}r+\frac{b^2}{4a^2}=\frac{c}{a}+\frac{b^2}{4a^2}$, by completing the square; and $r+\frac{b}{2a}=\sqrt{\frac{c}{a}+\frac{b^2}{4a^2}}=\sqrt{\frac{4ac+b^2}{4a^2}}$, by evolution, therefore $r=\pm\sqrt{\frac{4ac+b^2}{4a^2}}-\frac{b}{2a}$.

7. Given $ar^2-br+c=d$, to find r.

Here, $ar^2-br=d-c$, by transposition, and $r^2-\frac{b}{a}r=\frac{d-c}{a}$ by division.

Also, $r^2-\frac{b}{a}r+\frac{b^2}{4a^2}=\frac{d-c}{a}+\frac{b^2}{4a^2}$ by completing the square; and $r-\frac{b}{2a}=\pm\sqrt{\frac{d-c}{a}+\frac{b^2}{4a^2}}$, by evolution; therefore $r=\frac{b}{2a}\pm\sqrt{\frac{d-c}{a}+\frac{b^2}{4a^2}}$.

8. Given $r^4+2ar^2=b$, to find r.

Here, $r^4+2ar^2+a^2=b+a^2$, by completing the square, and $r^2+a=\sqrt{b+a^2}$, by evolution; whence $r^2=\sqrt{b+a^2}-a$, and consequently $r=\sqrt{:\sqrt{b+a^2}-a}$.

9. Given $ar^n-br^{\frac{n}{2}}-c=-d$, to find r.

First, $ar^n-br^{\frac{n}{2}}=c-d$, by transposition, and $rn-\frac{b}{a}r^{\frac{n}{2}}=\frac{c-d}{a}$, by division. Also, $r^n-\frac{b}{a}r^{\frac{n}{2}}+\frac{b^2}{4a^2}=\frac{c-d}{a}+\frac{b^2}{4a^2}$, by completing the square, and $r^{\frac{n}{2}}-\frac{b}{2a}=\sqrt{\frac{c-d}{a}+\frac{b^2}{4a^2}}$, by evolution; therefore $r^{\frac{n}{2}}=\frac{b}{2a}\pm\sqrt{\frac{c-d}{a}+\frac{b^2}{4a^2}}$ and consequently $r=\overline{\frac{b}{2a}\pm\sqrt{\frac{c-d}{a}+\frac{b^2}{4a^2}}}\Big|^{\frac{2}{n}}$.

QUESTIONS *producing* QUADRATIC EQUATIONS.

1. To find two numbers, whose difference is 8, and product 240.

Let $r=$ the least number, then will $r+8=$ the greater, and $r\times\overline{r+8}=r^2+8r=240$, by the question; whence $r^2+8r+16=240+16=256$ by completing the square; also $r+4=\sqrt{256}=16$, by evolution, and therefore $r=16-4=12=$ the least number, and $12+8=20=$ the greater.

2. To

2. To divide the number 60 into two such parts, as that their product may be 864.

Let $r =$ greater part, then will $60-r =$ the less, and $r \times \overline{60-r} = 60r-r^2 = 864$, by the question, that is, $r^2-60r = -864$; whence $r^2-60r+900 = -864+900 = 36$, by completing the square: Also $r-30 = \sqrt{36} = 6$, by extracting the root; therefore $r = 6+30 = 36 =$ greater part, and $60-r = 60-36 = 24 =$ the less part.

Sold a piece of cloth for 24*l.* and gained as much per cent. as the cloth cost me; What was the price of it?

Let $r =$ pounds the cloth cost, then $24-r =$ whole gain, but $100 : r :: r : 24-r$, by the question, or $r^2 = 100 \times \overline{24-r} = 2400-100r$, that is, $r^2+100r = 2400$; whence $r^2+100r+2500 = 2400+2500 = 4900$, by completing the square, and $r+50 = \sqrt{4900} = 70$, by extraction of roots, consequently $r = 70-50 = 20 =$ price of the cloth.

4. A person bought a number of oxen for 80*l.* and if he had bought 4 more for the same money, he would have paid 1*l.* less for each; How many did he buy?

Suppose he bought r oxen, then $\frac{80}{r} =$ price of each, and $\frac{80}{r+4} =$ price of each, if $r+4$ had cost 80*l.* But $\frac{80}{r} = \frac{80}{r+4} + 1$, by the question, or $80 = \frac{80r}{r+4} + r$, or $80r+320 = 80r+r^2+4r$, that is, $r^2+4r = 320$; whence $r^2+4r+4 = 320+4 = 324$, by completing the square, and $r+2 = \sqrt{324} = 18$, by evolution, consequently $r = 18-2 = 16 =$ number of oxen required.

5. What two numbers are those, whose sum, product and difference of their squares, are all equal to each other?

Let $r =$ greater number, and $u =$ less; then $\left\{\begin{array}{l} r+u = ru \\ r+u = r^2-u^2 \end{array}\right\}$, by the question, and $1 = \frac{r^2-u^2}{r+u} = r-u$, or $r = u+1$, from the 2d equation: Also $u+1+u = \overline{u+1} \times u$, from the first equation; or $2u+1 = u^2+u$, that is, $u^2-u = 1$; whence $u^2-u+\frac{1}{4} = 1\frac{1}{4}$, by completing the square: Also $u-\frac{1}{2} = \sqrt{1\frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$ by evolution, consequently $u = \frac{\sqrt{5}}{2} + \frac{1}{2} = \frac{\sqrt{5}+1}{2}$, and $r = u+1 = \frac{\sqrt{5}+3}{2}$.

6. There are four numbers in Arithmetical Progression whereof the product of the two extremes is 45, and that of the means 77; What are the numbers?

Let

Let $r =$ lefs extreme, and $u =$ common difference, then r, $r+u$, $r+2u$, $r+3u$ will be the 4 numbers, and

$$\left\{\begin{array}{l} r \times \overline{r+3u} = r^2+3ru = 45 \\ \overline{r+u} \times \overline{r+2u} = r^2+3ru+2u^2 = 77 \end{array}\right\} \text{by the queftion;}$$

whence $2u^2 = 77-45 = 32$, and $u^2 = \frac{32}{2} = 16$, by fubtraction and divifion, or $u = \sqrt{16} = 4$ by evolution, therefore $r^2+3ru = r^2+12r = 45$, by the firft equation; alfo $r^2+12r+36 = 45+36 = 81$, by completing the fquare, and $r+6 = \sqrt{81} = 9$, by the extraction of roots, confequently $r = 9-6 = 3$, and the numbers are 3, 7, 11 and 15.

RECAPITULATION *of the* PRINCIPLES *of* ARITHMETIC *and* ALGEBRA.

AXIOM 1. Since whole numbers increafe in a decuple proportion, 10 is the univerfal ratio of any feries of numbers whatever; and the reafon for carrying at 10 in Addition and Multiplication is felfevident, fince 10 in any place to the right is equal to 1 in the next place to the left. Hence alfo the reafon for carrying according to the fubdivifions of any integer when feveral denominations are to be added.

AXIOM 2. If two whole numbers be equally increafed, their difference is always the fame. Hence the reafon of borrowing 10 in one place to the right, and paying it back by carrying one to the next place. Hence likewife the reafon will be evident, for placing the firft figure to the right of the product of every particular multiplier directly below its own multiplier.

AXIOM 3. The multiplicand will be increafed or diminifhed in proportion to the multiplier, when the fame multiplicand is ufed. Hence the reafon why the multiplicand is increafed, when it is multiplied by any thing greater than unity, and decreafed, when it is multiplied by a fraction.

AXIOM 4. The dividend will be increafed or diminifhed in proportion to the divifor, when the fame dividend is ufed. Hence, to divide by any thing greater than unity, will quote a number lefs than the dividend; and, on the contrary, to divide by any thing lefs than unity, will quote a number greater than the dividend.

AXIOM 5. The whole is equal to all its parts taken together. Hence one fum may be made equal to feveral by Addition, and fubtraction may be proved by adding the difference to the leaft given fum.

AXIOM 6. If equal quantities be added to, taken from, multiplied or divided by, equal quantities, the fums, remainders, products, and quotients, will refpectively be equal. Hence the reafon of reducing equations by Addition, Subtraction, Multiplication, and Divifion, and of abridging commenfurable terms, and canceling equal quantities and numbers.

AXIOM

Axiom 7. To multiply, or divide, any quantity or number by other quantities or numbers continually, is the ſame as to multiply by the product of thoſe other numbers. Hence the reaſon of multiplying, or dividing by component parts.

Axiom 8. If four numbers or quantities be proportional, the rectangle or product of the extremes will be equal to the product of the means; and *vice verſa*, if the product of the extremes be equal to that of the means, the numbers or quantities are proportional.

Axiom 9. The quotient of any two ſucceeding powers, when the next higher is divided by the next lower, exhibits the root of theſe powers. On the contrary, if any power be multiplied by the root of that power, the product will be the next higher power of the root: And if a higher power be divided by the root, the quotient will exhibit the next lower power. Again, if a proportional part of a higher power be divided by a proportional part of the next lower power, the quotient will exhibit a proportional part of the root. Hence the firſt figure or figures in the root of any power being raiſed to the power next lower than that whoſe root is wanted, and that power multiplied by a number expreſſing the proportion, which the given power bears to its root, produces a proportional diviſor, whoſe ratio, compared with the dividual, is a proportional part of the root, which being annexed to the former part of the root, and raiſed to the full power of the given number, will be either the whole or a proportional part of the given power, diſcoverable by Subtraction, &c. Hence we have a general rule for extracting the root of any power whatever.

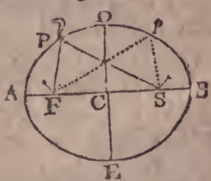
AN
INTRODUCTION
TO
CONIC SECTIONS.

SECTION I.

Of the Ellipsis.

Definition 1.

Fig. 1.



If two pins be fixed at the points F, S; and a thread PSFP, put about them and knotted at P; then if the thread be drawn tight, and the point P and the thread be moved about the fixed centres F, S; the point P will describe the curve PD*p*BEAP, called an *Ellipsis*. See Fig. 1.

Def. 2. The points or centres F, S, are called the *foci.*

Def. 3. The line A,B, drawn through the foci to the curve, is called the *transverse axis.*

Fig. 2.

Def. 4. The point C in the middle of the axis AB, is the *centre.* See Fig. 2.

Def. 5. The line DE, (drawn through the centre C) perpendicular to the transverse AB, is called the *conjugate axis.* See Fig. 2.

Def. 6. Any line TO, drawn through the centre C to the curve, is called a *diameter.* And the extremity T (or O) its *vertex.*

Def. 7. If TO be a diameter, then the diameter GK, drawn parallel to the tangent at its vertex T, is called its *conjugate.* And the two diameters TO, GK, are said to be *conjugates* to one another.

Def. 8. The line LR (drawn through the focus F, perpendicular to the transverse axis AB,) is called the *parameter* or *latus rectum.*

Def.

Def. 9. A line drawn from any point of the curve (as HI) perpendicular to the transverse axis, is called an *ordinate* to the transverse. And, in general, any line drawn from the curve to any diameter TO, parallel to its conjugate GK, (as HN,) is an *ordinate* to that principal diameter TO. If it go quite through the figure, as H*h*, it is called a *double ordinate.*

Def. 10. A right line meeting the ellipsis in one point M, but does not cut it, is called a *tangent* to it in that point, as TM.

Def. 11. The part of the diameter between the vertex and the ordinate, is called the *abscissa*, TN, AI. And the *vertex* is the extremity of any diameter.

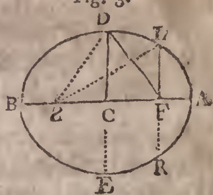
PROPOSITION I. The sum of the lines FP, SP drawn from the foci, to any point of the curve, is equal to the transverse axis AB. See Fig. 1.

For by construction, $PF+PS = AF+AS = AF+AF+FS = 2AF+FS$. And the same $PF+PS = 2BS+FS$; therefore $2AF+FS = 2BS+FS$, and $2AF = 2BS$, or $AF = BS$. Whence $PF+PS = 2AF+FS = AF+BS+FS = AB$.

COR. The two foci are equally distant from the vertices, and also from the centre: $AF = BS$; and $FC = SC$. For it is proved that $AF = BS$; and since $AC = CB$ (Def. 4,) therefore $AC-AF = CB-BS$, or $FC = SC$.

PROP. II. A line drawn from the end of the conjugate axis, to the focus, is equal to half the transverse; $DF = CA$. See Fig. 3.

Fig. 3.



Draw DS to the other focus. Then the two right angled triangles CDF and CDS are similar and equal. For $SC = CF$, the angles at C are right, and CD common: therefore $SD = DF$; and since the sum $SD+DF =$ the transverse (Prop. 1,) and of them $DF =$ half the transverse CA.

COR. The distance of the foci is a mean proportional between the sum and difference of the transverse and conjugate axis, $SF^2 = \overline{BA+DE} \times \overline{BA-DE}$: For $CA^2 = DF^2 = DC^2+CF^2$; and $CF^2 = CA^2-CD^2 = \overline{CA+CD} \times \overline{CA-CD}$; and $4CF^2$ or $SF^2 = \overline{2CA+2CD} \times \overline{2CA-2CD}$.

PROP. III. The rectangle of the focal distances, from either vertex, is equal to the square of the semiconjugate: $AF \times FB = DC^2$. See Fig. 3.

For $DC^2 = DF^2-CF^2 =$ (Prop. 2.) $CA^2-CF^2 = \overline{CA+CF} \times \overline{CA-CF} = \overline{BC+CF} \times \overline{CA-CF} = BF \times FA$.

PROP.

PROP. IV. As the transverse axis to the conjugate, so the conjugate to the latus rectum of the transverse : AB : DE : : DE : LR. See Fig. 3.

For $SL+LF=BA=2CA$ (Prop. 1.) ; and $SL=2CA-LF$, and by squaring $SL^2=4CA^2-4CA\times LF+LF^2$. And in the right angled triangle SLF, $SL^2=SF^2+LF^2$; whence $4AC^2-4CA\times LF+LF^2=SF^2+LF^2$, and $4AC^2-4CA\times LF=SF^2=4CF^2$, and $4AC^2=4CA\times LF+4CF^2=4CA\times LF+4DF^2-4DC^2$, and $4AC^2+4DC^2=4CA\times LF+4DF^2$; but $CA^2=DF^2$ (Prop. 2.) ; therefore $4DC^2=4CA\times LF=2CA\times 2LF$; that is, $DE^2=BA\times LR$.

COR. 1. As the semitransverse is to the semiconjugate, so the semiconjugate to half the latus rectum ; CA : DC : : DC : LF.

COR. 2. As the semitransverse, to the distance of the focus from the centre ; so is the same distance, to the difference between the semitransverse and half the latus rectum : $FC^2=CA\times\overline{CA-LF}$.

For $CF^2=DF^2-DC^2=CA^2-CD^2=CA^2-CA\times LF$.

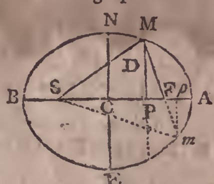
COR. 3. The rectangle BFA = half the transverse $\times$ half the latus rectum $=CA\times FL$. By Cor. 1. and Prop. 3. See Fig. 3.

SCHOLIUM. Since the transverse axis is to the conjugate, so the conjugate to the latus rectum, of the transverse axis. Therefore, in any other diameters, the third proportional, to the diameter and its conjugate, is called the *latus rectum* of that diameter.

PROP. V. From any point M in the curve, drawing the line MF, MS, to the two foci ; and the ordinate MP perpendicular to the transverse axis BA ; it will be,

As the semitransverse, CA :
To the distance of the focus from the centre, CF : :
So the distance of the ordinate from the centre, CP :
To half the difference of the lines MF, MS, or $\frac{MS-MF}{2}$.

Fig. 4.



For, make $SD=CA$, then $SM=CA+DM$, and $FM=2CA-SM=CA-DM$. In the right angled triangle SMP, SM^2 or $CA^2+2CA\times DM+DM^2=SP^2+PM^2=\overline{CF+CP}^2+PM^2=CF^2+2CF\times CP+CP^2+PM^2$, and in the right angled triangle FMP, FM^2 or $CA^2-2CA\times DM+DM^2=FP^2+PM^2=\overline{CF-CP}^2+PM^2=CF^2-2CF\times CP+CP^2+PM^2$; then subtracting the latter equation from the former, $SM^2-FM^2=4CA\times DM=4CF\times CP$, and $CF\times CP=CA\times DM$. But since $SM=CA+DM$, and $FM=CA-DM$; therefore $SM-FM=2DM$; therefore $CF\times CP=CA\times\frac{SM-FM}{2}$.

COR. 1. If F, S be the foci, MP an ordinate ; then it is CA : CF : : CP : $\overline{CA-MF}$ or $\overline{SM-CA}$. See Fig. 4.

For

For $CF \times CP = CA \times DM$, and $DM = SM - CA = CA - FM$.

Cor. 2. If F,S, be the foci, MP an ordinate; then the difference of the ſquares of the lines SM,FM; that is, $SM^2 - FM^2 = 4CF \times CP$.

Cor. 3. If F, S, be the foci, MP an ordinate; then $CA \times \overline{SM - FM} = 2CF \times CP$.

For $SM^2 - FM^2 = \overline{SM + FM} \times \overline{SM - FM} = 2CA \times \overline{SM - FM} = 4CF \times CP$, and $CA \times \overline{SM - FM} = 2CF \times CP$.

Scholium. If PM fall on the other ſide of F, as pm, then $pF = Cp - CF$, and its ſquare the ſame as before, and the reſt of the demonſtration the ſame.

Prop. VI. If an ordinate MP be drawn to the tranſverſe axis; it will be,

As the ſquare of the tranſverſe, BA^2 :

To the ſquare of the conjugate, NE^2 ::

So the rectangle of the ſegments of the tranſverſe BPA :

To the ſquare of the ordinate, PM^2. See Fig. 4.

For make $SD = CA$, then DM is half the difference of SM and MF; therefore by Prop. 5. $CA : CF :: CP : DM$, and $CA : CA + CF$ or $BF :: CP : CP + DM$, and $CA : CP :: BF : CP + DM$, and $CA : CA + CP$ or $BP :: BF : BF + CP + DM$. But $BF = BC + CF = SD + CF$; and $BF + CP + DM = SD + CF + CP + DM = SM + CS + CP = SM + SP$; whence $CA : BP :: BF : SM + SP$. Again, ſince $CA : CF :: CP : DM$; then $CA : (CA - CF)\ AF :: CP : CP - DM$; and $CA : CP :: AF : CP - DM$. And $CA : (CA - CP)\ PA :: AF : AF - CP + DM$. But $AF = CA - CF = SD - SC$; therefore $AF - CP + DM = SD - SC - CP + DM = SM - SP$; therefore $CA : PA :: AF : SM - SP$, and we had before, $CA : BP :: BF : SM + SP$; then multiplying theſe proportions together, we have $CA^2 : BP \times PA :: BF \times FA : SM^2 - SP^2$.

But (Prop. 3.) $BF \times FA = CN^2$; and $SM^2 - SP^2 = PM^2$; therefore $CA^2 : BPA :: CN^2 : PM^2$, or alternately, $CA^2 : CN^2 :: BPA : PM^2$, or $BA^2\ (4CA^2) : NE^2\ (4CN^2) :: BPA : PM^2$.

Cor. 1. $CA^2 : CN^2 :: BFA : PM^2$.

Cor. 2. As the tranſverſe BA : to its latus rectum :: So the rectangle BPA : to ſquare of the ordinate PM^2.

For (Prop. 4.) latus rectum $= \frac{NE^2}{AB}$, whence, ſince $BA^2 : EN^2 :: BFA : PM^2$, therefore, $BA : \frac{NE^2}{BA}$ or latus rectum :: $BFA : PM^2$.

Cor. 3. The rectangles of the ſegments of the tranſverſe are as the ſquares of the ordinates.

For every rectangle is to the ſquare of its ordinate, in the given ratio of CA^2 to CN^2, or of BA to the latus rectum.

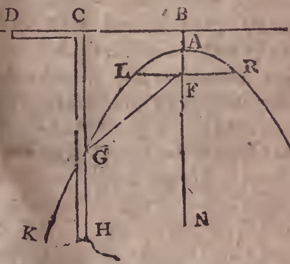
Cor.

Cor. 4. As the square of the semitransverse CA^2 :
Rectangle of the focal distances from vertex BFA : :
So rectangle of the segments BPA :
To square of the ordinate PM^2.

SECTION II.

Of the Parabola.

Fig. 5.



Definition 1. If one end of a thread, equal in length to CH, be fixed at the point F, and the other end fixed at H, the end of the square DCH. And if the side CD of the square be moved along the right line BD, and always coincide with it, then, if the string FGH be always kept tight, and close to the side GH of the square, the point or pin G (where it leaves the square) will describe a curve MRALGK called a Parabola. See Fig. 5.

Def. 2. The fixed point F is called the *focus*.

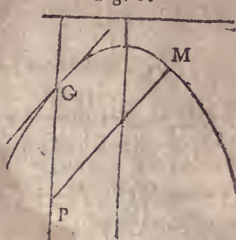
Def. 3. The right line BD is called the *directrix*.

Def. 4. If the line BN be drawn through the focus F, perpendicular to BD; then AN is called the *axis* of the parabola, and A the *vertex*.

Def. 5. A line drawn through the focus F, perpendicular to the axis, as LR, is called the *parameter* or *latus rectum*.

Def. 6. Any line drawn within the curve, parallel to the axis, as GH, is called a *diameter*. And the point G, where it cuts the curve, is the *vertex*.

Fig. 6.



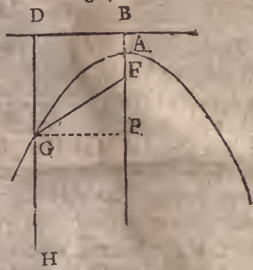
Def. 7. A right line drawn from any diameter to the curve, and parallel to the tangent at the vertex, as PM, is called an *ordinate*. If it go quite through the curve, it is called a *double ordinate*. See Fig. 6.

Def.

Def. 8. The part of the diameter between the vertex and ordinate, as GP, is called the *abſciſſa.*

Def. 9. A right line meeting the curve in one point G, but does not cut it, is called a *tangent* in that point.

Fig. 7.



PROPOSITION I. If BD be the directrix, G any point in the curve, the line GD drawn to the directrix, parallel to the axis, is equal to the line GF drawn from the ſame point G to the focus; GD = GF. See Fig. 7.

For HG+GF = length of the ſtring = HD; take away GH from both, and then GD = GF.

COR. 1. The diſtances of the focus, and of the directrix from the vertex are equal. AB = AF. For when D is at B, G will be at A; conſequently AB = AF.

COR. 2. If GP be an ordinate to the axis; then AP+AF = FG; For AP+AF = BP = GD.

COR. 3. FG—FP = half the latus rectum.

PROP. II. The diſtance of the focus from the vertex is $\frac{1}{4}$ the latus rectum: AF = $\frac{1}{4}$LR = $\frac{1}{2}$LF. See Fig. 5.

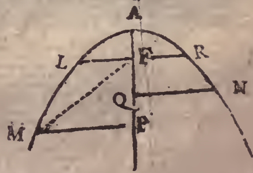
For when the pin G comes to L, then LF = FB (Prop. 1. Cor. 1.) = 2FA, and AF = $\frac{1}{2}$FL. For the ſame reaſon FA = $\frac{1}{2}$FR, therefore FA = $\frac{1}{4}$LR.

SCHOLIUM. As the latus rectum to the axis is four times the diſtance of the vertex A from the focus F: So in any other diameter GH, four times the diſtance of its vertex from the focus, or AFG, is called its *latus rectum.*

PROP. III. The ſquare of any ordinate to the axis is equal to the rectangle of the latus rectum and abſciſſa: $PM^2 = LR \times AP$. See Fig. 8.

For

Fig. 8.



For $MF = AF + AP =$ (Prop. 2.) $AP + \frac{1}{4}LR$, and $FP = AP - AF = AP - \frac{1}{4}LR$. And in the right angled triangle MFP, $MP^2 = MF^2 - FP^2 = \overline{MF+FP} \times \overline{MF-FP} = 2AP \times \frac{1}{2}LR = AP \times LR$.

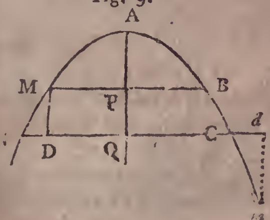
Cor. 1. If F be the focus, $MP^2 = AP \times 4AF$.

Cor. 2. The abſciſſas are as the ſquares of their ordinates. $AP : AQ :: PM^2 : QN^2$. For $AP : AQ :: AP \times LR : AQ \times LR :: PM^2 : QM^2$.

Cor. 3. The latus rectum is a third proportional to the abſciſſa and ordinate. $AP : PM : LR \div$.

Prop. IV. As the latus rectum to the ſum of any two ordinates; ſo their difference, to the difference of the abſciſſæ. Lat. rect. : $CD :: ND : PQ$. See Fig. 9.

Fig. 9.



Let L = latus rectum, then (Prop. 3.) $L \times AP = PM^2$; and $L \times AQ = NQ^2$. And by ſubtraction, $L \times AQ - L \times AP = NQ^2 - PM^2$; therefore $L : NQ + PM :: NQ - PM : AQ - AP$; that is, $L : DC :: ND : PQ$.

Cor. 1. If MD be the axis, NC an ordinate to it; then the rectangle NDC = MD × parameter.

Cor. 2. The rectangle NDC is every where as MD.

SECTION III.

Of the Hyperbola.

Fig. 10.

Definition 1. If the ends of two threads SPQ. FPQ, be faſtened at the points S, F, and be made to paſs through a ſmall bead, or pin P, and knotted together at Q; then taking hold of Q, and drawing the threads tight; if the bead be moved along the threads, the point P will deſcribe the curve mp APM, called an *hyperbola*. See Fig. 10

Def.

Def. 2. And if the end of the long thread be fixed at F, and that of the ſhort one at S; and the curve NBR be deſcribed after the ſame manner, that curve is called the *oppoſite hyperbola*; and both curves together, MA*m*, NBR, are called *oppoſite ſections*, or *oppoſite hyperbolas*.

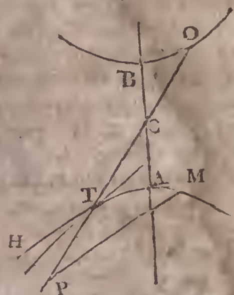
Def. 3. The two fixed points F,S, are called the *foci*.

Def. 4. The line AB (paſſing through the foci, when continued) contained between the two parts of the curve, is called the *tranſverſe axis*.

Def. 5. The middle point of AB, that is, C, is called the *centre* of the hyperbola, or of the oppoſite ſections.

Def. 6. If VY be drawn through the centre C perpendicular to AB; and with radius CF, and centre A, an arch be deſcribed, cutting VY in V, and Y; then VY is called the *conjugate axis*.

Fig. 11.



Def. 7. Any line TO drawn through the centre C, and terminated at the oppoſite ſections, is called a diameter; and the extremity T (or O) its *vertex*. And the line drawn through the centre, parallel to the tangent at the vertex, is called its *conjugate diameter*. See Fig. 10.

Def. 8. If any diameter OT be continued within the curve, the part within, TP, is called the *abſciſſa*.

Def. 9. Any line PM, drawn parallel to the tangent at the vertex T, and terminated at the abſciſſa and curve, is called an *ordinate* to that diameter TO. And if it go quite through the curve, it is called a *double ordinate*.

Def. 10. The line LI, drawn through the focus F, perpendicular to the tranſverſe axis AB, and terminating at the curve, is called the *parameter* or *latus rectum*. See Fig. 10.

Def. 11. If the ends of the two axes be joined by the lines BY, BV; and through the centre C, two lines CH, CG, be drawn parallel to BY, BV; or (which is the ſame) if VY be placed at A, perpendicular to BA; and the lines CH, CG, be drawn from the centre C, through the ends E,D; theſe lines CH, CG, are called the *aſſymptotes* of the hyperbola, or of the oppoſite hyperbolas.

Def.

b D—

Def. 12. When the tranſverſe and conjugate axes are equal, $AC = CV$ or AD, the curve is called an *equilateral hyperbola*, or *right angled hyperbola.*

Def. 13. A right line, which meets the hyperbola in one point T, but does not cut it, as TH, is called a *tangent* to it, in that point T. See Fig. 11.

Def. 14. If two oppoſite hyperbalas, KO, TW, be in like manner deſcribed to the tranſverſe VY ($=$ DE,) and conjugate AB; theſe are called *conjugate hyperbolas*, with regard to the former.

Proposition I. The difference of the lines SP, FP, drawn from the foci, to any point P of the curve, is equal to the tranſverſe axis AB. See Fig. 10.

For by conſtruction $PS-PF = AS-AF = AB+BS-AF =$ (becauſe $BS = AF$) AB.

Cor. Hence $CF = CS$, or the foci are equally diſtant from the centre.

Prop. II. The ſquare of the diſtance of the focus from the centre is equal to the ſum of the ſquares of the ſemitranſverſe and ſemiconjugate. $CF^2 = CA^2+CY^2$.

For, make AE equal and parallel to CY, then the radius $CE = CF$; and in the right angled triangle CAE, $CE^2 = CA^2+AE^2$; that is, $CF^2 = CA^2+AE^2 = CA^2+CY^2$.

Cor. $CF^2-AE^2 = CA^2$; and $CF^2-CA^2 = AE^2 = CY^2$.

Prop. III. The rectangle of the focal diſtances from either vertex is equal to the ſquare of the ſemiconjugate, $FA \times SA = CY^2$.

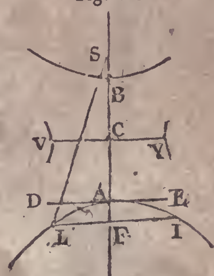
For, making $AE = CY$; by the property of the circle, $FA \times AS = AE^2 = CY^2$.

Cor. The rectangle if the diſtance of either focus from the two vertices is equal to the ſquare of the ſemiconjugate, $FA \times FB = AE^2 = CY^2$.

For $SB = FA$ and $SA = FB$, whence $FA \times FB = FA \times SA = AE^2$.

Prop. IV. As the tranſverſe axis is to the conjugate; ſo the conjugate, to the latus rectum of the tranſverſe; $AB : VY :: VY : LI$. See Fig. 12.

Fig. 12.



For (Prop. 1.) $SL-LF = BA = 2CA$; and $SL = 2CA+LF$; and $SL^2 = 4CA^2 +4CA \times LF+LF^2$; and in the right angled triangle SLF, $SL^2 = SF^2+LF^2$, and ſubtracting LF^2 from theſe two values of SL^2; then $4CA^2+4CA \times LF = SF^2 = 4CF^2$; and $CF^2 = CA^2+CA \times LF$. But (Prop. 2.) $CF^2 = CA^2+CY^2 = CA^2+CA \times LF$; therefore $CY^2 = CA \times LF$, and multiplying by 4, $VY^2 = BA \times LI$.

Cor.

Cor. 1. As the ſemitranſverſe, to the ſemiconjugate; ſo the ſemiconjugate to half the latus rectum, CA : CY : LF.

Cor. 2. As the ſemitranſverſe to the diſtance of the focus from the centre; ſo is the ſame diſtance, to the ſum of the ſemitranſverſe and half the latus rectum, CA : CF :: CF : CA+LF.

For, (Prop. 2.) $CF^2 = CA^2 + CY^2 = CA^2 + CA \times LF = CA \times \overline{CA \times FL}$.

Cor. 3. The rectangle BFA $= \frac{1}{2}$ tranſverſe $\times \frac{1}{2}$ latus rectum $=$ CA $\times$ FL. By Cor. 1. and Prop. 3.

Scholium. Since the tranſverſe axis is to the conjugate, as the conjugate to the latus rectum of the tranſverſe axis; therefore, in any other diameters, the third proportional, to any diameter and its conjugate, is called the *latus rectum of that diameter*. Therefore, in a right angled hyperbola, every diameter is equal to its latus rectum.

FINIS.